Working Paper Series 760
(ISSN 2788-0443)

Quantitative Easing in the Euro Area: Implications for Income and Wealth Inequality

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Prague, August 2023

ISBN 978-80-7343-567-7 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-689-5 (Národohospodářský ústav AV ČR, v. v. i.)

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Abstract

This study examines how and to what extent quantitative easing of the ECB affects household income and wealth inequality in the euro area. Previous theoretical models have investigated the dynamics of inequality measures through differential access of households to financial/capital market (the portfolio rebalancing channel), neglecting the labor market differential (the earnings heterogeneity channel). Although the portfolio rebalancing channel may provide insight into wealth inequality and non-labor income inequality, this is not the case with labor (and thus total) income inequality. To be in line with the empirical evidence on labor income inequality, this study also considers segmented labor market on the basis of capital-skill complementarity in production and asymmetric real wage rigidities. When only financial market segmentation is considered, the quantitative results indicate a drop in total income inequality that is diminished over time, while wealth inequality experiences a rise that gradually becomes weaker. The introduction of the segmented labor market significantly mitigates the observed drop in total income inequality, while a rise in wealth inequality is largely amplified. Given the possible broadening of the ECB's mandate towards distributional issues in the future, the analysis of segmented labor and financial markets can be more beneficial to the ECB as it provides a clearer picture of the inequality effects.

JEL Classification: E21, E22, E44, E52, E58.

Keywords: quantitative easing, capital-skill complementarity, asymmetric real wage rigidity, skill premium, portfolio rebalancing channel, earnings heterogeneity channel.

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1. Introduction

Following the outbreak of the global financial crisis of 2007-2008, the euro area (EA) experienced a severe liquidity shortage, while at the same time the conventional monetary policy of the European Central Bank (ECB) was constrained by the zero-lower bound (ZLB). To support price stability and the real economy as a whole, the ECB implemented unconventional monetary policy such as quantitative easing (QE)[1]. In addition to aggregate effects, Ampudia et al. (2018) and Lenza and Slacalek (2018) empirically show that QE generates distributional effects in the EA economy: (1) labor and total income inequality are reduced significantly and (2) wealth inequality[2] is decreased to a lesser extent. Using only the portfolio rebalancing channel, the previous model economies cannot capture the empirical evidence on labor income inequality[3], and thus only partially shed light on total income inequality and related wealth inequality. This paper contributes to the literature by incorporating the earnings heterogeneity channel that distinguishes labor income sources between the wealthy and the poor. Accordingly, the combination of these two channels is used to examine the extent to which income and wealth of poor and wealthy households are affected by QE over different time horizons, i.e. short, medium and long run.

This study develops a model that is characterized by the two types of household heterogeneity within a New Keynesian framework: financial (capital) and labor market segmentation. *Financial (capital) market segmentation* makes a distinction between wealthy and poor households in the sense that only wealthy households have access to financial/capital markets. This segmentation is related to the portfolio rebalancing channel, according to which households' rebalancing of their asset portfolio induces aggregate and distributional effects on the economy. Specifically, the QE policy implies that the central bank purchases long-term government bonds, and thus reduces its amount relative to short-term government bonds in the portfolio of households. In response to QE, households rebalance their asset portfolio as they are assumed to have a preference for holding a certain mix of assets with different maturities. In addition, the model economy includes the portfolio adjustment costs that make the assets with different maturities imperfect substitutes so that changes in the relative supply of long-term bonds affect the term spread and then the real economy through general equilibrium forces.

[1]The QE program of the ECB is defined as the Asset Purchase Program (APP). In January 2015, the ECB announced the introduction of the APP, but started its implementation in March 2015. The APP includes the combined purchases of public and private sector securities. Initially, total APP purchases amounted to as much as €60 billion a month until the end of September 2016. This paper focuses on the Public Sector Purchase Program (PSPP), the largest part of the APP that includes only the purchases of public sector securities (€50 billion) - sovereign bonds from euro-area governments. The Governing Council of the ECB expanded the initial purchases within the APP on multiple occasions so that in March 2016 the amount of monthly purchases was increased to €80 billion. A detailed discussion of the APP is provided in Gambetti and Musso (2017).

[2]The result about increasing wealth inequality is common to theoretical models that abstract from housing wealth in studying QE implications (see e.g., Hohberger et al., 2020).

[3]The empirical evidence on labor income inequlity of Lenza and Slacalek (2018) is shown in Appendix B.

Labor market heterogeneity refers to the existence of two distinct categories of workers: high-skilled workers and low-skilled workers. This segmentation is considered as labor income is an important component of total disposable income and as such plays an important role in driving income inequality (see e.g., Ampudia et al., 2018 and Lenza and Slacalek, 2018). The segmented labor market is associated with the earnings heterogeneity channel, which transmits its effects through capital intensive production and asymmetric real wage rigidities. To provide a clearer picture of the different role of high/low skilled workers in the production process, there exists capital-skill complementarity in the production process in the spirit of Krusell et al. (2000), which results in capital being more complementary with high-skilled labor[4]. Additionally, asymmetric real wage rigidities are introduced to acknowledge the markedly sluggish adjustment of real wages, which is a characteristic of the euro area labour market documented among others by Kollmann et al. (2016).

The novelty of this study lies in considering the interaction of labor and financial market segmentation, which leads to the separation of the euro area population into two distinct groups: wealthy households (70%) and poor households (30%). Wealthy households have access to financial/capital markets and provide high-skilled labor services. Poor households do not have access to financial/capital markets and supply low-skilled labor services. Accordingly, 30 per cent of the total population does not participate in financial and capital markets and has attained at most post-secondary education, which is in accordance with Sakkas and Varthalitis (2021). However, this setting is in contrast to Hohberger et al. (2020), who consider only financial market segmentation. In their study, the heterogeneity in households' labor income is neglected, which in turn provides a rather limited insight into the dynamics of total income inequality. The same conclusion applies to the dynamics of wealth inequality due to the close relationship between total income and wealth inequality, a finding that is also reported by Bilbiie et al. (2022b) but for conventional monetary policy.

The main quantitative results of this study are as follows. Purchasing long-term government bonds from wealthy households, the ECB reduces the term spread. In response to a lower term spread and to restore the duration of their portfolio, the wealthy increase investment in other long-term assets, such as physical capital, and redirect resources from short-term government bonds to consumption. A higher level of investment and consumption increases aggregate demand pressure, which stimulates higher employment and wages of both types of households. A larger upward real wage rigidity for poor households implies a rise in the wage premium, and also in unskilled employment inequality as their labor supply is more sensitive to the change in labor income. Given that the rise in employment of the poor is larger than the rise in wages of the wealthy, there is a drop in skilled labor income inequality. In addition, capital-skill complementarity (CSC) amplifies the drop in the said inequality. This is

[4]Despite a rise in capital in response to QE, a slow capital accumulation (due to capital adjustment costs) induces a smaller increase in the demand for complementary high-skilled labor compared to low-skilled labor. This is in line with Bilbiie et al. (2022a), who indicate that low-skilled workers are characterised with a more cyclical labor demand as they are more readily available for increasing production at the time of an aggregate demand expansion.

because the labor supply of the wealthy is more responsive than the capital stock, which leads to their lower marginal productivity and thus a lower wage premium. With higher wages under CSC relative to CD economy, the poor can enjoy a larger consumption, stimulating further aggregate demand and employment. However, in the medium/long run, CSC refers to increasing labor income inequality.

The results of this study also indicate a fall in non-labor income inequality. In addition to paying higher net lump-sum taxes after QE, wealthy households have losses on profit income and interest income on holding short- and long-term government bonds. However, the presence of real wage rigidity largely limits a drop in profit income, leading to a rise in the non-labor income of wealthy households. There are two important implications of higher non-labor income of the wealthy. First, a drop in total income inequality is mitigated compared to the economies with flexible wages, i.e. the economy with segmented labor and financial markets and the economy with only a segmented financial market. Second, wealth inequality rises as more resources are available for the accumulation of larger amount of assets. In addition, the shape of consumption inequality dynamics closely follows that of total income inequality. Specifically, the consumption of poor households exhibits a higher response than that of the wealthy in the short-run as the poor spend a much larger fraction of an increase in their income on consumption goods. However, this trend of consumption inequality reverses in favor of the wealthy in the medium/long run.

2. Related Literature

This paper relies on two strands of literature. The first highlights the importance of the portfolio rebalancing channel in studying the effects of QE. In theoretical models, the identification of this channel is mostly based on financial friction in the form of transaction costs that investors pay when they face portfolio changes. Transaction costs are associated with the assumption of imperfect substitutability of assets with different maturities, which allows central bank purchases of assets to affect the real economy. Andrés et al. (2004) are the first to introduce such financial friction in the standard DSGE model to make short- and long-term bonds imperfect substitutes. Similar to Andrés et al. (2004), Chen et al. (2012) introduce segmentation and transaction costs in bond markets to show the stimulative effects of the Federal Reserve LSAP program on GDP growth and inflation. Harrison (2012) uses a representative agent NK model amended with portfolio adjustment costs to indicate that QE scales up the aggregate demand and inflation in the UK. In addition to portfolio adjustment costs, Falagiarda (2014) introduces a secondary market for bond trading to indicate that QE2 in the US and the first phase of the APF in the UK exert upward pressure on output and inflation, with the effects more pronounced in the UK. What is common to all these papers is their focus on the aggregate effects of QE in the representative NK framework, while this study focuses on the distributional effects of QE in the (tractable) heterogeneous NK setting.

Hohberger et al. (2020) use the portfolio rebalancing channel to compare the distributional implications of expansionary conventional and unconventional monetary policy (QE) for the EA. The results of their estimated open-economy DSGE model indicate a fall in income and a rise in wealth inequality between the wealthy and the poor in the short and medium term, but the persistent inequality effects are largely absent in the long term. However, by means of the portfolio rebalancing channel, Hohberger et al. (2020) could account for household heterogeneity only in terms of financial income. As stated by Ampudia et al. (2018), household labor income in the EA is an important component of total income, which goes in favor of considering labor market heterogeneity and corresponding inequality in labor income. Although Cui and Sterk (2021) and Sims et al. (2022) study the distributional implications of QE in the US in the presence of household heterogeneity, they neglect labor market segmentation.

To acknowledge household heterogeneity in labor income, and thus to provide a clearer picture of the distributional effects of QE in the EA, this paper also considers a second strand of literature that focuses on the earnings heterogeneity channel. In this regard, Dolado et al. (2021) distinguish between high-skilled and low-skilled workers by introducing capital-skill complementarity (CSC) and asymmetric search and matching frictions within a New Keynesian model for the US. Dolado et al. (2021) show that expansionary conventional monetary policy leads to increasing income inequality between high- and low-skilled workers. Unlike Dolado et al. (2021), the present study introduces the earnings heterogeneity channel (EHC) through CSC and asymmetric wage rigidities such that EHC coexists with the portfolio rebalancing channel. This is in line with Sakkas and Varthalitis (2021), who indicate that households' savings and income can be associated with their skills and educational attainment. This further means that we could consider the joint heterogeneity of households where the wealthy are treated as high-skilled and the poor as low-skilled.

As regards the distributional effects of QE in the EA, the empirical studies that provide support for including the portfolio rebalancing channel are, among others, Krishnamurthy et al. (2018), Urbschat and Watzka (2020). In addition, Albertazzi et al. (2021) show that portfolio rebalancing is particularly distinct to vulnerable European economies such as Ireland, Greece, Spain, Italy, Cyprus, Portugal, and Slovenia. Ampudia et al. (2018) and Lenza and Slacalek (2018) provide empirical evidence that serves as a motivation in the present study for including the earnings heterogeneity channel. Coibion et al. (2017) refer to several factors that the earnings heterogeneity channel includes: unemployment risk, asymmetric wage rigidities, different complementarity with physical capital across the agents' skill sets, and different household-specific characteristics that underlie households' labor supply. Recent paper by Donggyu (2021) introduces the earnings heterogeneity channel on the basis of an idiosyncratic productivity shock and unemployment risk to examine the inequality effects of QE in the US. In contrast to Donggyu (2021), the current study focuses on CSC in production and asymmetric wage rigidities under the earnings heterogeneity channel.

This paper is organized as follows. Section 3 describes the model economy. Section 4 explains the transmission channels of QE. Section 5 refers to the calibration, while Section 6 indicates the simulation results of QE. Section 7 concludes.

3. Model Economy

This paper considers a closed-economy model whose demand side is characterized by two different types of infinitely-lived representative households[5]: a fixed fraction of wealthy households indexed by $w \in (0,1)$ and poor households indexed by $p \in (0,1)$. Wealthy households have access to financial markets and provide skilled labor services. Poor households do not participate in financial markets and supply unskilled labor services. Hence, the model incorporates two important sources of households heterogeneity: labor market services (high- and low-skilled workers) and access to financial markets (Ricardian and non-Ricardian households). On the production side, perfectly competitive intermediate goods producers rent capital and the two types of labor services from the households to produce a homogeneous intermediate output. In addition, capital-skill complementarity is incorporated in the production function à la Krusell et al. (2000) to capture the different roles of high- and low-skilled workers in the production at the time of increased capital stock due to QE. Intermediate output is then differentiated by monopolistically competitive final-goods producers. The final output is used for consumption, investment, and government expenditure.

The model also features nominal and real frictions to ensure that the main variables of interest respond smoothly to an exogenous QE shock. These frictions are sticky prices, sticky wages and quadratic costs for changes in the capital stock. The government conducts fiscal and monetary policy. Specifically, the fiscal authority follows the passive fiscal policy rule so that the lump-sum taxes/transfers respond to the deviation in the value of short- and long-term debt from their respective steady state. The monetary authority implements monetary policy at the exogenous ZLB and purchases long-term government bonds from wealthy households. To motivate the non-neutrality of the QE policy, the model includes the imperfect substitutability between assets of different maturities (short-term and long-term government bonds) by means of portfolio adjustment costs.

3.1 Households

3.1.1 Wealthy households

Wealthy households maximize their expected lifetime utility, which is a separably additive function of consumption $c_{w,t}$, real money holdings m_t and labor supply $n_{w,t}$:

$$\max_{c_{w,t},n_{w,t},m_t,b_t^s,b_t^{l,h},i_{\varsigma,t},k_{\varsigma,t}} \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c} (c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} + \frac{\varphi_m}{1-\chi} (m_\tau)^{1-\chi} - \varphi_{n,w} \frac{(n_{w,\tau})^{1+\eta}}{1+\eta} \Big\}$$

subject to the real budget constraint in every period t:

[5] In the present model, households are different when their types (poor and wealthy) are compared with each other. However, as idiosyncratic income risk is absent within types, there is a representative household within each type. Given that the focus of this paper is on the comparison of the two types of households, a less rich setting of heterogeneity than the Aiyagari-incomplete type model is used.

$$c_{w,t} + q_t b_t^s + q_{L,t} b_t^{l,h} \left(1 + \frac{\phi_b}{2}\left(\kappa \frac{b_t^s}{b_t^{l,h}} - 1\right)^2\right) + t_{w,t} + i_{s,t} + i_{e,t} + m_t \leq w_{w,t} n_{w,t} + \frac{b_{t-1}^s}{\pi_t} +$$

$$+ (1 + \varrho q_{L,t}) \frac{b_{t-1}^{l,h}}{\pi_t} + r_{s,t}^k k_{s,t-1} + r_{e,t}^k k_{e,t-1} + \frac{m_{t-1}}{\pi_t} + tr_{w,t} + \frac{\Pi_t^{int}}{s_w} + \frac{\Pi_t^r}{s_w}$$

where $\mathbb{E}_t$ is the expectation operator conditional on information in period t, $\beta \in (0,1)$ is the subjective discount factor, $c_{w,t}(C_{w,t})$ is the time-t individual level of consumption (aggregate consumption), σ_c is the inverse of the intertemporal elasticity of substitution, $h < 1$ is the parameter for external habit formation in consumption, s_w is the population share of the wealthy, $\chi > 0$ is the inverse of the elasticity of real money balances, $\eta > 0$ is the inverse Frisch elasticity of labour supply, $\varphi_m > 0$ and $\varphi_{n,w} > 0$ are the relative utility weights on real money holdings and labor supply, respectively.

Total resources of wealthy households include real labor income $w_{w,t} n_{w,t}$, real payoff on previous period short-term government bonds $\frac{b_{t-1}^s}{\pi_t}$ and long-term government bonds $\frac{b_{t-1}^{l,h}}{\pi_t}$ (where $\pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation rate), rental income on capital stock $r_{s,t}^k k_{s,t-1} + r_{e,t}^k k_{e,t-1}$, real money holdings m_{t-1}, real transfers from the government $tr_{w,t}$, and real profits in the form of dividends $\Pi_t^{int} + \Pi_t^r$ from ownership of intermediate and final goods firms. These total resources can be used for purchasing consumption goods $c_{w,t}$, investment in short-term government bonds b_t^s and long-term government bonds $b_t^{l,h}$, and for paying real lump-sum taxes $t_{w,t}$ to the government. The wealthy also make investment decisions regarding (structure and equipment) physical assets:

$$k_{\varsigma,t} = (1 - \delta_\varsigma) k_{\varsigma,t-1} - S\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\Big) k_{\varsigma,t} + i_{\varsigma,t}, \quad \text{for} \quad \varsigma \in \{s, e\}$$

subject to quadratic capital adjustment costs defined as in Hayashi (1982):

$$S\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\Big) = \frac{\phi_k}{2}\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\right)^2,$$

where ϕ_k is the capital adjustment cost and $S(\cdot)$ is the capital adjustment cost function that satisfies the following properties: $S' \geq 0, S'' \geq 0$ and $S(1) = 0$.

To solve the maximization problem of wealthy household, the Lagrangian function is set up:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ \frac{1}{1-\sigma_c}(c_{w,\tau} - hC_{w,\tau-1})^{1-\sigma_c} + \frac{\varphi_m}{1-\chi}(m_\tau)^{1-\chi} - \varphi_{n,w}\frac{(n_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}\Bigg(c_{w,\tau} + q_\tau b_\tau^s +$$

$$+ q_{L,\tau} b_\tau^{l,h}\left(1 + \frac{\phi_b}{2}(\kappa \frac{b_\tau^s}{b_\tau^{l,h}} - 1)^2\right) + t_{w,\tau} + (k_{s,\tau} - (1-\delta_s)k_{s,\tau-1}) + (k_{e,\tau} - (1-\delta_e)k_{e,\tau-1}) +$$

$$+ m_\tau - w_{w,\tau} n_{w,\tau} - \frac{b_{\tau-1}^s}{\pi_\tau} - (1 + \varrho q_{L,\tau})\frac{b_{\tau-1}^{l,h}}{\pi_\tau} + \frac{\phi_k}{2}\Big(\frac{k_{s,\tau}}{k_{s,\tau-1}} - 1\Big)^2 k_{s,\tau} + \frac{\phi_k}{2}\Big(\frac{k_{e,\tau}}{k_{e,\tau-1}} - 1\Big)^2 k_{e,\tau} -$$

$$- r_{s,\tau}^k k_{s,\tau-1} - r_{e,\tau}^k k_{e,\tau-1} - \frac{m_{\tau-1}}{\pi_\tau} - tr_{w,\tau} - \frac{\Pi_t^{int}}{s_w} - \frac{\Pi_t^r}{s_w}\Bigg)\Bigg\}$$

Taking the FOCs, we have the following optimality conditions:

$$[c_{w,t}]: \quad \lambda_{w,t} = \frac{1}{(c_{w,t} - hC_{w,t-1})^{\sigma_c}} \tag{1}$$

$$[n_{w,t}]: \quad \lambda_{w,t} w_{w,t} = \varphi_{n,w} (n_{w,t})^{\eta} \tag{2}$$

$$[m_t]: \quad \varphi_m m_t^{-\chi} + \mathbb{E}_t\, \beta \frac{\lambda_{w,t+1}}{\pi_{t+1}} = \lambda_{w,t} \tag{3}$$

$$[b_t^s]: \quad \mathbb{E}_t\, \beta \left(\frac{\lambda_{w,t+1}}{\pi_{t+1}} \right) = q_t \lambda_{w,t} + q_{L,t} \lambda_{w,t} \phi_b \Big(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \Big) \kappa \tag{4}$$

$$[b_t^{l,h}]: \quad \mathbb{E}_t\, \beta \left(\frac{\lambda_{w,t+1}}{\pi_{t+1}} (1 + \varrho q_{L,t+1}) \right) = q_{L,t} \lambda_{w,t} + q_{L,t} \lambda_{w,t} \frac{\phi_b}{2} \Big(k \frac{b_t^s}{b_t^{l,h}} - 1 \Big)^2 - q_{L,t} \lambda_{w,t} \phi_b \Big(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \Big) \kappa \frac{b_t^s}{b_t^{l,h}} \tag{5}$$

$$\begin{aligned} [k_{\varsigma,t}]: \quad & \lambda_{w,t} \left(1 + \frac{\phi_k}{2} \Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \Big)^2 + \phi_k \Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \Big) \frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} \right) = \\ & = \mathbb{E}_t\, \beta \lambda_{w,t+1} \left((1 - \delta_\varsigma) + r_{\varsigma,t+1}^k + \phi_k \Big(\frac{k_{\varsigma,t+1}}{k_{\varsigma,t}} - 1 \Big) \Big(\frac{k_{\varsigma,t+1}}{k_{\varsigma,t}} \Big)^2 \right), \quad \text{for } \varsigma \in \{s, e\} \end{aligned} \tag{6}$$

At the beginning of period t, the portfolio of the wealthy includes nominal (one-period) short-term risk-less government bonds b_{t-1}^s and (perpetual) long-term government bonds $b_{t-1}^{l,h}$. One-period bonds issued in period t are purchased at the real price $q_t = \frac{1}{R_t}$ and deliver the payoff one in period $t+1$, where R_t is a one-period nominal risk-free interest rate that is controlled by the central bank. As in Woodford (2001), long-term government bonds are modeled as perpetual nominal bonds that pay a nominal coupon that starts from one unit in the first period after issuance and decays over time geometrically at the rate $\varrho \in [0, 1]$. The real price of long-term government bonds issued in period t is given by $q_{L,t} = \frac{1}{R_t^L - \varrho}$, where R_t^L is the gross yield-to-maturity on a perpetual bond in period t and ϱ is the coupon decay factor. The duration (maturity) of long-term bonds is $d_t = \frac{R_t^L}{R_t^L - \varrho}$, where ϱ is used to match the average duration of long-term government bonds.

Wealthy households have a preference or target $\kappa = \frac{b^{l,h}}{b^s}$ for holding a mix of short-term and long-term government bonds. Deviation from this target value triggers portfolio adjustment cost $\phi_b > 0$, which makes two assets of different maturities imperfect substitutes, and thus opens the space for the portfolio rebalancing channel to function.

3.1.2 Poor households

Poor households maximize their lifetime utility, which is a separably additive function of consumption $c_{p,t}$ and labor supply $n_{p,t}$:

$$\max_{c_{p,t}, n_{p,t}} \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Big\{ \frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(n_{p,\tau})^{1+\eta}}{1+\eta} \Big\}$$

subject to the real budget constraint in every period t:

$$c_{p,t} + t_{p,t} \leq w_{p,t}n_{p,t} + tr_{p,t}$$

The Lagrangian function associated with the maximization problem of a poor household is:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{1}{1-\sigma_c} (c_{p,\tau} - hC_{p,\tau-1})^{1-\sigma_c} - \varphi_{n,p} \frac{(n_{p,\tau})^{1+\eta}}{1+\eta} - \lambda_{p,\tau} \Big(c_{p,\tau} + t_{p,\tau} - w_{p,\tau}n_{p,\tau} - tr_{p,\tau} \Big) \right)$$

Taking the FOCs, we have the following optimality conditions:

$$[c_{p,t}]: \quad \lambda_{p,t} = \frac{1}{(c_{p,t} - hC_{p,t-1})^{\sigma_c}} \tag{7}$$

$$[n_{p,t}]: \quad \lambda_{p,t}w_{p,t} = \varphi_{n,p}(n_{p,t})^{\eta} \tag{8}$$

Total income of the poor includes real labor income $w_{p,t}n_{p,t}$ from supplying unskilled labor services to intermediate goods firms and real transfers $tr_{p,t}$ received from the government. The poor spend their disposable income on consumption goods $c_{p,t}$ and on paying real lump-sum taxes $t_{p,t}$. Following Kaplan et al. (2014), poor households in the present model fit the definition of hand-to-mouth households because they hold no liquid and illiquid wealth, and as such spend all of their disposable income every period. As hand-to-mouth households have larger marginal propensity to consume than the other type of households, they are expected to be more sensitive to small and temporary changes in income.

3.2 Producers

3.2.1 Intermediate (wholesale) goods producers

There is a continuum of measure one of perfectly competitive firms that take prices $P_{int,t}$ as given and produce a homogeneous good $Y_{int,t} = Y_t$. To produce output, firms use the aggregate stock of structure capital $K_{s,t-1}$ and equipment capital $K_{e,t-1}$, aggregate skilled labor from wealthy households $N_{w,t}$, and aggregate unskilled labor from poor households, $N_{p,t}$. In the spirit of Krusell et al. (2000), the production function is given in the form of a nested CES composite of factor inputs:

$$Y_{int,t} = F(K_{s,t-1}, K_{e,t-1}, N_{w.t}, N_{p,t}) = AK_{s,t-1}^{\iota} \Big[m(N_{p,t})^{\sigma} + (1-m)\Big(\rho(K_{e,t-1})^{\nu} + (1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}} \tag{9}$$

where $A > 0$ stands for aggregate productivity, $m, \rho < 1$ determine the income shares of unskilled labor, equipment capital and skilled labor. The parameter ι indicates the income share of structure capital. Two parameters $\sigma, \nu \leq 1$ govern factor inputs elasticities. The elasticity of substitution between equipment capital and skilled labor is defined as $\varepsilon_1 = \frac{1}{1-\nu}$, while the elasticity of substitution between equipment capital and unskilled labor and between skilled and uskilled labor is defined as $\varepsilon_2 = \frac{1}{1-\sigma}$.

Intermediate goods producers seek to maximize their nominal profits, which are distributed as dividends to wealthy households, subject to the production function given by the equation (9):

$$P_t \Pi_t^{int} = P_{int,t} Y_{int,t} - W_{w,t} N_{w,t} - W_{p,t} N_{p,t} - R^k_{s,t} K_{s,t-1} - R^k_{e,t} K_{e,t-1},$$

while the real profit of the intermediate goods firms is expressed as:

$$\Pi_t^{int} = \frac{Y_{int,t}}{x_t} - w_{w,t} N_{w,t} - w_{p,t} N_{p,t} - r^k_{s,t} K_{s,t-1} - r^k_{e,t} K_{e,t-1},$$

where $x_t = \frac{P_t}{P_{int,t}}$ is the markup of the price of the final consumption good over the price of the intermediate good, while $\frac{1}{x_t}$ is the real marginal cost for retailers or the real price of the intermediate goods.

Taking the first order conditions of the real profit function with respect to capital and (skilled and unskilled) labor inputs, we have the following demands for capital and labor:

$$[K_{s,t-1}] : r^k_{s,t} \equiv \frac{1}{x_t} F^s_{k,t} = \frac{1}{x_t} A \cdot \iota \cdot K^{\iota-1}_{s,t-1} \Big[m(N_{p,t})^\sigma + (1-m) \Big(\rho (K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu \Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}} \tag{10}$$

$$[K_{e,t-1}] : r^k_{e,t} \equiv \frac{1}{x_t} F^e_{k,t} = \frac{1}{x_t} A K^\iota_{s,t-1} (1-\iota) \Big[m(N_{p,t})^\sigma + (1-m) \Big(\rho (K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu \Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}-1}$$
$$(1-m)\rho \cdot \Big(\rho (K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu \Big)^{\frac{\sigma}{\nu}-1} (K_{e,t-1})^{\nu-1} \tag{11}$$

$$[N_{w,t}] : w_{w,t} \equiv \frac{1}{x_t} F^w_{n,t} = \frac{1}{x_t} A K^\iota_{s,t-1} (1-\iota) \Big[m(N_{p,t})^\sigma + (1-m) \Big(\rho (K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu \Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}-1}$$
$$(1-m)(1-\rho) \Big(\rho (K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu \Big)^{\frac{\sigma}{\nu}-1} (N_{w,t})^{\nu-1} \tag{12}$$

$$[N_{p,t}] : w_{p,t} \equiv \frac{1}{x_t} F^p_{n,t} = \frac{1}{x_t} A K^\iota_{s,t-1} (1-\iota) \Big[m(N_{p,t})^\sigma + (1-m) \Big(\rho (K_{e,t-1})^\nu + (1-\rho)(N_{w,t})^\nu \Big)^{\frac{\sigma}{\nu}} \Big]^{\frac{1-\iota}{\sigma}-1} m (N_{p,t})^{\sigma-1} \tag{13}$$

The optimal demand for labor and capital inputs equates real prices (wage and rental rate) to their marginal products times the real marginal cost.

Combining equations (12) and (13), the so-called skill premium, which is a function of labor input ratios, can be expressed as:

$$\frac{w_{w,t}}{w_{p,t}} \equiv \frac{F^w_{n,t}}{F^p_{n,t}} = \frac{(1-m)}{m} (1-\rho) \Big(\rho \Big(\frac{K_{e,t-1}}{N_{w,t}} \Big)^\nu + (1-\rho) \Big)^{\frac{\sigma}{\nu}-1} \Big(\frac{N_{p,t}}{N_{w,t}} \Big)^{1-\sigma} \tag{14}$$

As shown in Krusell et al. (2000), capital-skill complementarity in the production function is present if $1 > \sigma > \nu$. This implies that $\varepsilon_2 > \varepsilon_1$. The skill premium increases with a rise in the equipment capital stock $\partial(\frac{F^w_{n,t}}{F^p_{n,t}})/\partial K_{e,t-1} > 0$, keeping all the other factors constant. However, the skill premium decreases in the skilled to unskilled labor ratio, $\partial(\frac{F^w_{n,t}}{F^p_{n,t}})/\partial(\frac{N_{w,t}}{N_{p,t}}) < 0$, under the assumption that all other factors remain unchanged.

To evaluate the quantitative importance of CSC in driving the QE distributional effects, the model in this paper also considers an alternative benchmark economy where CSC is not present. This model economy is characterized with a standard CD structure:

$$Y_t = AK_t^\theta \left(\varkappa N^\gamma_{w,t} + (1-\varkappa)N^\gamma_{p,t}\right)^{\frac{1-\theta}{\gamma}}$$

There are two types of CD economy: (1) where there is capital and two types of labor that are imperfect substitutes; (2) where there is capital and two types of labor that are perfect substitutes, with parameters $\varkappa = 0.5$ and $\gamma = 1$. The second type of CD economy features only the portfolio rebalancing channel, while for the first type of CD economy, the EHC is still present due to the asymmetric real wage rigidities. In addition, in the first type of CD economy, the changes in the skill premium are not a result of the changes in the stock of physical capital, i.e. there is only the relative quantity effect while the capital-skill complementarity effect is not present:

$$\frac{w_{w,t}}{w_{p,t}} \equiv \frac{F^w_{n,t}}{F^p_{n,t}} = \frac{\varkappa}{1-\varkappa}\left(\frac{N_{p,t}}{N_{w,t}}\right)^{1-\gamma}$$

As in Kina et al. (2020), the calibration procedure for the first type of CD economy is the same as the CSC economy except for the two internally calibrated parameters. The first parameter is A, which is calibrated to make output Y equivalent in the two economies, while the second parameter $\varkappa$ is chosen to have the same skill premium in the two economies. The same calibration procedure is used for both CSC and CD economies to guarantee that any differences in QE effects (skill premium) between the two economies cannot be ascribed to their initial conditions.

3.2.2 Final (retail) goods producers

There exists a continuum $j \in [0,1]$ of monopolistically competitive retail firms. Each firm buys an amount $Y_t(j)$ of the homogeneous intermediate good $Y_{int,t}$, and produces a variety of the final good $Y^f_t(j)$ which is an imperfect substitute for varieties produced by other final goods firms. The technology used in the production process is linear, $Y^f_t(j) = Y_t(j)$ (see e.g., Dolado et al., 2021). These differentiated products are then aggregated into a homogeneous final good Y^f_t by the following CES aggregator:

$$Y^f_t = \left[\int_0^1 Y^f_t(j)^{\frac{\epsilon}{\epsilon-1}} dj\right]^{\frac{\epsilon}{\epsilon-1}} = \left[\int_0^1 Y_t(j)^{\frac{\epsilon}{\epsilon-1}} dj\right]^{\frac{\epsilon}{\epsilon-1}} = Y_t$$

where $\epsilon > 1$ is the exogenous elasticity of substitution between the different types of goods, Y^f_t stands for final goods, and Y_t refers to intermediate goods. Final good could be used for consumption, investment, and government expenditure.

Retail firms purchase intermediate goods from wholesale producers at the wholesale price $P_{int,t}$, which is equal to the nominal marginal cost $mc^n_{int,t}$ in the intermediate goods sector. The fact that wholesale producers are perfectly competitive implies that $P_{int,t} = mc^n_{int,t}$. Purchased intermediate goods are differentiated by the retailers at no cost, so that the nominal marginal cost of producing final goods coincides with that of wholesale goods. Then, each retail firm sells its unique variety at a retail mark-up over the wholesale price in a monopolistically competitive market. Although retailers have monopolistic power by setting the price for their own products $P_t(j)$, as in Dolado et al. (2021) they take aggregate price P_t and the price of intermediate good $P_{int,t}$ as given.

The retail sector plays the role of introducing the nominal price rigidity into the economy as it has to pay quadratic price adjustment costs when changing prices. Price stickiness is important for ensuring the real effects of monetary policy on the economy. To motivate price stickiness, the Rotemberg (1982) price adjustment costs model is used. This means that final goods firms maximize their current and expected discounted profits subject to quadratic price adjustment costs measured in terms of the final good. Specifically, each retailer indexed by j pays an increasing and convex cost measured in terms of Y_t when the size of its price increase, $P_t(j)/P_{t-1}(j)$, deviates from the steady state inflation rate π:

$$\frac{\phi_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t \tag{15}$$

where $\phi_p \geq 0$ measures the degree of price stickiness. Higher values of ϕ_p indicate greater price stickiness, while $\phi_p = 0$ implies perfectly flexible prices of final goods.

Given the equation (15), each final good firm j chooses $P_t(j)$ to maximize the present discounted value of real profits for its owners (wealthy households):

$$\max_{P_t(j)} \ \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Pi^r_t(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\lambda_{w,\tau}}{\lambda_{w,t}} \left(\left(\frac{P_\tau(j)}{P_\tau} - \frac{P_{int,\tau}}{P_\tau} \right) Y_t(j) - \frac{\phi_p}{2}\left(\frac{P_\tau(j)}{\pi P_{\tau-1}(j)} - 1\right)^2 Y_\tau \right)$$

subject to the price-elastic demand of households[6]

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

where $\Lambda_{t,\tau}$ is the stochastic discount factor in period t for real payoffs in period τ, $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate ($\pi = 1$) and $mc^r_t = \frac{P_{int,t}}{P_t}$ is the real marginal cost of producing an additional unit of output (or the Lagrange multiplier from the cost minimization problem of the intermediate firm producer[7]).

By substituting the constraint related to demand of households for final goods into the objective function, we have:

$$\max_{P_t(j)} \ \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \Pi^r_t(j) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\lambda_{w,\tau}}{\lambda_{w,t}} \left(\left(\frac{P_\tau(j)}{P_\tau} - \frac{P_{int,\tau}}{P_\tau} \right) \left(\frac{P_\tau(j)}{P_\tau}\right)^{-\epsilon} Y_\tau - \frac{\phi_p}{2}\left(\frac{P_\tau(j)}{\pi P_{\tau-1}(j)} - 1\right)^2 Y_\tau \right)$$

[6]Derivation of the price-elastic demand of households and the aggregate price level is provided in Appendix A.1 and Appendix A.2.

[7]Derivation of the real marginal cost is provided in Appendix A.3.

and solving the resulting profit maximization problem with respect to $P_t(j)$ yields

$$[P_t(j)]: \ (1-\epsilon)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}\frac{Y_t}{P_t}-(-\epsilon)\left(\frac{P_{int,t}}{P_t}\right)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon-1}\frac{Y_t}{P_t}-\phi_p\left(\frac{P_t(j)}{\pi P_{t-1}(j)}-1\right)\frac{Y_t}{\pi P_{t-1}(j)}$$

$$+\mathbb{E}_t\,\beta\frac{\lambda_{w,t+1}}{\lambda_{w,t}}\phi_p\left(\frac{P_{t+1}(j)}{\pi P_t(j)}-1\right)\frac{P_{t+1}(j)Y_{t+1}}{\pi P_t(j)^2}=0$$

Since $mc_t^r = P_{int,t}/P_t$ and $Y_t(j) = Y_t$ are identical for all final goods firms, every firm sets the same price. The combination of that result and $P_t = (\int_0^1 P_t(j)^{1-\epsilon}dj)^{\frac{1}{1-\epsilon}}$ indicates that $P_t^*(j) = P_t^*$. In a symmetric equilibrium, the first-order condition for the retailers' problem becomes:

$$(1-\epsilon)+\epsilon\frac{P_{int,t}}{P_t}-\phi_p\left(\frac{P_t(j)}{\pi P_{t-1}(j)}-1\right)\frac{P_t}{\pi P_{t-1}(j)}+\mathbb{E}_t\,\Lambda_{t,t+1}\phi_p\left(\frac{P_{t+1}(j)}{\pi P_t(j)}-1\right)\frac{P_{t+1}(j)}{\pi P_t(j)}\frac{Y_{t+1}}{Y_t}=0$$

$$\Leftrightarrow \quad (1-\epsilon)+\epsilon mc_t^r-\phi_p\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}+\mathbb{E}_t\,\beta\frac{\lambda_{w,t+1}}{\lambda_{w,t}}\phi_p\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}\frac{Y_{t+1}}{Y_t}=0$$

If the cost of price adjustment is $\phi_p = 0$, i.e. when prices are fully flexible, the above equation reduces to the standard markup rule: $P_t = \frac{\epsilon}{\epsilon-1}mc_t^n$, where prices are set as a markup over nominal marginal costs. When $\phi_p > 0$, changes in marginal costs translate only gradually into changes in prices.

Rearranging terms and log-linearizing the above equation around a symmetric steady state, we obtain the expression known as the log-linearized New Keynesian Phillips Curve

$$\widetilde{\pi}_t = \frac{(\epsilon-1)}{\phi_p}\widetilde{mc}_t^r + \beta\,\mathbb{E}_t\,\widetilde{\pi}_{t+1}$$

As for the aggregate real profit that the continuum of unit mass retailers makes, the symmetric equilibrium ($P_t(j) = P_t,\ Y_t(j) = Y_t$ for $\forall j$) yields:

$$\Pi_t^r = \int_0^1 \Pi_t^r(j)dj = \int_0^1\left(\frac{P_t(j)}{P_t}Y_t(j) - mc_t^r\cdot Y_t(j) - \frac{\phi_p}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)}-1\right)^2 Y_t\right)dj$$

$$\Leftrightarrow \quad \Pi_t^r = \left(1 - mc_t^r - \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2\right)Y_t$$

3.3 Monetary and fiscal policies

The central bank monetary policy sets the short term nominal interest rate following the standard Taylor rule, which includes an interest rate smoothing component and a potential reaction to the deviations of inflation and output from their respective steady states:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\theta_r}\left[\left(\frac{\pi_t}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y}\right)^{\theta_y}\right]^{1-\theta_r} exp(\epsilon_t^r) \qquad (16)$$

where R is the steady-state value of the (gross) nominal policy rate, $0 \le \theta_r \le 1$ is the parameter associated with interest rate smoothing, $\theta_\pi > 0$ and $\theta_y > 0$ measure the interest rate response to inflation and output, respectively. The monetary policy shock ϵ_t^r is an *i.i.d.* with zero mean and standard deviation σ_R, $ln(\epsilon_t^r) = \rho_r ln(\epsilon_{t-1}^r) + \nu_t^r$.

The central bank also performs the asset purchases that have been previously issued by the government. Following Hohberger et al. (2019), QE policy is simulated as an AR(2) process that provides a hump-shape path of the central bank holdings of long-term government bonds:

$$lnB_t^{l,cb} = (\phi_{cb1} + \phi_{cb2})lnB_{t-1}^{l,cb} - (\phi_{cb1}\phi_{cb2})lnB_{t-2}^{l,cb} + \epsilon_t^{l,cb}$$

The specification of QE as an AR(2) process is important for capturing the initial purchase of long-term government bonds by the ECB in 2015q1, followed by further extension of the central bank holdings for three years, and a gradual exit from QE.

Total government debt on the basis of issued bonds includes short-term (B_t^s) and long-term (B_t^l) government bonds:

$$B_t = B_t^s + B_t^l,$$

where B_t^l is further decomposed into long-term bonds held by the central bank ($B_t^{l,cb}$) and by the household sector ($B_t^{l,h}$):

$$B_t^l = B_t^{l,cb} + B_t^{l,h} = f_t^l \cdot B_t^l + (1 - f_t^l) \cdot B_t^l$$

When the central bank conducts the QE program, it purchases long-term government bonds from the private sector, which in turn increases the amount of long-term bonds in the asset side of its balance sheet. The liability side of the central bank's balance sheet also increases as the central bank pays for the purchased bonds by the newly created money provided to the private sector, $(M_t - M_{t-1}/\pi_t)$.

The real operational profit of the central bank is:

$$\Pi_t^{cb} = M_t - \frac{M_{t-1}}{\pi_t} - \left(q_{L,t}B_t^{l,cb} - (1 + \varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t} \right)$$

As for the fiscal policy, in each period the fiscal authority purchases the final consumption good, G_t, issues government bonds to refinance its outstanding debt, B_t^s and B_t^l, distributes lump-sum transfers TR_t and raises lump-sum taxes T_t.

The consolidated government budget constraint (in aggregate real terms) is:

$$T_t + q_t B_t^s + q_{L,t}B_t^l + M_t - \frac{M_{t-1}}{\pi_t} - \left(q_{L,t}B_t^{l,cb} - (1 + \varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t} \right) = \frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t})\frac{B_{t-1}^l}{\pi_t} + G_t + TR_t \quad (17)$$

The real government spending G_t follows a serially correlated process

$$G_t = (Y\Gamma)^{1-\phi_g}(G_{t-1})^{\phi_g} exp(\epsilon_t^g)$$

where $\Gamma = G/Y$ is the steady state share of government consumption in output.

Similarly to G_t, lump-sum transfers TR_t are assumed to follow a serially correlated process:

$$TR_t = TR^{(1-\phi_{tr})} TR_{t-1}^{\phi_{tr}} exp(\epsilon_t^{tr})$$

Lump-sum taxes T_t are adjusted as a result of discrepancies in the value of long- and short-term government bonds from their steady-state. The passive fiscal policy rule that the lump-sum taxes follow can be written as:

$$T_t = \Phi \left(\frac{q_{L,t-1} B_{t-1}^l + q_{t-1} B_{t-1}^s}{q_L B^l + q B^s} \right)^{\rho_1}$$

The rationale behind the passive fiscal policy rule is to prevent the emergence of inflation as a fiscal phenomenon and the explosive path of government debt. The parameter Φ makes the fiscal rule an identity in steady state (see e.g., Chen et al., 2012), while the parameter $\rho_1 > 0$ determines the response of taxes to total government debt.

3.4 Aggregate variables and market clearing

The aggregate per-capita quantity of any household specific variable $x_t(i)$ is given by

$$x_t = \int_0^1 x_t(i) di = s_w \cdot x_{w,t} + s_p \cdot x_{p,t}$$

as households within each of the two types (i.e. wealthy and poor) are identical.

The aggregate resource constraint or the goods market clearing condition[8] is given by:

$$Y_t = C_t + I_t + G_t + \sum_{\varsigma \in \{s,e\}} s_w \frac{\phi_k}{2} \left(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \right)^2 k_{\varsigma,t} + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t - q_{L,t} s_w b_t^{l,h} \frac{\phi_b}{2} \left(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \right)^2$$

This condition implies that final output is used for consumption, investment, government expenditures and covering adjustment costs.

4. Transmission channels of QE

This section is dedicated to explaining two important channels for the transmission of QE effects to the real economy and to the inequality measures: the portfolio rebalancing channel and the earnings heterogeneity channel.

The portfolio rebalancing channel of QE can be illustrated with the analysis of the term spread, which is the difference between the long-term and short-term interest rates. We combine the log-linearised first-order conditions for short-term and long-term bond holdings ($\widetilde{b}_t^s$ and $\widetilde{b}_t^{l,h}$) of wealthy households to express the gross yield-to-maturity on a perpetual bond:

$$\widetilde{R}_t^L = \frac{\varrho}{R^L} \mathbb{E}_t \widetilde{R}_{t+1}^L + \frac{R^L - \varrho}{R^L} \left(\widetilde{R}_t - \phi_b (1 + \frac{\pi}{\beta} q_L \kappa)(\widetilde{b}_t^s - \widetilde{b}_t^{l,h}) \right) \tag{18}$$

[8]Derivation of the goods market clearing is provided in Appendix A.6.

where $\frac{R^L-\varrho}{R^L} > 0$ and $(1 + \frac{\pi}{\beta} q_L \kappa) > 0$.

Iterating on (18), we obtain the expression for long-term yields as the sum of (current and) expected future short-term interest rates and changes in relative bond holdings:

$$\widetilde{R}_t^L = \left(\frac{R^L - \varrho}{R^L}\right) \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\varrho}{R^L}\right)^s \left(\widetilde{R}_{t+s} - \phi_b (1 + \frac{\pi}{\beta} q_L \kappa)(\widetilde{b}_{t+s}^s - \widetilde{b}_{t+s}^{l,h})\right)$$

It follows that the term premium depends on changes in relative bond holdings:

$$\widetilde{R}_t^L - \left(\frac{R^L - \varrho}{R^L}\right) \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\varrho}{R^L}\right)^s \widetilde{R}_{t+s} = \left(\frac{R^L - \varrho}{R^L}\right) \phi_b (1 + \frac{\pi}{\beta} q_L \kappa) \, \mathbb{E}_t \sum_{s=0}^{\infty} \left(\frac{\varrho}{R^L}\right)^s \left(\widetilde{b}_{t+s}^{l,h} - \widetilde{b}_{t+s}^s\right) \quad (19)$$

The term spread is a positive function of long-term bonds held by wealthy households, but a negative function of short-term bonds. Accordingly, by purchasing the long-term bonds of households (QE), the central bank induces a fall in the long-term yield relative to the short-term yield. The term spread experiences a fall, which is actually the way to stimulate households to hold a relatively larger amount of short-term bonds in their portfolio (the preferred habitat theory). In equation (19), the transaction costs parameter ϕ_b controls for how the changes in the relative size of bond holdings with different maturities influence the term spread. Higher parameter ϕ_b means that households are less motivated to equalise returns through arbitrage behaviour. Although QE makes the short-term bonds more attractive as R_t^L reduces relative to R_t, the parameter ϕ_b discourages households from equalising returns via reallocation of portfolio funds to short-term bonds. In this way, larger portfolio adjustment costs ϕ_b refer to lower substitutability between assets of different maturities and thus a stronger response of the term spread. However, if the transaction costs are absent $\phi_b = 0$, the portfolio rebalancing channel cannot be identified. In this case, the central bank long-term bond purchases do not affect the term spread (and the real economy) because long-term and short-term bonds become perfect substitutes. Specifically, the term spread remains unchanged as households may compensate for the smaller supply of long-term bonds in their portfolio by purchasing short-term bonds in the same amount.

To show the relationship between the term spread and the real economy, we combine the log-linearised first-order condition for consumption of wealthy households and the term spread equation (18):

$$\widetilde{\lambda}_{w,t} = \left(\frac{q_L \kappa}{q_L \kappa + q}\right) \left(1 + \frac{\beta q}{\pi q_L \kappa}\right) \widetilde{R}_t + \left(\frac{q_L \kappa}{q_L \kappa + q}\right) \left(\frac{R^L}{R^L - \varrho}\right) \left(\widetilde{R}_t^L - \frac{R^L - \varrho}{R^L} \widetilde{R}_t\right)$$
$$+ \left(\frac{q_L \kappa}{q_L \kappa + q}\right) \left(\frac{\beta \varrho}{\pi} \mathbb{E}_t \widetilde{q}_{L,t+1} + \left(1 + \frac{\beta}{\pi} \frac{1}{q_L \kappa}\right) (\mathbb{E}_t \widetilde{\lambda}_{w,t+1} - \mathbb{E}_t \widetilde{\pi}_{t+1})\right)$$

The above expression indicates that a fall in the term spread, which is triggered by the QE program, leads to a higher consumption of the wealthy and, through the general equilibrium forces, to a higher consumption of the poor. This result comes from:

$$\frac{\partial \widetilde{\lambda}_{w,t}}{\partial \left(\widetilde{R}_t^L - \frac{R^L - \varrho}{R^L} \widetilde{R}_t\right)} = \frac{q_L \kappa}{q_L \kappa + q} \left(\frac{R^L}{R^L - \varrho}\right) > 0,$$

$$\text{and} \quad \frac{\partial \widetilde{\lambda}_{w,t}}{\partial \widetilde{c}_{w,t}} = -\frac{\sigma_c c_w}{c_w - hC_w} < 0$$

In response to a higher consumption, aggregate demand experiences a rise. Given that the two different types of workers respond differently regarding their respective consumption (due to different income sources), there is a change in consumption inequality. Similarly, the other inequality measures, such as total income and wealth inequality, also experience a change in response to the changes in the real economy induced by QE. The derived expressions related to the inequality measures can be found in Appendix A.9.3.

Through the portfolio rebalancing channel, previous studies could account for household heterogeneity in terms of financial income. However, the labor market in their model economy is specified such that different types of workers work the same number of hours and receive the same wage. In this regard, the previous model economies cannot explain the differences between the two types of households regarding their labor income. Ampudia et al. (2018) emphasize that labor income is an important component of total income in the EA. To acknowledge the labor market heterogeneity and corresponding labor income inequality, this paper introduces *the earnings heterogeneity channel*. Following Dolado et al. (2021), the present study makes a distinction between the roles of high-skilled and low-skilled workers in the production process, but within the scope of implemented QE by the ECB. In addition to CSC, the earnings heterogeneity channel includes asymmetric wage rigidity that is distinctive to the labor market in the EA. Given the interaction between labor and financial markets, wage rigidity affects both labor and non-labor income inequality (wage is a cost part of the wealthy's profit).

For the analysis of the skill premium dynamics, we log-linearize the equation (14) to obtain:

$$\widetilde{w}_{w,t} - \widetilde{w}_{p,t} = (\sigma - \nu)\rho\Big(\frac{K_e}{N_w}\Big)^{\nu}\Big(\rho\Big(\frac{K_e}{N_w}\Big)^{\nu} + (1-\rho)\Big)^{-1}(\widetilde{K}_{e,t-1} - \widetilde{N}_{w,t}) + (\sigma - 1)(\widetilde{N}_{w,t} - \widetilde{N}_{p,t}) \quad (20)$$

The growth rate of the skill premium is decomposed into two parts. Taking as given the influence of the first component, the second component $(\sigma - 1)(\widetilde{N}_{w,t} - \widetilde{N}_{p,t})$ indicates that under $\sigma < 1$ the faster growth rate of the relative supply of skilled labor reduces the skill premium. Krusell et al. (2000) calls this part the "relative quantity effect". Taking as given the second component, the first component $(\sigma - \nu)\rho\Big(\frac{K_e}{N_w}\Big)^{\nu}\Big(\rho\Big(\frac{K_e}{N_w}\Big)^{\nu} + (1-\rho)\Big)^{-1}(\widetilde{K}_{e,t-1} - \widetilde{N}_{w,t})$ indicates that under $\sigma > \nu$ the faster growth rate of equipment capital than that of skilled labor leads to higher skill premium. This second component is called the "capital-skill complementarity effect". The trade-off between those two effects determines the dynamics of the skill premium.

5. Calibration

In Table 1, the calibrated values of structural parameters of the model are summarized. The model is calibrated at a quarterly frequency to match the average Euro Area data for the period 2000-2014, before the implementation of the QE program. These parameter values are chosen (calibrated) either based directly on data (including existing econometric evidence) or by ensuring that the model's long run solution targets key macroeconomic ratios of the EA-19 economy (see Table 6).

Households are different in terms of their access to financial/capital markets and their labor services offered to the labor market. Following Sakkas and Varthalitis (2021), population shares are set to $s_p = 0.3$ and $s_w = 0.7$ so that 30 percent of the total population in the EA-19 does not participate in capital and financial markets and provides low-skilled labor services. A similar treatment of wealthy and poor households in the US can be found in Bhattarai et al. (2022) and Bilbiie et al. (2022a).

The subjective discount factor, $\beta = 0.9995$, is set to match a net annualised money-market interest rate of around 2.21 percent (or a quarterly gross money-market rate of around $R = 1 + \frac{2.21}{4 \cdot 100} = 1.0055$). The coefficient of relative risk aversion of consumption σ_c is set to 1, giving the log utility function in consumption. The Frisch elasticity of labor supply is set to 1. The parameters for labor disutility, $\varphi_{n,w}$ and $\varphi_{n,p}$, are calibrated to obtain the average of skilled and unskilled hours worked per week of $0.247(= 41.5h/168h)$, and to acknowledge that wealthy households work 8.27% more than poor households in the steady state. The elasticity of utility with respect to real money holdings $\chi = 3.42$ is taken from Neiss and Pappa (2005), who estimate this value on the basis of UK data. The choice of $\chi = 3.42$ implies an interest elasticity of money demand of $-1/\chi = -0.29$. The preference parameter for real money holdings in the utility function φ_m is chosen to obtain the steady state real money-to-consumption ratio of 1.905 per quarter. As in Coenen et al. (2008), the money-to-consumption ratio is computed as a ratio of monetary aggregate held by the household sector M1 and nominal consumption expenditure for the period 2000–2014.

The steady state gross inflation rate is set to 1.005, which is in line with the mandate of the ECB (2% annualised inflation). The elasticity of substitution among differentiated retail goods ϵ is set to 6 as in Gerali et al. (2010), which refers to the gross price markup of 20% over marginal cost ($\mu^p = \frac{\epsilon}{\epsilon - 1} = 1.2$). The Rotemberg adjustment cost parameter is set to 59.0259 so that the slope of the Phillips curve in the model corresponds to that in a Calvo staggered price-setting model with four quarters of an average price rigidity. In the Calvo (1983) model, the percent of reoptimizing firms or the average time for which firms set the new prices is $1 - \theta$. This means that the average frequency of price changes is $\frac{1}{1-\theta}$, leading to the value of the Rotemberg (1982) parameter:

$$\phi_p = \frac{(\epsilon - 1)\theta}{(1-\theta)(1-\beta\theta)} = \frac{5 \cdot 0.75}{(1-0.75)(1-0.75 \cdot 0.9945)} = 59.0259$$

The capital adjustment costs parameter ϕ_k is set to 5.28 so that the elasticity of the investment to capital ratio with respect to Tobin's q is 13.33 (see Matheron, 2018). For the sake of simplicity, the same parameter value ϕ_k is chosen for both types of capital[9].

The steady-state level of the technological process A is normalized to 1 for the CSC economy. The depreciation rate of equipment capital δ_e and structures δ_s are used from Krusell et al. (2000). We also use the estimates of the key substitution parameters $\sigma = 0.401$ and $\nu = -0.495$ from Krusell et al. (2000). The choice of $\sigma = 0.401$ implies the elasticity of substitution between equipment capital (or skilled labor) and unskilled labor of $1/(1 - \sigma) = 1.67$, while $\nu = -0.495$ implies the elasticity of substitution between equipment capital and skilled labor of $1/(1 - \nu) = 0.67$. Thus, the skilled households are more complementary with equipment capital in the production than the unskilled households or the production function exhibits capital–skill complementarity. The parameters corresponding to income shares $m = 0.2977$, $\rho = 0.5685$ and $\iota = 0.1679$ are simultaneously calibrated to match a skill (wage) premium of 1.55 and a labor income share of 65 percent, and the share of equipment capital in total capital of 1/3. The calibrated values of parameters in the production function are in line with those estimated or calibrated in the related literature.

To be in line with the average historical EA data, we set government spending to output ratio at 18%, while government debt to output ratio is set to 2.96 or at 74% of annual output. Similar to Albonico and Tirelli (2020), transfers to non-Ricardian households are calibrated to obtain a steady-state consumption ratio between the two groups of households (c_p/c_w) around 0.8. The steady-state difference between aggregate transfers and lump-sum taxes to output ratios (net government transfers/taxes) is then calculated as a residual from the steady state government budget constraint.

The parameters of the fiscal and monetary policy rules are calibrated following Coenen et al. (2008). Specifically, fiscal policy responses to both short-term and long-term debt are set to 0.1. In addition, interest rate sensitivity to inflation gap and output gap are set to 2 and 0.10, respectively. The interest rate smoothing parameter is chosen very close to one as in Falagiarda (2014) to indicate the presence of the ZLB under which the monetary policy (short-term) interest rate is restricted to respond to fluctuations in inflation and output. Cui and Sterk (2021) also assume the ZLB by pegging the nominal interest rate at $R_t = R$ in the model version with QE.

The steady state values of the key variables related to the ECB asset purchase program are summarised in Table 4. Data for short-term and long-term bonds outstanding relative to annual GDP is taken from Eurostat Government Finance Statistics. The ECB provides data for 'Securities held for monetary policy purposes - ILM' that serve as a measure for long-term bonds held by the ECB. The amount of long-term bonds held by wealthy households is the difference between total long-term bond

[9]Derivation for the the elasticity of the investment to capital ratio with respect to Tobin's q is provided in Appendix A.8.

supply and long-term government bonds of the EBC. The parameter ϱ is set to match the average duration of 25 quarters long-term government debt, $d = \frac{1}{1-\frac{\beta}{\pi}\varrho} = 25$.

Table 1: Parameter values in the baseline analysis of the CSC economy with no real wage rigidity

Notation	Description	Value	Source
Households			
β	Subjective discount factor	0.9995	Calibration
χ	Elasticity of money demand	3.42	Neiss and Pappa (2005)
η	Elasticity of labor supply	1	Convention
σ_c	Coefficient of relative risk aversion	1	Convention
$\varphi_{n,w}$	Relative utility weight on labor-wealthy	16.917	Calibration
$\varphi_{n,p}$	Relative utility weight on labor-poor	14.771	Calibration
s_w	Population share of the wealthy	0.7	Sakkas and Varthalitis (2021)
s_p	Population share of the poor	0.3	Sakkas and Varthalitis (2021)
Intermediate goods firms			
A	Scale parameter	1	Convention
δ_s	Structure capital depreciation rate	0.014	Krusell et al. (2000)
δ_e	Equipment capital depreciation rate	0.031	Krusell et al. (2000)
ι	Structure capital income share	0.1679	Calibration
m	Low-skilled labor income share	0.2977	Calibration
ρ	Equipment capital income share	0.5685	Calibration
σ	Measure of elas of subs between K_e and N_p	0.401	Krusell et al. (2000)
ν	Measure of elas of subs between K_e and N_w	-0.495	Krusell et al. (2000)
ϕ_k	Capital adjustment cost	5.28	Matheron (2018)
Final goods firms			
ϕ_p	Price adjustment cost	59.0259	Gerali et al. (2010)
ϵ	Elasticity of substitution between retail goods	6	Gerali et al. (2010)
Fiscal and monetary policy			
ρ_1	Fiscal policy response to debt	0.1	Coenen et al. (2008)
θ_π	Monetary policy response to inflation	2	Coenen et al. (2008)
θ_y	Monetary policy response to output	0.1	Coenen et al. (2008)
θ_r	Monetary policy inertia	0.997	Falagiarda (2014)
Π	Gross inflation rate	1.005	Convention
Autoregressive parameters			
ϕ_g	Governement spending	0.9	Coenen et al. (2008)
ϕ_{tr}	Lump-sum transfers	0.9	Coenen et al. (2008)
Standard deviation			
σ_g	Government spending shock	0.18	Coenen et al. (2008)
σ_{tr}	Lump-sum transfers	0.195	Coenen et al. (2008)
σ_r	Monetary policy shock	0.1	Hohberger et al. (2019)

Table 2: Parameter values in the baseline analysis of the CD1 economy with real wage rigidity

Notation	Description	Value	Source
A	Total factor productivity	0.9962	Target output of CSC
θ	Income share of capital	0.35	Data
$\varkappa$	Income share of high-skilled labor	0.7494	Target $\frac{w_w}{w_p} = 1.55$
γ	Measure of elas of subs between N_w and N_p	0.2908	Katz and Murphy (1992)
δ_k	Depreciation rate of capital	0.025	Data

Table 3: Parameter values in the baseline analysis of the CD2 economy with no real wage rigidity

Notation	Description	Value	Source
A	Total factor productivity	1	Convention
θ	Income share of capital	0.35	Data
$\varkappa$	Income share of high-skilled labor	0.5	Convention
γ	Measure of elas of subs between N_w and N_p	1	Convention
δ_k	Depreciation rate of capital	0.025	Data

Table 4: Calibration in the analysis of asset purchase policy, CSC and CD economies

Notation	Description	Value
$B/Y = B^s/Y + B^l/Y$	Total debt to GDP ratio	0.740
B^s/Y	Total short-term debt to GDP ratio	0.063
$B^l/Y = B^{l,h}/Y + B^{l,cb}/Y$	Total long-term debt to GDP ratio	0.677
$B^{l,h}/Y$	LT debt held by households	0.622
$B^{l,cb}/Y$	LT debt held by the central bank	0.055
$f^l = \frac{B^{l,cb}}{B^l}$	Fraction of LT debt by CB in total LT debt	0.0818
ϕ_b	Portfolio adjustment cost parameter	0.0015
$\sigma_{l,cb}$	Magnitude of the asset purchases	0.01
ϕ_{cb1}	Persistence of the asset purchases	0.89
ϕ_{cb2}	Persistence of the asset purchases	0.97
ϱ	Bonds payoff decay factor	0.9653

Table 5: Selected steady-state values in the CSC economy

Notation	Description	Value
c_p	Consumption of the poor	0.2415
c_w	Consumption of the wealthy	0.3019
n_p	Labor of the poor	0.2335
n_w	Labor of the wealthy	0.2528
w_p	Wage of the poor	0.8329
w_w	Wage of the wealthy	1.2909
r_s^k	Real return to structures	0.0145
r_e^k	Real return to equipment	0.0315
Y	Total output	0.5294

Table 6: Selected Steady-state ratios in the CSC economy

Notation	Description	Value
C/Y	Consumption as a share of GDP	0.536
K_s/Y	Structure capital as a share of GDP	9.628
K_e/Y	Equipment capital as a share of GDP	4.814
I_s/Y	Structure investment as a share of GDP	0.135
I_e/Y	Equipment investment as a share of GDP	0.149
G/Y	Government expenditure to GDP ratio	0.180
B/Y	Total debt to GDP ratio	0.740
$T/Y - TR/Y$	Net lump sum tax as a share of GDP	0.184
M/C_w	Money-to-consumption ratio	1.905
w_w/w_p	Skill premium	1.55
li_share	Labor income share	0.65

6. Results

The ECB started with the implementation of the QE program in March 2015. Figure 1 shows the impulse responses of selected endogenous variables after a one-standard-deviation QE shock in the Euro Area. Following Hohberger et al. (2019), QE shock is simulated as an AR(2) process so that the initial purchase is followed by a further accumulation of long-term assets by the ECB for another 12 quarters, after which a gradual exit takes place. This paper examines the distributional effects of QE by means of two channels: the portfolio rebalancing channel and the earnings heterogeneity channel.

The portfolio rebalancing channel establishes the relationship between the central bank QE and the whole economy through changes in investors' portfolios. Given that only wealthy households have access to financial markets, QE starts having the effects through their portfolio. Specifically, by purchasing long-term government bonds, the central bank expands its balance sheet (the asset side of the balance sheet) and increases liquidity provision to the wealthy (the liability side of the balance sheet). As the wealthy receive central bank reserves (short-term assets) in exchange for long-term government bonds, QE changes the portfolio duration of the wealthy. According to the equation (19), a lower supply of $b_t^{l,h}$ relative to b_t^s implies an increase in price $q_{L,t}$ and a reduction in R_t^L. Given that the short-term interest rate is constrained at the (exogenous) ZLB, a smaller R_t^L causes the term-spread to decline. In response to a lower term-spread and to restore the portfolio duration, wealthy households increase investment in other long-term assets such as physical capital, reduce savings in short-term bonds and increase current consumption. Cui and Sterk (2021) highlight the importance of both household consumption and investment in transmitting the QE effects to the real economy. They also show that the increase in investment demand is driven by the need of investors to replace government bonds (direct channel) and by the rise in goods demand (indirect equilibrium channel).

Stimulating aggregate demand, QE has positive effects on the real economy and inflation[10]. To produce a larger amount of final goods, retail firms increase their demand for inputs in production (intermediate goods), which in turn causes a rise in the relative price of intermediate goods mc_t^r. A higher mc_t^r is associated with a higher demand of intermediate goods firms for capital and labor. Investment and employment increase, which is also accompanied with higher wages and rental rate on capital. However, high- and low-skilled workers do not enjoy the same rise in wages and employment, an observation that can be explained by *the earnings heterogeneity channel*. Figure 1 shows a fall in unskilled employment inequality and in the skill-premium in the short run. The employment of high-skilled workers is more pronounced for two reasons. First, capital-skill complementarity implies a larger demand for skilled labor on the back of increased capital stock. Second, high-skilled workers increase their labor supply to compensate the loss in non-labor income induced by negative profits[11]

[10] Boeckx et al. (2017), among others, estimate that an exogenous expansion of the ECB's balance sheet has significant stimulative effects on the economic activity and inflation in the EA.

[11] The countercyclical markups or negative profits are a standard feature of the model economies with only sticky prices but absent sticky wages.

and lower interest payments on long-term government bonds. As stated by Angelopoulos et al. (2014), a higher real return to capital can also stimulate the skilled to collect larger labor income resources that will be used for capital accumulation. There can be overshooting in labor supply ($\widetilde{K}_{t-1} - \widetilde{N}_{w,t} < 0$) due to a slower adjustment of capital, which results in decreasing CSC effects and the skill premium[12]. However, in the medium/long run, the relative supply of skilled labor decreases while the complementarity between capital stock and skilled labor increases. Both factors give rise to an increasing skill premium and decreasing relative skilled labor income in the medium/long run.

According to Christoffel et al. (2009), the labor market in the EA is highly rigid in many aspects. This particularly applies to wages that are less prone to instantaneous changes, and as such have a substantial degree of rigidity. The authors argue that the collective wage bargaining process lies behind the sluggish adjustment of wages. To be in line with the empirical findings of Lenza and Slacalek (2018) regarding the skill premium in favor of high-skilled labor and higher employment growth for low-skilled workers after QE, this paper incorporates real wage rigidity[13] as a second component of labor market segmentation. Following Blanchard and Galí (2007), ad-hoc real wage rigidity is introduced such that the slow adjustment of real wages is a result of (unmodelled) distortions instead of preferences in labor markets. For the same wage setting, Kollmann et al. (2016) provide the estimated value of 0.97 (0.96) for real wage rigidity in the EA (US) over the period 1999q1–2014q4. The current study uses the value of 0.97 for the wage of poor households, while the value of 0.8 applies to wealthy households[14]. Wealthy households face lower labor market friction in the form of (upward) real wage rigidity as they are a more valuable labor source for intermediate goods firms and as such enjoy larger (implicitly assumed) bargaining power in the wage determination.

Figure 2 reports the dynamic responses of selected variables when CSC and CD1 production functions are interacted with asymmetric real wage rigidity. Compared to the case with flexible wages, both types of workers increase their labor supply to smooth their level of consumption. However, poor households work harder relative to their richer counterparts as they do not have wealth to be protected against the changes in disposable income. This causes a rise in unskilled labor inequality $\widetilde{N}_{p,t} - \widetilde{N}_{w,t} > 0$, which outweighs a fall in capital to skill labor inequality $\widetilde{K}_t - \widetilde{N}_{w,t} < 0$, referring to the stronger relative quantity effects than the CSC effects. The presence of capital-skill complementarity in the economy where labor supply is more responsive than the capital stock implies a mitigated rise in the skill premium, which stimulates poor households to supply even more labor services, pushing up the inequality $\widetilde{N}_{p,t} - \widetilde{N}_{w,t} > 0$.

[12]To prove that the skill-premium decreases in the short-run, we could use the equation (20). The first component of the skill-premium decreases as $(\sigma - \nu) > 0$ and $\rho\left(\frac{K}{N_w}\right)^{\nu}\left(\rho\left(\frac{K}{N_w}\right)^{\nu} + (1-\rho)\right)^{-1} > 0$ and $(\widetilde{K}_{t-1} - \widetilde{N}_{w,t}) < 0$. The second component also decreases as $(\sigma - 1) < 0$ and $(\widetilde{N}_{w,t} - \widetilde{N}_{p,t}) > 0$.

[13]The equation for ad-hoc real wage rigidity is more elaborated in Appendix A.7. Interestingly, only asymmetric real wage rigidity is in line with empirical evidence, while symmetric real wage rigidity generates the same qualitative results as the flexible wage setting.

[14]The value of the wage rigidity parameter of 0.8 for wealthy households corresponds approximately to the average of values used by Dolado et al. (2021) and Komatsu (2022). Specifically, Dolado et al. (2021) indicate 33% while Komatsu (2022) refers to 10% lower real wage rigidity for wealthy households.

As for the skilled labor income inequality, it experiences a fall up to seven quarters since the beginning of QE, which is consistent with the empirical evidence of Lenza and Slacalek (2018). Although in the short-run poor households are winners regarding total labor income, the model predicts a reversing and less pronounced trend of labor income inequality in favour of wealthy households in the medium and long run.

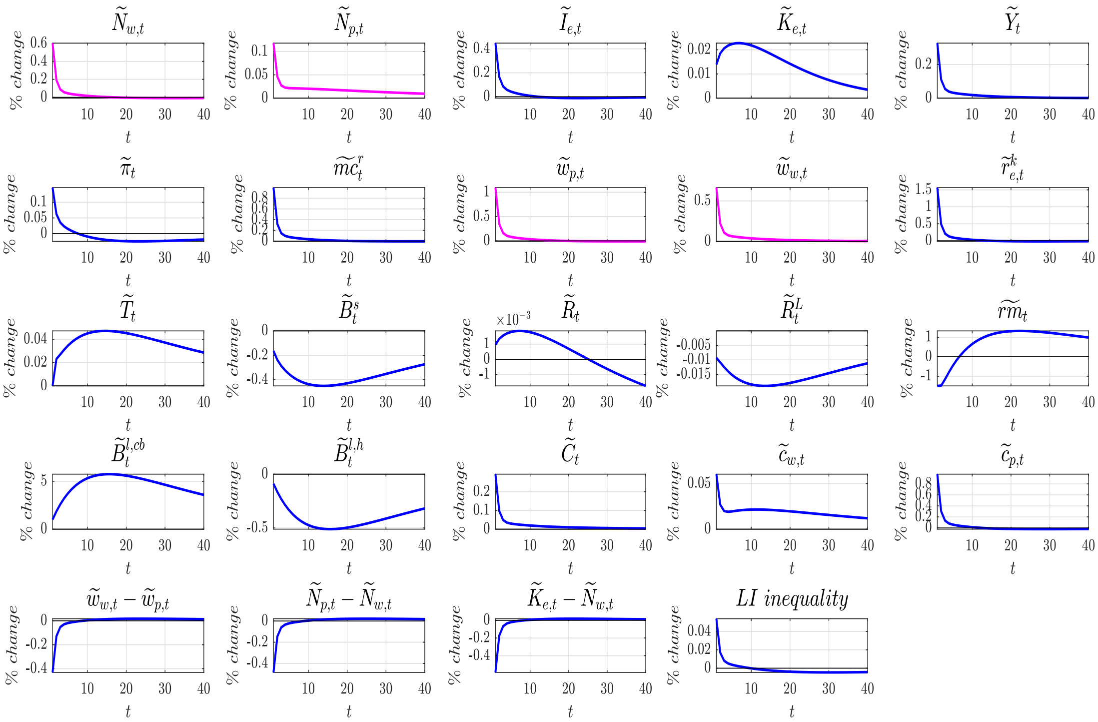

Figure 1: IRFs for the quantitative easing shock: CSC (the case of flexible wages)

In Figure 3, the economies with segmented labor and financial markets are compared to the economy with only segmented financial market (blue solid line) in terms of four inequality measures. The presence of real wage rigidity induces a drop in labor income inequality (see purple solid and red dashed lines), which becomes mitigated over time. Total income inequality experiences a fall, which is the most pronounced for the economy with only segmented financial market. Similar dynamics can be observed for consumption inequality. Although there is a rise in wealth inequality for all types of economies that persistently remains above the baseline, the segmented labor market generates a larger increase in wealth inequality. As a measure of wealth inequality, we use any increase in the value of asset holdings of wealthy households as a poorer part of the population is excluded from financial/capital markets. Given that the total income of wealthy households can be important for the dynamics of wealth inequality, we next examine the components of the total income.

Figure 4 shows that labor and non-labor income go in opposite directions except for the economy CD2+NRW, where poor households enjoy an increase in both components of total income. Generally, households tend to benefit from the rise in labor income, while non-labor income steadily declines. A

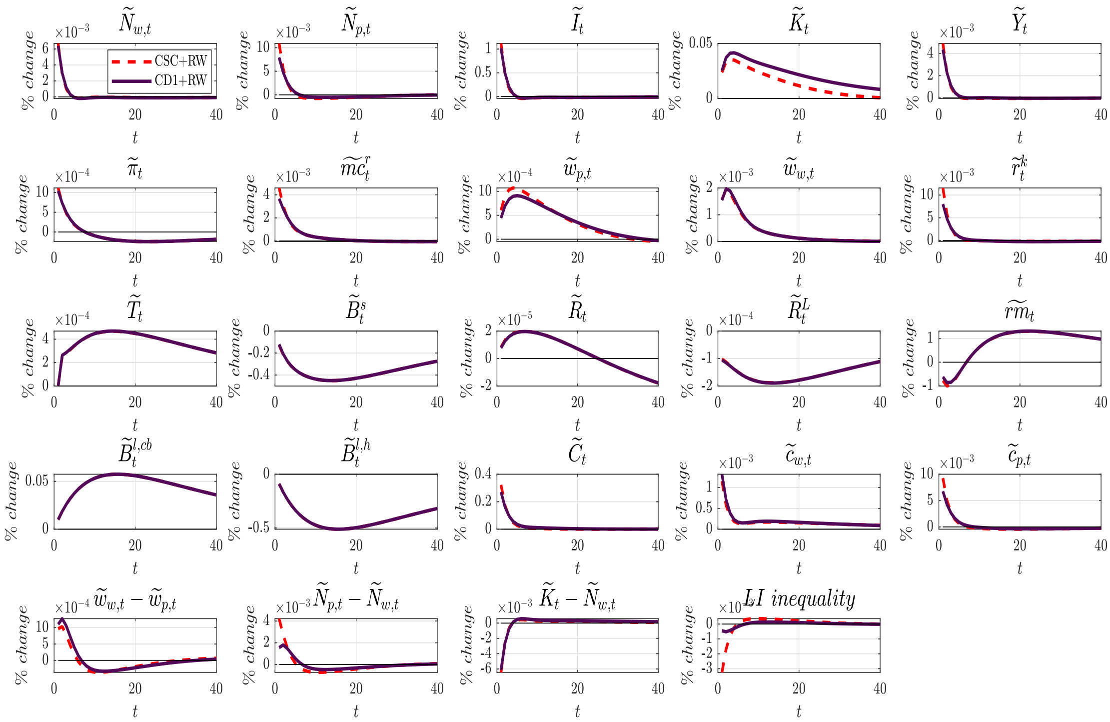

Figure 2: IRFs for the quantitative easing shock: CSC vs CD1 (the case of rigid wages)

Notes: For the CSC economy, the variables $\widetilde{I}_t$, $\widetilde{k}_t$, $\widetilde{K}_t$ - $\widetilde{N}_{w,t}$ and $\widetilde{r}^k_t$ stand for equipment investment, equipment capital, equipment to skilled labor ratio and equipment rental rate, respectively.

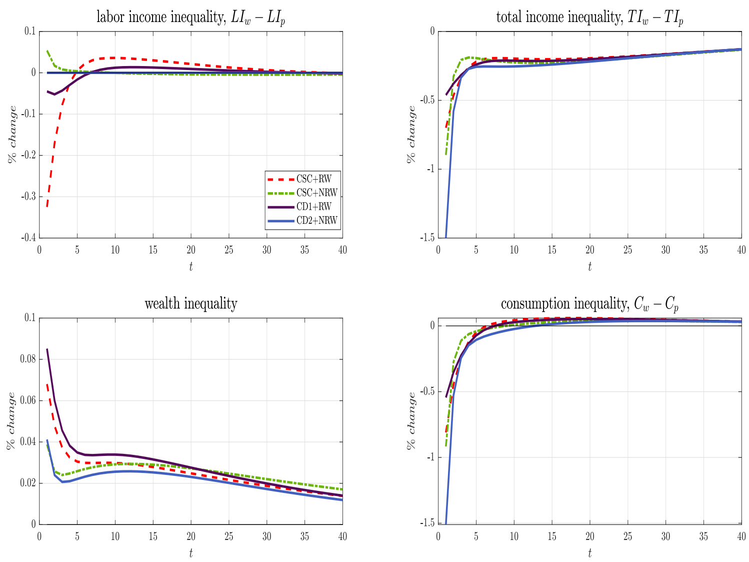

Figure 3: Inequality measures: The comparison of CSC and CD economies

Notes: Blue color indicates the portfolio rebalancing channel, while the other colors refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel.

drop in non-labor income of wealthy households is noticeably mitigated in the economies with real wage rigidity (see green dashed lines for CSC+RW and CD1+RW) due to its counteracting effects on

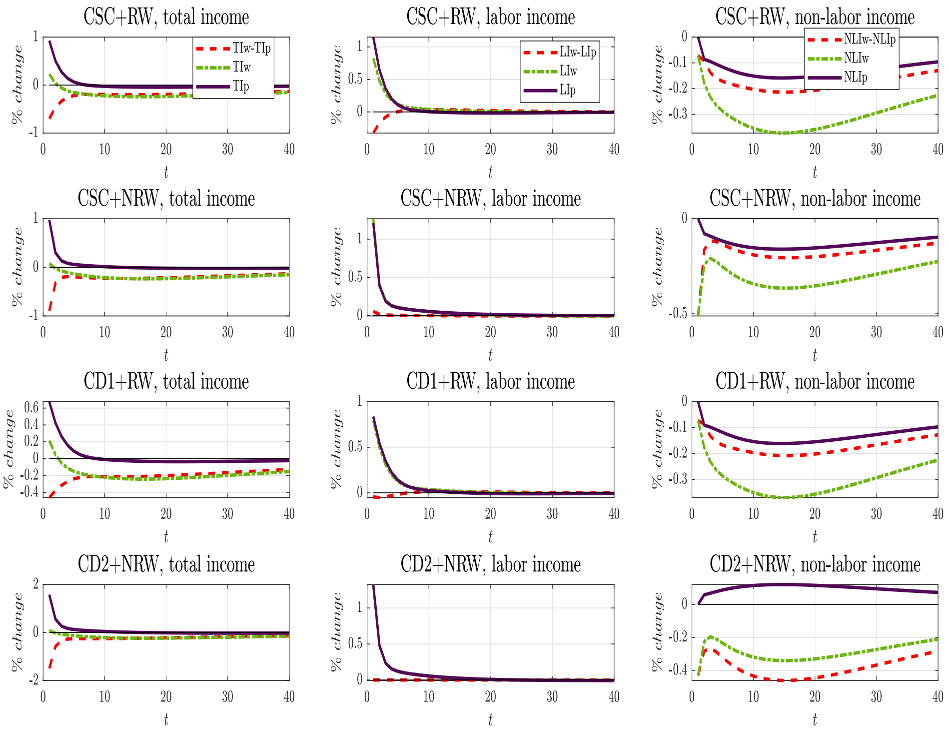

Figure 4: Total income components: The comparison of CSC and CD economies

Notes: The economy CD2+NRW includes the portfolio rebalancing channel, while the other economies refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel.

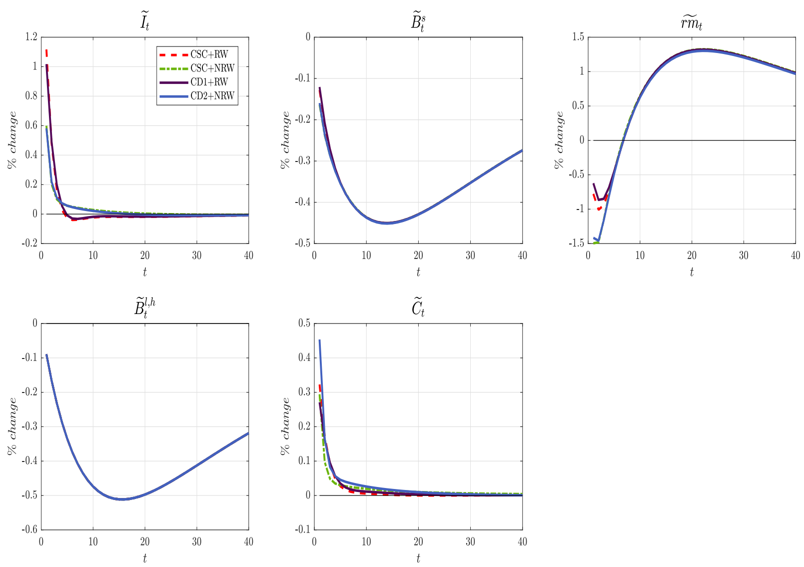

Figure 5: Investment sources: The comparison of CSC and CD economies

Notes: Blue color indicates the portfolio rebalancing channel, while the other colors refer to the interaction of the earnings heterogeneity channel and the portfolio rebalancing channel. $\widetilde{I}_t$ is the sum of investment in structure and equipment capital, $I \cdot \widetilde{I}_t = I_s \cdot \widetilde{I}_{s,t} + I_e \cdot \widetilde{I}_{e,t}$.

declining profit income. The total income of the wealthy becomes higher, which allows a larger accumulation of assets and thus causes greater wealth inequality. In Figure 5, we observe higher investment in capital and a larger amount of real money holdings in the economies CSC+RW and CD1+RW. The presence of real wage rigidity motivates the wealthy to increase capital investment, which enables higher rental income and compensates for lower wage payments, and increases real money holdings due to lower inflation.

7. Conclusion

In response to the global financial crisis of 2007-2008, the ECB implemented the QE program by injecting central bank reserves into the economic system in exchange for purchased long-term government securities. The main objective of the QE program is to bring the euro area back to its potential in periods when the traditional monetary policy instrument (the short-term policy interest rate) is unavailable due to the zero lower bound. Although QE may be successful in achieving its main goal, there might be side effects of QE such that a certain fraction of the EA population benefits more from QE than the rest of the population. Given that the QE effects may go in opposite directions along different household heterogeneity dimensions, the overall distributional effects of QE could be better examined within a framework that includes joint household heterogeneity.

To have a clearer picture of the inequality effects of QE, this study considers a framework with two dimensions of household heterogeneity. First, we introduce financial market segmentation that separates the EA population of households into two distinct groups on the basis of different access to financial/capital markets. Additionally, labor market segmentation is considered in the form of capital-skill complementarity in the production process and asymmetric real wage rigidities. This segmentation implies that differently skilled workers work a different number of hours and receive different wages. Compared to the model economy with only financial market segmentation, the results indicate that the interaction of labor and financial market segmentation significantly mitigates a decrease in total income inequality and amplifies a rise in wealth inequality. Casiraghi et al. (2018) state that in the future the ECB will broaden its mandate, focusing on both price stability and the distributional effects of QE. Accordingly, this paper suggests that the ECB could benefit more from the analysis of labor and financial market segmentation as it provides a clearer picture of the inequality effects.

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Appendix A: Model Derivation

A.1 The price-elastic demand of households

Final goods Y_t^f are expressed as the CES aggregate production function according to the equation called the "aggregate output index":

$$Y_t^f = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 1$ is the elasticity of substitution among final or intermediate goods due to a linear technology in differentiation process, $Y_t^f(j) = Y_t(j)$.

A demand curve for final goods of each retailer can be derived by referring to the profit maximization problem of retail firms:

$$\max_{Y_t(j)} \int_0^1 P_t(j) Y_t(j) dj - \int_0^1 P_{int,t} Y_t(j) dj$$

Given that the CES aggregate production function makes exact aggregation difficult, Iacoviello (2005) suggests a linear aggregator of the form $Y_t^f = \int_0^1 Y_t(j) dj = Y_t$ within a local region of the steady state.

$$\max_{Y_t(j)} \int_0^1 P_t(j) Y_t(j) dj - P_{int,t} Y_t$$

$$s.t. \;\; Y_t^f = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

We set the Lagrangian function to solve the maximization problem:

$$\mathcal{L} = \int_0^1 P_t(j) Y_t(j) dj - P_{int,t} Y_t - \lambda_t^p \left[\left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - Y_t^f \right]$$

Taking the FOC with respect to $Y_t(j)$ gives:

$$\int_0^1 P_t(j) dj - \lambda_t^p \left[\frac{\epsilon}{\epsilon-1} \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon-1}{\epsilon} \int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}-1} dj \right] = 0$$

$$\int_0^1 P_t(j) dj - \lambda_t^p (Y_t^f)^{\frac{1}{\epsilon}} \int_0^1 Y_t(j)^{-\frac{1}{\epsilon}} dj = 0$$

$$P_t(j) - \lambda_t^p (Y_t^f)^{\frac{1}{\epsilon}} Y_t(j)^{-\frac{1}{\epsilon}} = 0$$

$$Y_t(j)^{\frac{1}{\epsilon}} = \lambda_t^p \frac{(Y_t^f)^{\frac{1}{\epsilon}}}{P_t(j)} \tag{21}$$

$$\Leftrightarrow Y_t(j)^{\frac{\epsilon-1}{\epsilon}} = \left(\lambda_t^p \frac{(Y_t^f)^{\frac{1}{\epsilon}}}{P_t(j)} \right)^{\epsilon-1}$$

$$\Leftrightarrow Y_t^f = \left(\int_0^1 \frac{(\lambda_t^p)^{\epsilon-1} (Y_t^f)^{\frac{\epsilon-1}{\epsilon}}}{P_t(j)^{\epsilon-1}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$\Leftrightarrow \lambda_t^p = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \equiv P_t$$

Plugging P_t into equation (21) gives the expression that refers to a downward sloping demand function of each retailer:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t^f$$

A.2 The aggregate price level

As in Dolado et al., 2021, due to differentiation, retailers have pricing power and thus can set the price for their products $P_t(j)$ but take the aggregate price level P_t as given. To derive the aggregate price index, we express the nominal value of output as follows:

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$$

Plugging in the demand for each variety $Y_t(j)$ yields:

$$P_t Y_t = \int_0^1 P_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t dj$$

Pulling out the integral things that are independent of j:

$$P_t Y_t = P_t^{\epsilon} Y_t \int_0^1 P_t(j)^{1-\epsilon} dj$$

Simplifying, we obtain an expression for the aggregate price level:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

A.3 Marginal costs for intermediate goods firms

The (nominal) cost minimization problem of intermediate goods firms for the case of having one type of capital in production:

$$\min_{N_{w,t}, N_{p,t}, K_{t-1}} \quad TC(Y_{i,t}) = W_{w,t} N_{w,t} + W_{p,t} N_{p,t} + R_t^k K_{t-1}$$

subject to the production technology:

$$A \left[m(N_{p,t})^{\sigma} + (1-m)\Big(\rho(K_{t-1})^{\nu} + (1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}} \right]^{\frac{1}{\sigma}} \geq Y_{i,t}$$

The Lagrangian function related to the cost minimization problem:

$$\begin{aligned} \mathcal{L} = {} & W_{w,t} N_{w,t} + W_{p,t} N_{p,t} + R_t^k K_{t-1} \\ & - \lambda_t \left(A \left[m(N_{p,t})^{\sigma} + (1-m)\Big(\rho(K_{t-1})^{\nu} + (1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}} \right]^{\frac{1}{\sigma}} - Y_{i,t} \right) \end{aligned}$$

where λ_t is the Lagrange multiplier from the cost minimization problem. The Lagrange parameter related to the technological constraint is the shadow price of change in the ratio of the use of capital and labor services. This means that the Lagrange parameter measures the nominal marginal cost, $\lambda_t = mc_t^n$.

The first order conditions of the minimization problem:

$$R_t^k = \lambda_t F_{k,t} = \lambda_t A\Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}-1}(1-m)\rho\cdot$$
$$\cdot\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(K_{t-1})^{\nu-1}$$

$$W_{w,t} = \lambda_t F_{n,t}^w = \lambda_t A\Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}-1}\cdot$$
$$\cdot(1-m)(1-\rho)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(N_{w,t})^{\nu-1}$$

$$W_{p,t} = \lambda_t F_{n,t}^p = \lambda_t A\Big[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\Big]^{\frac{1}{\sigma}-1}m(N_{p,t})^{\sigma-1}$$

The first order conditions of the minimization problem can be rewritten as:

$$R_t^k = \lambda_t A^\sigma Y_{i,t}^{1-\sigma}(1-m)\rho\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(K_{t-1})^{\nu-1}$$
$$W_{w,t} = \lambda_t A^\sigma Y_{i,t}^{1-\sigma}(1-m)(1-\rho)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}-1}(N_{w,t})^{\nu-1}$$
$$W_{p,t} = \lambda_t A^\sigma Y_{i,t}^{1-\sigma}m(N_{p,t})^{\sigma-1}$$

where the substitution is expressed as

$$A^\sigma Y_{i,t}^{1-\sigma} = A\left[m(N_{p,t})^\sigma + (1-m)\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}}\right]^{\frac{1-\sigma}{\sigma}}$$

We express $N_{p,t}$ from the optimality condition related to the labor supply of the poor:

$$N_{p,t} = \left(\frac{W_{p,t}}{A^\sigma\lambda_t Y_t^{1-\sigma}m}\right)^{\frac{1}{\sigma-1}}$$

and combine the optimality conditions for K_{t-1} and $N_{w,t}$:

$$\frac{K_{t-1}}{N_{w,t}} = \left(\frac{R_t^k(1-\rho)}{W_{w,t}\rho}\right)^{\frac{1}{\nu-1}}$$

so that the second part of the RHS in the production function becomes:

$$\Big(\rho(K_{t-1})^\nu + (1-\rho)(N_{w,t})^\nu\Big)^{\frac{\sigma}{\nu}} \Leftrightarrow K_{t-1}^\sigma\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^\nu\right)^{\frac{\sigma}{\nu}}$$

Now we have the following in the production function

$$\left(\frac{Y_{i,t}}{A}\right)^\sigma = m\left(\frac{W_{p,t}}{A^\sigma\lambda_t Y_t^{1-\sigma}m}\right)^{\frac{\sigma}{\sigma-1}} + (1-m)K_{t-1}^\sigma\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^\nu\right)^{\frac{\sigma}{\nu}}$$

Next, we express K_{t-1}^σ from its optimality condition

$$K_{t-1}^\sigma = \frac{R_t^k K_{t-1}}{A^\sigma\lambda_t Y_t^{1-\sigma}(1-m)\rho\left(\rho + (1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^\nu\right)^{\frac{\sigma-\nu}{\nu}}}$$

to obtain

$$\left(\frac{Y_{i,t}}{A}\right)^{\sigma} = m\left(\frac{W_{p,t}}{A^{\sigma}\lambda_t Y_t^{1-\sigma} m}\right)^{\frac{\sigma}{\sigma-1}} +$$

$$+ (1-m)\frac{R_t^k K_{t-1}}{A^{\sigma}\lambda_t Y_t^{1-\sigma}(1-m)\rho\left(\rho+(1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma-\nu}{\nu}}}\left(\rho+(1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma}{\nu}}$$

In the above expression, we plug in K_{t-1}/Y_t:

$$\left(\frac{K_{t-1}}{Y_t}\right) = \left(A^{\sigma}(R_t^k)^{-1}\lambda_t(1-m)\rho\left(\rho+(1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma-\nu}{\nu}}\right)^{\frac{1}{1-\sigma}}$$

which yields

$$\left(\frac{Y_t}{A}\right)^{\sigma} = \frac{A^{\frac{\sigma^2}{1-\sigma}}Y_t^{\sigma}}{\lambda_t^{\frac{\sigma}{\sigma-1}}}\left(m^{\frac{1}{1-\sigma}}W_{p,t}^{\frac{\sigma}{\sigma-1}} + (1-m)^{\frac{1}{1-\sigma}}\left(\rho+(1-\rho)\left(\frac{N_{w,t}}{K_{t-1}}\right)^{\nu}\right)^{\frac{\sigma}{1-\sigma}\frac{1-\nu}{\nu}}\left(\frac{\rho}{R_t^k}\right)^{\frac{\sigma}{1-\sigma}}\right)$$

Given the optimal allocation, the nominal marginal costs for intermediate goods firms for the case of having one type of capital in production are:

$$\lambda_t = \frac{1}{A}\left(m^{\frac{1}{1-\sigma}}W_{p,t}^{\frac{\sigma}{\sigma-1}} + (1-m)^{\frac{1}{1-\sigma}}\left(\rho^{\frac{1}{1-\nu}}(R_t^k)^{\frac{\nu}{\nu-1}} + (1-\rho)^{\frac{1}{1-\nu}}(W_{w,t})^{\frac{\nu}{\nu-1}}\right)^{\frac{\sigma}{1-\sigma}\frac{1-\nu}{\nu}}\right)^{\frac{\sigma-1}{\sigma}}$$

The marginal cost represents the cost, relative to each production factor, of producing an additional unit of the intermediate goods. All intermediate goods firms have the same marginal costs as they share the same technology and have the same prices of the production factors.

A.4 Labor income share (li_share)

$$\frac{x_t\left(w_{w,t}N_{w,t}+w_{p,t}N_{p,t}\right)}{Y_{i,t}} = \frac{x_t}{Y_{i,t}}\left(\frac{Y_{i,t}}{x_t}(1-\iota)\Big(\frac{Y_{i,t}}{AK^{\iota}_{s,t-1}}\Big)^{\frac{\sigma}{\iota-1}}(1-m)(1-\rho)\Big(\rho(K_{e,t-1})^{\nu}+(1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}-1}(N_{w,t})^{\nu}\right.$$

$$\left. + \frac{Y_{i,t}}{x_t}(1-\iota)\Big(\frac{Y_{i,t}}{AK^{\iota}_{s,t-1}}\Big)^{\frac{\sigma}{\iota-1}}m(N_{p,t})^{\sigma}\right)$$

$$= (1-\iota)\Big(\frac{Y_{i,t}}{AK^{\iota}_{s,t-1}}\Big)^{\frac{\sigma}{\iota-1}}\Big((1-m)(1-\rho)\Big(\rho(K_{e,t-1})^{\nu}+(1-\rho)(N_{w,t})^{\nu}\Big)^{\frac{\sigma}{\nu}-1}N^{\nu}_{w,t}+mN^{\sigma}_{p,t}\Big)$$

A.5 Long-Term Bond prices

If long-term government bonds are treated as perpetuities, we can keep track of the stock of total long-term government bonds rather than individual issues. In addition, we obtain the information about total payments that investors can receive in period t by purchasing perpetuities issued s periods ago, $b^{l,h}_{t-s}$. In this regard, the budget constraint of wealthy households where the focus is on the nominal long-term government bonds is:

$$q_{L,t}b_t^{l,h} + \ldots = \frac{1}{\pi_t}\sum_{s=1}^{\infty}\varrho^{s-1}b^{l,h}_{t-s} + \ldots$$

Following Niestroj et al. (2013), we define $\mathcal{B}^{l,h}_{t-1}$, the stock of long-term bonds in period t, as the sum of all nominal payments accumulated on past bond purchases in period t:

$$\mathcal{B}^{l,h}_{t-1} = \sum_{s=1}^{\infty} \varrho^{s-1} b^{l,h}_{t-s}$$

while corresponding $\mathcal{B}^{l,h}_{t}$ is defined as:

$$\mathcal{B}^{l,h}_{t} = \sum_{s=1}^{\infty} \varrho^{s-1} b^{l,h}_{t+1-s}$$

Given the definition of $\mathcal{B}^{l,h}_{t}$:

$$\mathcal{B}^{l,h}_{t} = \sum_{s=1}^{\infty} \varrho^{s-1} b^{l,h}_{t+1-s} = b^{l,h}_{t} + \sum_{s=2}^{\infty} \varrho^{s-1} b^{l,h}_{t+1-s} = b^{l,h}_{t} + \sum_{s=1}^{\infty} \varrho^{(s+1)-1} b^{l,h}_{t+1-(s+1)} = b^{l,h}_{t} + \varrho \sum_{s=1}^{\infty} \varrho^{s-1} b^{l,h}_{t-s},$$

we can relate the above two terms as follows:

$$\mathcal{B}^{l,h}_{t} = b^{l,h}_{t} + \varrho \mathcal{B}^{l,h}_{t-1}$$

and the budget constraint becomes:

$$q_{L,t}(\mathcal{B}^{l,h}_{t} - \varrho \mathcal{B}^{l,h}_{t-1}) + \ldots = \frac{1}{\pi_t} \mathcal{B}^{l,h}_{t-1} + \ldots$$

$$q_{L,t}\mathcal{B}^{l,h}_{t} + \ldots = \frac{1}{\pi_t}(1 + \varrho q_{L,t}) \mathcal{B}^{l,h}_{t-1} + \ldots$$

The LHS term can be written as:

$$q^{L}_{t,t}\mathcal{B}^{l,h}_{t,t} = q^{L}_{t,t}(\varrho \mathcal{B}^{l,h}_{t,t-1} + b^{l,h}_{t,t}) = q^{L}_{t,t} b^{l,h}_{t} + q^{L}_{t,t}\varrho \sum_{s=1}^{\infty} \varrho^{s-1} b^{l,h}_{t,t-s} = q^{L}_{t,t} b^{l,h}_{t,t} + \sum_{s=1}^{\infty} q^{L}_{t,t-s} b^{l,h}_{t,t-s}$$

where in the last part of the above expression we use $q^{L}_{t,t-s} = \varrho^{s} q^{L}_{t,t}$.

The RHS term can be written as:

$$\frac{1}{\pi_{t,t}}(1 + \varrho q^{L}_{t,t}) \mathcal{B}^{l,h}_{t-1,t-1} = \frac{1}{\pi_{t,t}}\left(1 + \varrho \frac{q^{L}_{t,t-s}}{\varrho^{s}}\right) \mathcal{B}^{l,h}_{t-1,t-1} =$$

$$= \frac{1}{\pi_{t,t}} \sum_{s=1}^{\infty} (1 + \varrho^{1-s} q^{L}_{t,t-s}) \varrho^{s-1} b^{l,h}_{t-1,t-s} = \frac{1}{\pi_{t,t}} \sum_{s=1}^{\infty} (\varrho^{s-1} + q^{L}_{t,t-s}) b^{l,h}_{t-1,t-s}$$

where we use $\mathcal{B}^{l,h}_{t-1,t-1} = \sum_{s=1}^{\infty} \varrho^{s-1} b^{l,h}_{t-1,t-s}$.

The budget constraint of the wealthy becomes:

$$q^{L}_{t,t} b^{l,h}_{t,t} + \sum_{s=1}^{\infty} q^{L}_{t,t-s} b^{l,h}_{t,t-s} + \ldots = \frac{1}{\pi_{t,t}} \sum_{s=1}^{\infty} (\varrho^{s-1} + q^{L}_{t,t-s}) b^{l,h}_{t-1,t-s} + \ldots$$

Following Niestroj et al. (2013), assume that nominal debt in period $t \geq 0$ is $\sum_{s=1}^{\infty} q^{L}_{t,t-s} b^{l,h}_{t,t-s} = 0$. This assumption is used to prove that the term on the RHS of the budget constraint related to the nominal long-term government bonds can be transformed into $(1 + \varrho q_{L,t}) b^{l,h}_{t-1}$:

$$\sum_{s=1}^{\infty}(\varrho^{s-1}+q^L_{t,t-s})b^{l,h}_{t-1,t-s} = (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1}+\sum_{s=2}^{\infty}(\varrho^{s-1}+q^L_{t,t-s})b^{l,h}_{t-1,t-s} =$$

$$= (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1}+\sum_{s=1}^{\infty}(\varrho^{s}+q^L_{t,t-s-1})b^{l,h}_{t-1,t-s-1} =$$

$$= (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1}+\sum_{s=1}^{\infty}\left(\frac{\varrho^s q^L_{t,t-s-1}}{q^L_{t-1,t-s-1}}\right)q^L_{t-1,t-s-1}b^{l,h}_{t-1,t-s-1} =$$

$$= (1+q^L_{t,t-1})b^{l,h}_{t-1,t-1}+\left(\frac{\varrho q^L_{t,t}+1}{q^L_{t-1,t-1}}\right)\sum_{s=1}^{\infty}q^L_{t-1,t-s-1}b^{l,h}_{t-1,t-s-1} = (1+\varrho q^L_{t,t})b^{l,h}_{t-1,t-1}$$

Given the above expression for the payments on long-term government bonds, the budget constraint of the wealthy is written in a more convenient recursive way. Long-term government bonds are treated as perpetuities that pay coupon payments of 1, ϱ, ϱ^2,... in periods $t+1$, $t+2$, $t+3$,..., respectively. This assumption implies that a payoff of one unit from holding a bond issued s periods ago is equivalent to a payoff of ϱ^s from holding a bond issued today. As in Carlstrom et al. (2017), $q_{L,t}$ is the new issue price that summarizes the prices at all maturities, while $\varrho q_{L,t}$ is the time-t price of the perpetuity issued in period $t-1$.

A.6 The aggregate resource constraint

If the budget constraint of households and government are satisfied, and the market clearing condition holds for $n-1$ markets, then Walras's law implies that the $n-th$ (goods) market will also be in equilibrium.

1. The real budget constraint of wealthy household:

$$s_w\left(c_{w,t}+q_t b^s_t+q_{L,t}b^{l,h}_t\left(1+\frac{\phi_b}{2}\left(\kappa\frac{b^s_t}{b^{l,h}_t}-1\right)^2\right)+t_{w,t}+\sum_{\varsigma\in\{s,e\}}(k_{\varsigma,t}-(1-\delta_\varsigma)k_{\varsigma,t-1})+m_t = w_{w,t}n_{w,t}+\frac{b^s_{t-1}}{\pi_t}+\right.$$

$$\left.+(1+\varrho q_{L,t})\frac{b^{l,h}_{t-1}}{\pi_t}-\sum_{\varsigma\in\{s,e\}}\frac{\phi_k}{2}\left(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}}-1\right)^2 k_{\varsigma,t}+\sum_{\varsigma\in\{s,e\}}r^k_{\varsigma,t}k_{\varsigma,t-1}+\frac{m_{t-1}}{\pi_t}+tr_{w,t}+\frac{\Pi^{int}_t}{s_w}+\frac{\Pi^r_t}{s_w}\right)$$

Real profits are distributed as dividends to wealthy household:

$$\Pi^{int}_t = \frac{Y_{int,t}}{x_t}-w_{w,t}N_{w,t}-w_{p,t}N_{p,t}-r^k_{s,t}K_{s,t-1}-r^k_{e,t}K_{e,t-1},$$

$$\Pi^r_t = \left(1-\frac{1}{x_t}-\frac{\phi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2\right)Y_t,$$

$$Y_t = Y_{int,t}$$

2. The real budget constraint of poor household:

$$s_p\Big(c_{p,t} + t_{p,t} = w_{p,t}n_{p,t} + tr_{p,t}\Big)$$

3. The consolidated government budget constraint (in aggregate real terms):

$$T_t + q_t B_t^s + q_{L,t}B_t^l + M_t - \frac{M_{t-1}}{\pi_t} - \left(q_{L,t}B_t^{l,cb} - (1+\varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t}\right) = \frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t})\frac{B_{t-1}^l}{\pi_t} + G_t + TR_t$$

The distribution of lump-sum taxes is:

$$T_t = s_w t_{w,t} + s_p t_{p,t}$$

The distribution of lump-sum transfers is:

$$TR_t = s_w tr_{w,t} + s_p tr_{p,t}$$

To derive the aggregate resource constraint, we start with the government budget constraint and express the distribution of lump-sum taxes:

$$T_t \equiv s_w t_{w,t} + s_p t_{p,t} = \frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t})\frac{B_{t-1}^l}{\pi_t} + G_t + TR_t - q_t B_t^s - q_{L,t}B_t^l - M_t + \frac{M_{t-1}}{\pi_t} + \left(q_{L,t}B_t^{l,cb} - (1+\varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t}\right)$$

Then, we express the lump-sum taxes from the household budget constraints and substitute them into the government budget constraint to obtain:

$$s_w w_{w,t} n_{w,t} + s_w \frac{b_{t-1}^s}{\pi_t} + s_w(1+\varrho q_{L,t})\frac{b_{t-1}^{l,h}}{\pi_t} - \sum_{\varsigma\in\{s,e\}} s_w \frac{\phi_k}{2}\Big(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1\Big)^2 k_{\varsigma,t} + \sum_{\varsigma\in\{s,e\}} s_w r_{\varsigma,t}^k k_{\varsigma,t-1} + s_w \frac{m_{t-1}}{\pi_t} + s_w tr_{w,t} +$$

$$+ \Pi_t^{int} + \Pi_t^r - s_w c_{w,t} - s_w q_t b_t^s - s_w q_{L,t} b_t^{l,h}\left(1 + \frac{\phi_b}{2}\Big(\kappa\frac{b_t^s}{b_t^{l,h}} - 1\Big)^2\right) - \sum_{\varsigma\in\{s,e\}} s_w(k_{\varsigma,t} - (1-\delta_\varsigma)k_{\varsigma,t-1}) - s_w m_t +$$

$$+ s_p w_{p,t} n_{p,t} + s_p tr_{p,t} - s_p c_{p,t} = \frac{B_{t-1}^s}{\pi_t} + (1+\varrho q_{L,t})\frac{B_{t-1}^l}{\pi_t} + G_t + TR_t - q_t B_t^s - q_{L,t}B_t^l - M_t + \frac{M_{t-1}}{\pi_t} +$$

$$+ \left(q_{L,t}B_t^{l,cb} - (1+\varrho q_{L,t})\frac{B_{t-1}^{l,cb}}{\pi_t}\right)$$

Aggregating terms in the previous expression and given the market clearing conditions, we obtain the expression for the aggregate resource constraint.

The following market clearing conditions are satisfied

in the labour market:

$$N_{w,t} = s_w n_{w,t}, \quad N_{p,t} = s_p n_{p,t}$$

in the capital market:

$$K_{s,t} = s_w k_{s,t}, \quad K_{e,t} = s_w k_{e,t}$$

in the bond market:

$$B_t = B_t^s + B_t^l = B_t^s + B_t^{l,h} + B_t^{l,cb}$$

and in the money market:

$$M_t = s_w m_t$$

The aggregate resource constraint or the goods market clearing is:

$$Y_t = C_t + I_t + G_t + \sum_{\varsigma \in \{s,e\}} s_w \frac{\phi_k}{2} \left(\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} - 1 \right)^2 k_{\varsigma,t} + \frac{\phi_p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t - q_{L,t} s_w b_t^{l,h} \frac{\phi_b}{2} \left(\kappa \frac{b_t^s}{b_t^{l,h}} - 1 \right)^2$$

A7. Ad-hoc real wage rigidity

As in Blanchard and Galí (2007), the real wage rigidity equation indicates that the current period real rigid wage is a function of the previous period real rigid wage and the household's marginal rate of substitution between consumption and leisure:

$$w_{k,t} = w_{k,t-1}^{\rho_w^k} \left(\varphi_{n,w} \cdot n_{k,t}^{\eta} \cdot c_{k,t} \right)^{1-\rho_w^k}$$

where ρ_w^k can be interpreted as an index of real wage rigidity for skill level $k \in \{w, p\}$ and $mrs_{k,t} = w_{k,t} = \varphi_{n,w} \cdot n_{k,t}^{\eta} \cdot c_{k,t}$ is the household's marginal rate of substitution between consumption and leisure for the case of $h = 0$ and $\sigma_c = 1$. This is a modified intertemporal optimality condition related to household's labor supply.

From the firm's side, we have the expression for the labor demand (*the market/contract wage*) as a function of the real marginal cost and marginal product of labor:

$$w_{k,t} = mc_t^r \cdot F_{n,t}^k$$

From the consumer-worker's side, we have labor supply relation (*the desired wage*):

$$w_{k,t} = mrs_{k,t} = \varphi_{n,w} \cdot n_{k,t}^{\eta} \cdot c_{k,t}, \quad \text{for } \sigma_c = 1 \text{ and } h = 0.$$

The log-linearized ad-hoc real wage rigidity for wealthy and poor households are as follows:

$$\widetilde{w}_{w,t} = \rho_w^w \cdot \widetilde{w}_{w,t-1} + (1 - \rho_w^w) \cdot \widetilde{mrs}_{w,t}, \qquad \widetilde{mrs}_{w,t} = \widetilde{w}_{w,t} = \eta \widetilde{n}_{w,t} + \widetilde{c}_{w,t}$$

$$\widetilde{w}_{p,t} = \rho_w^p \cdot \widetilde{w}_{p,t-1} + (1 - \rho_w^p) \cdot \widetilde{mrs}_{p,t}, \qquad \widetilde{mrs}_{p,t} = \widetilde{w}_{p,t} = \eta \widetilde{n}_{p,t} + \widetilde{c}_{p,t}$$

In the steady state we have

$$w_w = mrs_w, \qquad w_p = mrs_p$$

This paper focuses on the comparison of inequality measures between asymmetric real wage rigidity and flexible (symmetric) real wages. Although the response of inequality measures is somewhat dampened with the symmetric real wage setting, CSC and CD economies indicate the same qualitative results for flexible and symmetric real wage frameworks. The introduction of symmetric wage rigidity makes the poor work harder, but strong income effects have an influence on the wealthy to work even more than the poor.

The case of asymmetric real wage rigidity refers to

$$\rho_w^w = 0.8 \text{ and } \rho_w^p = 0.97,$$

while the case of flexible real wages indicates

$$\rho_w^w = 0 \text{ and } \rho_w^p = 0.$$

A.8 The elasticity of the investment to capital ratio with respect to Tobin's q

The Lagrangean function for the wealthy household's maximization problem in real terms:

$$\mathcal{L} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Bigg\{ log c_{w,\tau} + \frac{\varphi_m}{1-\chi}(m_\tau)^{1-\chi} - \varphi_{n,w}\frac{(n_{w,\tau})^{1+\eta}}{1+\eta} - \lambda_{w,\tau}\Bigg(c_{w,t} + \frac{b_t^s}{R_t} +$$

$$+ \frac{b_t^{l,h}}{R_t^L}\Bigg(1 + \frac{\phi_b}{2}\Big(\kappa\frac{b_t^s}{b_t^{l,h}} - 1\Big)^2\Bigg) + t_{w,t} + i_{s,t} + i_{e,t} + m_t - w_{w,t}n_{w,t} -$$

$$- \frac{b_{t-1}^s}{\pi_t} - \frac{b_{t-1}^{l,h}}{R_t\pi_t} - r_{s,t}^k k_{s,t-1} - r_{e,t}^k k_{e,t-1} - \frac{m_{t-1}}{\pi_t} - tr_{w,t} - \Pi_t^{int} - \Pi_t^r\Bigg) -$$

$$- \sum_{\varsigma\in\{s,e\}} Q_{\varsigma,t}\Big((k_{\varsigma,t} - (1-\delta_\varsigma)k_{\varsigma,t-1}) + \frac{\phi_k}{2}\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\Big)^2 k_{\varsigma,t} - i_{\varsigma,t}\Big)\Bigg\}$$

where $\lambda_{w,t}$ is the Lagrangean multiplier associated with the budget constraint of the wealthy household (i.e. the marginal utility of having extra consumption); $q_{\varsigma,t} = Q_{\varsigma,t}/\lambda_{w,t}$ is the Tobin's q marginal ratio with $Q_{\varsigma,t}$ being the Lagrangean multiplier associated with the law of motion of capital stock (i.e. the marginal utility from having additional installed capital). This ratio provides a measure of how much the wealthy household needs to sacrifice current consumption to have additional future capital.

The FOC for investment of the type $\varsigma \in \{s, e\}$ gives

$$-\lambda_{w,t} - Q_{\varsigma,t}\phi_k\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\Big)\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}} + Q_{\varsigma,t} = 0$$

$$\Leftrightarrow \frac{\lambda_{w,t}}{Q_{\varsigma,t}} = 1 - \phi_k\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\Big)\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}}$$

$$\Leftrightarrow \frac{1}{q_{\varsigma,t}} = 1 - \phi_k\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} - \delta_\varsigma\Big)\frac{k_{\varsigma,t}}{k_{\varsigma,t-1}}$$

$$\Leftrightarrow \frac{i_{\varsigma,t}}{k_{\varsigma,t-1}} = \Big(-\frac{1}{q_{\varsigma,t}} + 1\Big)\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}} + \delta_\varsigma$$

$$\Leftrightarrow log\Big(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\Big) = log\Big(\Big(-e^{-log(q_{\varsigma,t})} + 1\Big)\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}} + \delta_\varsigma\Big)$$

The elasticity of the investment to capital-ratio with respect to Tobin's q is

$$\frac{\partial log\left(\frac{i_{\varsigma,t}}{k_{\varsigma,t-1}}\right)}{\partial log(q_{\varsigma,t})} = \frac{1}{\left(-e^{-log(q_{\varsigma,t})}+1\right)\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}}+\delta_\varsigma}\left(-\frac{k_{\varsigma,t-1}}{\phi_k k_{\varsigma,t}}e^{-log(q_{\varsigma,t})}(-1)\right)$$

In the steady state, the above expression becomes

$$\frac{\partial log\left(\frac{i_\varsigma}{k_\varsigma}\right)}{\partial log(q_\varsigma)} = \frac{1}{\delta_\varsigma}\frac{1}{\phi_k}$$

If the elasticity of the investment to structure capital ratio with respect to Tobin's marginal q is $\varrho_{s,k} = 1/(\delta_s \cdot \phi_k) = 13.33$, then the elasticity of the investment-capital adjustment cost is:

$$\phi_k = \frac{1}{\delta_s \cdot \varrho_{s,k}} = \frac{1}{0.0142 \cdot 13.33} = 5.283$$

For simplicity, we assume that ϕ_k is the same for two types of capital, which implies $\varrho_{e,k} = 6.11$.

A.9 The log-linearised system of equations

This section specifies the log-linearised equations derived as first-order approximations around the model's nonstochastic steady state.

A.9.1 Wealthy households

1. FOC with respect to consumption:

$$\widetilde{\lambda}_{w,t} = \frac{-\sigma_c(c_w\widetilde{c}_{w,t} - hC_w\widetilde{C}_{w,t-1})}{(c_w - hC_w)}$$

2. FOC with respect to labor supply:

$$\eta\widetilde{n}_{w,t} = \widetilde{\lambda}_{w,t} + \widetilde{w}_{w,t}$$

3. FOC with respect to real money balances:

$$\widetilde{m}_t = \frac{1}{\chi}\left(-\frac{\pi}{\pi-\beta}\widetilde{\lambda}_{w,t} + \frac{\beta}{\pi-\beta}\,\mathbb{E}_t(\widetilde{\lambda}_{w,t+1} - \widetilde{\pi}_{t+1})\right)$$

4. FOC with respect to short-term bond holdings:

$$\mathbb{E}_t\,\frac{\beta}{\pi}(\widetilde{\lambda}_{w,t+1} - \widetilde{\pi}_{t+1}) = q(\widetilde{\lambda}_{w,t} + \widetilde{q}_t) + q_L\phi_b\kappa(\widetilde{b}_t^s - \widetilde{b}_t^{l,h})$$

5. FOC with respect to long-term bond holdings:

$$\widetilde{q}_{L,t} = \widetilde{\lambda}_{w,t+1} - \widetilde{\lambda}_{w,t} - \widetilde{\pi}_{t+1} + \frac{\beta\varrho}{\pi}\widetilde{q}_{L,t+1} + \phi_b(\widetilde{b}_t^s - \widetilde{b}_t^{l,h})$$

6. FOC with respect to physical capital:

$$\widetilde{\lambda}_{w,t} + \phi_k\widetilde{k}_{\varsigma,t} - \phi_k\widetilde{k}_{\varsigma,t-1} = \mathbb{E}_t\,\beta\Big((1-\delta_\varsigma)\widetilde{\lambda}_{w,t+1} + r_\varsigma^k(\widetilde{\lambda}_{w,t+1} + \widetilde{r}_{\varsigma,t+1}^k) + \phi_k\widetilde{k}_{\varsigma,t+1} - \phi_k\widetilde{k}_{\varsigma,t}\Big), \quad \text{for } \varsigma \in \{s,e\}$$

7. The price of long-term government bonds:

$$\widetilde{q}_{L,t} = -\frac{R^L}{R^L - \varrho}\widetilde{R}_t^L$$

8. The price of short-term government bonds:

$$\widetilde{q}_t = -\widetilde{R}_t$$

9. The budget constraint:

$$c_w\widetilde{c}_{w,t} + qb^s(\widetilde{q}_t + \widetilde{b}^s_t) + q_L b^{l,h}(\widetilde{q}_{L,t} + \widetilde{b}^{l,h}_t) + t_w\widetilde{t}_{w,t} + \sum_{\varsigma\in\{s,e\}} k_\varsigma(\widetilde{k}_{\varsigma,t} - (1-\delta_\varsigma)\widetilde{k}_{\varsigma,t-1}) + m\widetilde{m}_t$$

$$= w_w n_w(\widetilde{w}_{w,t} + \widetilde{n}_{w,t}) + \frac{b^s}{\pi}(\widetilde{b}^s_{t-1} - \widetilde{\pi}_t) + \frac{b^{l,h}}{\pi}(\widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t) + \varrho q_L\frac{b^{l,h}}{\pi}(\widetilde{q}_{L,t} + \widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t)$$

$$+ \sum_{\varsigma\in\{s,e\}} k_\varsigma r^k_\varsigma(\widetilde{k}_{\varsigma,t-1} + \widetilde{r}^k_{\varsigma,t}) + \frac{m}{\pi}(\widetilde{m}_{t-1} - \widetilde{\pi}_t) + tr_w\widetilde{tr}_{w,t} + \frac{\Pi^{int}}{s_w}\widetilde{\Pi}^{int}_t + \frac{\Pi^r}{s_w}\widetilde{\Pi}^r_t$$

10. Law of motion of capital:

$$\widetilde{i}^k_{\varsigma,t} = \frac{1}{\delta_\varsigma}(\widetilde{k}_{\varsigma,t} - (1-\delta_\varsigma)\widetilde{k}_{\varsigma,t-1}), \quad \text{for} \quad \varsigma \in \{s,e\}$$

A.9.2 Poor households

1. FOC with respect to consumption:

$$\widetilde{\lambda}_{p,t} = \frac{-\sigma_c(c_p\widetilde{c}_{p,t} - hC_p\widetilde{C}_{p,t-1})}{(c_p - hC_p)}$$

2. FOC with respect to labor supply:

$$\eta\widetilde{n}_{p,t} = \widetilde{\lambda}_{p,t} + \widetilde{w}_{p,t}$$

3. The budget constraint:

$$c_p\widetilde{c}_{p,t} + t_p\widetilde{t}_{p,t} = w_p n_p(\widetilde{w}_{p,t} + \widetilde{n}_{p,t}) + tr_p\widetilde{tr}_{p,t}$$

A.9.3 Intermediate goods firms

1. Production function:

$$\widetilde{Y}_{int,t} = \iota\widetilde{K}_{s,t-1} + (1-\iota)\left(mN_p^\sigma + (1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}}\right)^{-1} mN_p^\sigma\widetilde{N}_{p,t} +$$

$$+(1-\iota)\left(mN_p^\sigma + (1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}}\right)^{-1}(1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}-1}\left(\rho K_e^\nu\widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu\widetilde{N}_{w,t}\right)$$

2. FOC with respect to structure capital:

$$\widetilde{r}^k_{s,t} = \widetilde{mc}^r_t + (\iota-1)\widetilde{K}_{s,t-1} + (1-\iota)\left(mN_p^\sigma + (1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}}\right)^{-1} mN_p^\sigma\widetilde{N}_{p,t} +$$

$$+(1-\iota)\left(mN_p^\sigma + (1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}}\right)^{-1}(1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}-1}\left(\rho K_e^\nu\widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu\widetilde{N}_{w,t}\right)$$

3. FOC with respect to equipment capital:

$$\widetilde{r}^k_{e,t} = \widetilde{mc}^r_t + \iota\widetilde{K}_{s,t-1} + \left(\frac{1-\iota}{\sigma} - 1\right)\left(mN_p^\sigma + (1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}}\right)^{-1} m\sigma N_p^\sigma\widetilde{N}_{p,t} +$$

$$+\left(\frac{1-\iota}{\sigma} - 1\right)\left(mN_p^\sigma + (1-m)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}}\right)^{-1}(1-m)\sigma\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{\frac{\sigma}{\nu}-1}\left(\rho K_e^\nu\widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu\widetilde{N}_{w,t}\right) +$$

$$+(\frac{\sigma}{\nu} - 1)\left(\rho K_e^\nu + (1-\rho)N_w^\nu\right)^{-1}\nu\left(\rho K_e^\nu\widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu\widetilde{N}_{w,t}\right) + (\nu-1)\widetilde{K}_{e,t-1}$$

4. FOC with respect to demand for skilled labor:

$$\widetilde{w}_{w,t} = \widetilde{mc}_t^r + \iota \widetilde{K}_{s,t-1} + \Big(\frac{1-\iota}{\sigma} - 1\Big)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1} m\sigma N_p^\sigma \widetilde{N}_{p,t} +$$

$$+\Big(\frac{1-\iota}{\sigma} - 1\Big)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1}(1-m)\sigma\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) +$$

$$+(\frac{\sigma}{\nu} - 1)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{-1}\nu\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) + (\nu-1)\widetilde{N}_{w,t}$$

5. FOC with respect to demand for unskilled labor:

$$\widetilde{w}_{p,t} = \widetilde{mc}_t^r + \iota \widetilde{K}_{s,t-1} + \Big(\frac{1-\iota}{\sigma} - 1\Big)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1} m\sigma N_p^\sigma \widetilde{N}_{p,t} +$$

$$+\Big(\frac{1-\iota}{\sigma} - 1\Big)\left(mN_p^\sigma + (1-m)\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}}\right)^{-1}(1-m)\sigma\Big(\rho K_e^\nu + (1-\rho)N_w^\nu\Big)^{\frac{\sigma}{\nu}-1}\Big(\rho K_e^\nu \widetilde{K}_{e,t-1} + (1-\rho)N_w^\nu \widetilde{N}_{w,t}\Big) +$$

$$+(\sigma-1)\widetilde{N}_{p,t}$$

6. Skill premium:

$$s\_premium = \widetilde{w}_{w,t} - \widetilde{w}_{p,t}$$

7. Unskilled to skilled labor ratio:

$$unskilled\_ls = \widetilde{n}_{p,t} - \widetilde{n}_{w,t}$$

8. Capital to skilled labor ratio:

$$k_e\_to\_l = \widetilde{k}_{e,t-1} - \widetilde{n}_{w,t}$$

9. Relative skilled labor income share or labor income inequality:

$$LI\_inequality = \widetilde{w}_{w,t} + \widetilde{n}_{w,t} - (\widetilde{w}_{p,t} + \widetilde{n}_{p,t})$$

10. Consumption inequality:

$$C\_inequality = \widetilde{c}_{w,t} - \widetilde{c}_{p,t}$$

11. Total income inequality:

$$TI\_inequality = \widetilde{TI}_{w,t} - \widetilde{TI}_{p,t}$$

$$\widetilde{TI}_{w,t} = \frac{NLI_w}{TI_w}\widetilde{NLI}_{w,t} + \frac{LI_w}{TI_w}\widetilde{LI}_{w,t}, \quad \widetilde{TI}_{p,t} = \frac{NLI_p}{TI_p}\widetilde{NLI}_{p,t} + \frac{LI_p}{TI_p}\widetilde{LI}_{p,t}$$

$$\widetilde{NLI}_{w,t} = \frac{1}{NLI_w}\Big(b^s\widetilde{b}_t^s - \frac{b^s}{R}(\widetilde{b}_t^s - \widetilde{R}_t) + \frac{b^{l,h}}{\pi}(\widetilde{b}_{t-1}^{l,h} - \widetilde{\pi}_t) + \sum_{\varsigma\in\{s,e\}} r_\varsigma^k k_\varsigma(\widetilde{r}_{\varsigma,t}^k + \widetilde{k}_{\varsigma,t-1}) + tr_w\widetilde{tr}_{w,t} - t_w\widetilde{t}_{w,t} + \frac{\Pi^r}{s_w}\widetilde{\Pi}_t^r\Big)$$

$$\widetilde{NLI}_{p,t} = \frac{1}{NLI_p}(tr_p\widetilde{tr}_{p,t} - t_p\widetilde{t}_{p,t})$$

$$\widetilde{LI}_{w,t} = \widetilde{w}_{w,t} + \widetilde{n}_{w,t}, \quad \widetilde{LI}_{p,t} = \widetilde{w}_{p,t} + \widetilde{n}_{p,t}$$

$$TI_p = NLI_p + LI_p, \quad LI_p = w_p n_p, \quad NLI_p = tr_p - t_p$$

$$TI_w = NLI_w + LI_w, \quad LI_w = w_w n_w, \quad NLI_w = b^s - \frac{b^s}{R} + \frac{b^{l,h}}{\pi} + \sum_{\varsigma \in \{s,e\}} r^k_\varsigma k_\varsigma + tr_w - t_w + \frac{\Pi^r}{s_w}$$

12. Wealth inequality:

$$W\_inequality = \left(\frac{b^s}{\pi} + \varrho q_L \frac{b^{l,h}}{\pi} + \frac{m}{\pi} + \sum_{\varsigma \in \{s,e\}} (1-\delta_\varsigma) k_\varsigma \right)^{-1}$$

$$\left(\frac{b^s}{\pi}(\widetilde{b}^s_{t-1} - \widetilde{\pi}_t) + \varrho q_L \frac{b^{l,h}}{\pi}(\widetilde{q}_{L,t} + \widetilde{b}^{l,h}_{t-1} - \widetilde{\pi}_t) + \frac{m}{\pi}(\widetilde{m}_{t-1} - \widetilde{\pi}_t) + \sum_{\varsigma \in \{s,e\}} k_\varsigma (1-\delta_\varsigma) \widetilde{k}_{\varsigma,t-1} \right)$$

A.9.4 Final goods firms

1. The New Keynesian Phillips Curve:

$$\widetilde{\pi}_t = \frac{(\epsilon - 1)}{\phi_p} \widetilde{mc}^r_t + \beta \, \mathbb{E}_t \, \widetilde{\pi}_{t+1}$$

2. Real profit of final goods firms:

$$\widetilde{\Pi}^r_t = \widetilde{Y}_t - \frac{mc^r}{1 - mc^r} \widetilde{mc}^r_t$$

A.9.5 The aggregate resource constraint

$$Y\widetilde{Y}_t = C\widetilde{C}_t + I\widetilde{I}_t + G\widetilde{G}_t$$

A.9.6 Fiscal policy

1. Fiscal Policy Rule:

$$\widetilde{T}_t = \rho_1 \left(\frac{q_L B^l}{q_L B^l + q B^s} (\widetilde{q}_{L,t-1} + \widetilde{B}^l_{t-1}) + \frac{q B^s}{q_L B^l + q B^s} (\widetilde{q}_{t-1} + \widetilde{B}^s_{t-1}) \right)$$

2. The real government budget constraint:

$$T\widetilde{T}_t + qB^s(\widetilde{q}_t + \widetilde{B}^s_t) + q_L B^l (\widetilde{q}_{L,t} + \widetilde{B}^l_t) + M\widetilde{M}_t - \frac{M}{\pi}(\widetilde{M}_{t-1} - \widetilde{\pi}_t) -$$

$$- \left(q_L B^{l,cb} (\widetilde{q}_{L,t} + \widetilde{B}^{l,cb}_t) - \frac{B^{l,cb}}{\pi} (\widetilde{B}^{l,cb}_{t-1} - \widetilde{\pi}_t) - \varrho q_L \frac{B^{l,cb}}{\pi} (\widetilde{q}_{L,t} + \widetilde{B}^{l,cb}_{t-1} - \widetilde{\pi}_t) \right) =$$

$$= \frac{B^s}{\pi} (\widetilde{B}^s_{t-1} - \widetilde{\pi}_t) + \frac{B^l}{\pi} (\widetilde{B}^l_{t-1} - \widetilde{\pi}_t) + \varrho q_L \frac{B^l}{\pi} (\widetilde{q}_{L,t} + \widetilde{B}^l_{t-1} - \widetilde{\pi}_t) + G\widetilde{G}_t + TR\widetilde{TR}_t$$

3. The distribution of lump-sum taxes:

$$\widetilde{T}_t = s_w \widetilde{t}_{w,t} + s_p \widetilde{t}_{p,t} \text{ and } \widetilde{T}_t = \widetilde{t}_{p,t}$$

4. The distribution of lump-sum transfers:

$$\widetilde{TR}_t = \widetilde{tr}_{p,t} \text{ and } \widetilde{tr}_{w,t} = 0$$

5. The decomposition of long-term government bonds:

$$B^l \widetilde{B}^l_t = B^{l,cb} \widetilde{B}^{l,cb}_t + B^{l,h} \widetilde{B}^{l,h}_t$$

A.9.7 The exogenous process

1. Central bank (nominal) money-market rate:

$$\widetilde{R}_t = \theta_r \widetilde{R}_{t-1} + (1-\theta_r)\left[\theta_\pi \widetilde{\pi}_t + \theta_y \widetilde{Y}_t\right] + \epsilon^r_t$$

2. The supply of long-term bonds:

$$\widetilde{B}^l_t = \phi_{b,l} \widetilde{B}^l_{t-1} + \epsilon^{b,l}_t$$

3. The central bank asset purchases:

$$\widetilde{B}^{l,cb}_t = (\phi_{cb1} + \phi_{cb2})\widetilde{B}^{l,cb}_{t-1} - (\phi_{cb1}\phi_{cb2})\widetilde{B}^{l,cb}_{t-2} + \epsilon^{l,cb}_t$$

4. Government expenditure:

$$\widetilde{G}_t = \phi_g \widetilde{G}_{t-1} + \epsilon^g_t$$

5. Transfers:

$$\widetilde{TR}_t = \phi_{tr} \widetilde{TR}_{t-1} + \epsilon^{tr}_t$$

A.9.8 Aggregate variables

1. Aggregate consumption:

$$C\widetilde{C}_t = C_w \widetilde{C}_{w,t} + C_p \widetilde{C}_{p,t} = s_w c_w \widetilde{c}_{w,t} + s_p c_p \widetilde{c}_{p,t}$$

2. Labor supply of the wealthy:

$$\widetilde{N}_{w,t} = \widetilde{n}_{w,t}$$

3. Labor supply of the poor:

$$\widetilde{N}_{p,t} = \widetilde{n}_{p,t}$$

4. Aggregate capital stock:

$$\widetilde{K}_{\varsigma,t} = \widetilde{k}_{\varsigma,t}, \quad \text{for} \;\; \varsigma \in \{s, e\}$$

5. Aggregate money holdings:

$$\widetilde{M}_t = \widetilde{m}_t$$

6. Aggregate long-term bond holdings:

$$\widetilde{B}^{l,h}_t = \widetilde{b}^{l,h}_t$$

7. Aggregate short-term bond holdings:

$$\widetilde{B}^s_t = \widetilde{b}^s_t$$

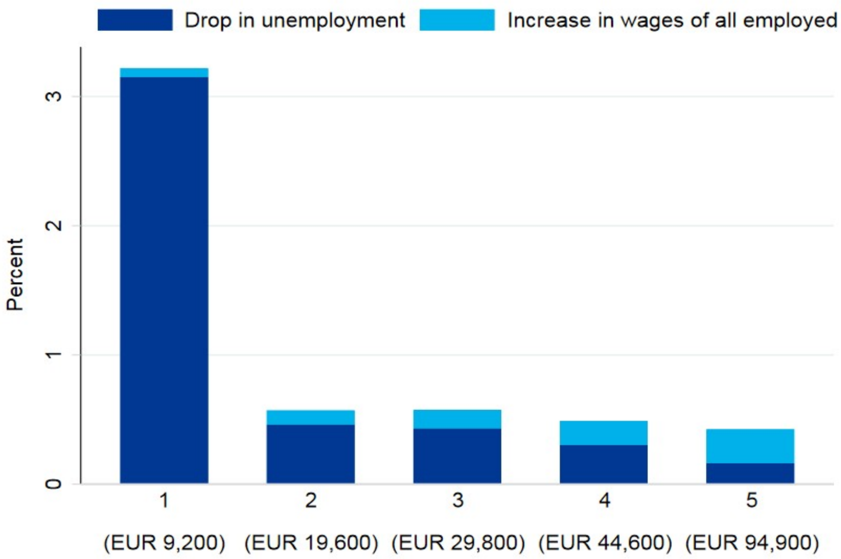

Figure 6: Decomposition of the Total Effect on Mean Income into the Extensive and the Intensive Margin

Appendix B: Empirical evidence by Lenza and Slacalek (2018)

Figure 6 shows the percentage change in mean income across income quintiles in the EA four quarters after the impact of the QE shock in the EA. Although the whole EA population benefits from the rise in employment and wages, there is the drop in labor income inequality between wealthy and poor households. The poor experience a larger increase in employment after QE, while the wealthy benefit more from the rise in wages. Labor income inequality declines due to a stronger rise in employment than that in wages.

Abstrakt

Tato studie zkoumá jak a do jaké míry kvantitativní uvolňování ECB ovlivňuje příjmy domácností a majetkovou nerovnost v eurozóně. Předchozí teoretické modely zkoumaly dynamiku měření nerovnosti prostřednictvím rozdílného přístupu domácností k finančnímu/kapitálovému trhu (kanál rebalancování portfolia), ale přitom opomíjely rozdíly na trhu práce (kanál heterogenity příjmů). Ačkoli kanál rebalancování portfolia může poskytnout pohled na majetkovou nerovnost a nepracovní příjmovou nerovnost, není tomu tak v případě pracovní (a tedy celkové) příjmové nerovnosti. Aby byla v souladu s empirickými důkazy o nerovnosti pracovních příjmů, uvažuje tato studie také o segmentovaném trhu práce na základě komplementarity kapitálu a dovedností ve výrobě a asymetrických rigiditách reálných mezd. Vezmeme-li v úvahu pouze segmentaci finančního trhu, kvantitativní výsledky naznačují pokles celkové příjmové nerovnosti, která se v průběhu času zmenšuje, zatímco majetková nerovnost zažívá nárůst, který postupně slábne. Zahrnutí segmentovaného trhu práce výrazně zmírňuje pozorovaný pokles celkové příjmové nerovnosti, přičemž nárůst majetkové nerovnosti je do značné míry zesílen. Vzhledem k možnému rozšíření mandátu ECB na otázky distribuce v budoucnu může být analýza segmentovaných trhů práce a finančních trhů pro ECB přínosnější, protože poskytuje jasnější obraz o dopadech na nerovnost.

Working Paper Series
ISSN 2788-0443

Individual researchers, as well as the on-line version of the CERGE-EI Working Papers (including their dissemination) were supported from institutional support RVO 67985998 from Economics Institute of the CAS, v. v. i.

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute of the CAS, v. v. i. (EI)
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
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Editor: Byeongju Jeong

The paper is available online at https://www.cerge-ei.cz/working-papers/.

ISBN 978-80-7343-567-7 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-689-5 (Národohospodářský ústav AV ČR, v. v. i.)