Working Paper Series 734
(ISSN 2788-0443)

Setting Interim Deadlines to Persuade

Maxim Senkov

CERGE-EI
Prague, August 2022

ISBN 978-80-7343-541-7 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-649-9 (Národohospodářský ústav AV ČR, v. v. i.)

Setting Interim Deadlines to Persuade*

Maxim Senkov †
CERGE-EI‡

August 12, 2022

Abstract

This paper studies the optimal design of self-reporting on the progress of a project by a rent-seeking agent reporting to a principal who is concerned with accomplishing the project before an exogenous deadline. The project has two stages: completing the first stage serves as a milestone and completing the second stage accomplishes the project. I show that if the project is sufficiently promising ex ante, then the agent commits to provide only the good news that the project is accomplished. If the project is not promising enough ex ante, the agent persuades the principal to start the funding by committing to provide not only good news but also the bad news that the milestone of the project has not been reached by an interim deadline.

Keywords: dynamic Bayesian persuasion, informational incentives, interim deadline, multistage project.

JEL Classification Numbers: D82, D83, G24, 031.

*Thanks to Pavel Kocourek, Jan Zápal, Inés Moreno de Barreda, Jeff Ely, Péter Esö, Margaret Meyer, Colin Stewart, Ansgar Walther, Ole Jann, Egor Starkov, Ludvig Sinander, Maxim Bakhtin, Rastislav Rehák, Vladimír Novák, Arseniy Samsonov, and the audiences at OLIGO 2022, SING17, and CMiD2022 for helpful comments.

†Email: msenkov@cerge-ei.cz. The author acknowledges financial support from the Lumina Quaeruntur fellowship (LQ300852101) of the Czech Academy of Sciences and from Czech Science Foundation (GAČR) under grant no. 20-27630S.

‡CERGE-EI, a joint workplace of Center for Economic Research and Graduate Education, Charles University and the Economics Institute of the Czech Academy of Sciences, Politických vězňů 7, P.O. Box 882, 111 21 Prague 1, Czech Republic.

1 Introduction

A characteristic feature of contractual relationships in venture finance is the existence of interim reporting deadlines. In particular, a startup will often face interim deadlines for reporting on the progress of a project to the venture capitalist. I study the economic rationale for the emergence of interim reporting deadlines in a setting in which the startup has the power to propose the terms for self-reporting to the venture capitalist.[1]

I study a game between a startup and an investor. The rent-seeking startup controls the information on the progress of the project, and commits to providing the investor with self-reports on its progress towards completion.[2] In return, the investor continuously provides funds over time and chooses when to stop funding the project. The project has two stages and evolves stochastically over time toward completion, conditional on continuous investment in it. The completion of the first stage serves as a milestone, while completion of the second stage achieves the project. The investor gets a lump-sum payoff if and only if it stops investing after the project is completed and before an exogenous project completion deadline, and the startup prefers to prolong the funding of the project for as long as possible.[3]

As the investor receives the reward only after a prolonged period of investment, it initially invests without being able to see if the investment is worthwhile. Hence, it is individually rational for the investor to start investing only if it is sufficiently optimistic regarding the future of the project. An important feature of the setting that I consider is that at the outset not only the investor, but also the startup is unable to find out if the project will bring profit to the investor or not - this can be inferred only as time goes on and some evidence is accumulated. The only tool that the startup has for persuading the investor to start investing is the promise

[1] There are multiple factors that can contribute to startups having the bargaining power to propose terms of self-reporting on the progress of a project to an investor. First, the startup could have a unique understanding of the innovative technology and of the project lifecycle. Second, the innovation proposed by the startup might have a high market value and be attractive to many potential investors in the market.

[2] In the spirit of dynamic Bayesian persuasion literature, I assume that choosing the information policy, the startup has an intertemporal commitment power.

[3] The assumption that the investor needs to have the project completed in finite time is natural in many economic settings. One possible rationale for the exogenous project completion deadline is an expected change in market conditions that renders the project unprofitable.

of future reports on the progress of the project.

Clearly, the good news about the completion of the project is valuable to the investor as it helps it to stop investing in a timely manner. Further, as evidence regarding the project accumulates over time, failure to pass the milestone in a reasonable time serves as credible evidence of the project being poor - and the investor prefers to stop investing after observing such bad news. When designing the information policy, the startup chooses optimally between the provision of these two types of evidence in order to postpone the investor's stopping decision for as long as possible.

When the project is sufficiently attractive ex ante to the investor, promises to provide information only on the completion of the project serve as a sufficiently strong incentive device to motivate the investor to start the funding at the outset.[4] Further, the future news on the completion of the project does not harm the total expected surplus generated by the interaction of the startup and investor, while the future news on the project being poor decreases the surplus that the startup can potentially extract from the investor. Accordingly, the startup commits to providing only the good news, but not the bad news on the project in the future: it *discloses the completion of the project and postpones the disclosure* in order to ensure the extraction of as much surplus as possible from the investor.

The situation changes when the project does not look promising to the investor ex ante. In that case, if the startup commits to disclosing only the completion of the project, the investor will not have the sufficient motivation to start investing in it. Thus, the startup extends the information policy to encompass not only the good news but also the bad. As in the case of the promising project, the startup discloses the project's completion and does so without any postponement, thereby fully exploiting its preferred incentive tool. In addition, the startup sets a deterministic date at which the bad news is released if the milestone of the project has not yet been reached - this date is *the interim reporting deadline*. The startup sets the interim deadline as late in time as possible in order to extract all the surplus from the investor.

The results of my research pin down the *necessary and sufficient conditions*

[4]The project is sufficiently attractive ex ante when the ratio of flow investment cost to completion benefit, normalized using the rate at which completion of one stage of the project occurs, is sufficiently low.

for the emergence of an interim self-reporting deadline when a startup offers the self-reporting conditions to the investor. The first condition is the presence of a hard exogenous project-completion deadline for the investor. The second condition is a sufficiently high cost-benefit ratio of the project, i.e., the project should be sufficiently unattractive to the investor ex ante.

2 Related literature

My paper is related to the *literature on dynamic Bayesian persuasion.* The closest paper in this strand of literature is by Ely and Szydlowski (2020). Similarly to my paper, they study the optimal persuasion of a receiver facing a lump-sum payoff to incur costly effort for a longer time. In my model, as in theirs, the sender is concerned to satisfy the beginning-of-the-game individual rationality constraint of the receiver when choosing the information policy. Further, the "leading on" information policy in Ely and Szydlowski (2020) has a similar flavor to the "postponed disclosure of completion" information policy in my paper: promises of news on completion of the project serve as an incentive device sufficient to satisfy the receiver's individual rationality constraint.

However, there are several substantial differences between Ely and Szydlowski (2020) and my paper. While in their model the state of the world is static and drawn at the beginning of the game, in my model it evolves endogenously over time, given the receiver's investment. As a result, the initial disclosure used in the "moving goalposts" policy in Ely and Szydlowski (2020) cannot provide additional incentives for the receiver in my model. The sender in my model uses another incentive device to incentivize the receiver to opt in at the initial period: she commits to an interim deadline at which she discloses that the first stage of the project is not completed.

Another closely related paper is by Orlov et al. (2020). The main similarity to my paper lies in the sender's incentive to postpone the receiver's irreversible stopping decision. The sender in their paper prefers to backload the information provision, which is in line with the properties of the optimal information policy in my paper. However, there are a number of substantial differences between our papers. In Orlov et al. (2020), the sender does not have the intertemporal commitment power. Further, the receiver obtains a payoff at each moment of time, and thus the sender does not need to persuade the receiver to opt in at the beginning of the game.

Ely (2017); Renault et al. (2017); Ball (2019) also analyze dynamic Bayesian persuasion models. However, their papers focus on persuading a receiver who chooses an action and receives a payoff at each moment of time, whereas in my paper the receiver takes an irreversible action and receives a lump-sum project completion payoff. Henry and Ottaviani (2019) consider a dynamic Bayesian persuasion model in which, similarly to my model, the receiver needs to take an irreversible decision. However, the incentives of the sender and receiver differ from my model: the receiver wants to match the static state of the world and the sender is concerned with both the receiver's action choice and with the timing of that choice.

My paper is also related to the *literature on the dynamic provision of incentives for experimentation* (Bergemann and Hege, 1998; Wolf, 2017; Curello and Sinander, 2020; Madsen, 2020). The closest paper in this strand of literature is by Wolf (2017). Similarly to my model, his model considers a multistage project and uses a discrete milestone to capture the endogenous progress of the project towards completion. However, in contrast to my paper, his paper focuses on a moral-hazard problem. In his model the investor (receiver) rather than the startup (sender) has the bargaining power to design the contract according to which the reporting happens. Green and Taylor (2016) also consider a related model and give the contracting power to the investor rather than to the startup. Similarly to my model, the optimal contract in their model demonstrates the emergence of an interim deadline.[5]

[5] In a broad sense, my paper also relates to the small strand of theoretical literature on dynamic startup-investor and startup-worker relations under information asymmetry (Kaya, 2020; Ekmekci et al., 2020). However, while these papers focus on the signaling of the type of startup, I study the provision of information by the startup on the progress of the project.

3 The model

I consider a game between an agent (she, sender) and a principal (he, receiver). Time is continuous and there is a publicly observable deadline T, $t \in [0, T]$.[6] For each t, *the principal* chooses sequentially to invest in the project ($a_t = 1$) or not ($a_t = 0$).[7] The flow cost of the investment is constant and given by c. The choice of $a_t = 0$ at some t is irreversible and induces the end of the game.

The *state of the world* at time t is captured by the number of stages of the project completed by t, x_t, and the project has two stages, $x_t \in \{0, 1, 2\}$. The state process begins at the state $x_0 = 0$ and, conditional on the continuation of the investment by the principal, it increases at a Poisson rate $\lambda > 0$. The formal definition of the state process is given in Appendix A. Information on the number of stages completed is controlled by *the agent*. Thus, when making investment decisions, the principal relies on the information provided by the agent.

There is a conflict of interest between the agent and the principal. The principal obtains the lump sum payoff for the completion of the project v if and only if the *second stage of the project* has been completed by the time of stopping, and a payoff of 0, otherwise. The agent is rent-seeking: she gets the flow payoff of c, and thus wants the principal to postpone his irreversible decision to stop as long as possible.

I study the agent's choice of information provision to the principal. The agent chooses an information policy to maximize her expected long-run payoff. I assume that the agent has the power to announce and commit to a policy. An information policy σ is a rule that for each date t and for each past history $h(t)$ specifies a probability distribution on the set of messages M. When choosing an information policy, the agent faces a rich strategy space. First, she can choose if the information on the completion of the first, or second, stage of the project will be disclosed by the policy. Second, she can choose how to disclose the completion of a stage of the project: for instance, to do so immediately or to postpone the disclosure.

The timing of the game is as follows. First, at $t = 0$, the agent publicly commits to an information policy σ. Second, at each t the principal observes the message

[6] The results for the setting without a deadline are easily obtained by considering $T \to \infty$. They are presented in Appendix F.

[7] The principal does not commit to an investment policy.

generated by the information policy and makes her investment decision. The game ends when the principal chooses to stop investing or at T, if he keeps investing until T. I assume that whenever indifferent about investing or not, the principal chooses to invest, and whenever indifferent about disclosing information or not, the agent chooses not to disclose.

Throughout the paper, I use the following intuitive notational convention with respect to the principal's time of stopping: I denote the time at which the principal stops investing by τ when it is a random variable and by S when it is deterministic. For any two times at which the principal stops investing S and τ,

$$S \wedge \tau \coloneqq \min(S, \tau).$$

4 No-information and full-information benchmarks

4.1 No-information benchmark

First, I consider the simple case when the information policy is given by σ^{NI}: the same message m is sent for all histories $h(t)$ and all dates t. Thus, the agent provides no information regarding the progress of the project. As I demonstrate, in this case the principal starts investing in the project if and only if it is sufficiently promising for the principal from the ex ante perspective and invests until a deterministic interior date.

As no news arrives, the principal bases his decision about when to stop investing on his unconditional belief regarding the completion of the second stage of the project. I denote the unconditional belief that n stages of the project were completed by t, by $p_n(t)$, i.e. $p_n(t) \coloneqq \mathrm{P}(x_t = n)$. The state of the world is fully determined by $p(t)$ given by

$$\begin{aligned} p_0(t) &= e^{-\lambda t}, \\ p_1(t) &= \lambda t e^{-\lambda t}, \\ p_2(t) &= 1 - e^{-\lambda t} - \lambda t e^{-\lambda t}. \end{aligned}$$

The principal's sequential choice of $a_t \in \{0, 1\}$ can be restated equivalently as the choice of deterministic stopping time $S^{NI} \in [0, T]$ chosen at $t = 0$.[8] Given

[8]Note that the dynamic belief system that he faces is deterministic in a sense of being fully

the principal's continuous investment, the probability of completion of the second stage of the project, $p_2(t)$, increases monotonously over time, making obtaining the payoff v more likely. However, postponing the stopping is costly.

To decide on S^{NI}, the principal trades off the flow benefits and flow costs of postponing the stopping decision, while keeping the individual rationality constraint in mind. The flow cost of postponing the stopping for Δt is given by $c \cdot \Delta t$ and the flow benefit is given by $v \cdot p_1(t) \lambda \Delta t$.[9] Thus, the necessary condition for the principal's stopping at some interior moment of time $(0 < S < T)$ is given by

$$v \cdot p_1(S) \lambda = c. \tag{1}$$

Let

$$\kappa := \frac{c}{v\lambda},$$

the ratio of the flow cost of investment c to the gross project payoff v normalized using λ, the rate at which a project stage is completed in expectation. The intuitive interpretation of κ is the *normalized cost-benefit ratio of the project*. κ is an inverse measure of how ex ante promising the project is for the principal. (1) is equivalently given by[10]

$$\underbrace{p_1(S)}_{\text{flow benefit of waiting}} = \underbrace{\kappa}_{\text{flow cost of waiting}} \tag{2}$$

and presented graphically in Figure 1. As the state process transitions monotonously from 0 to 1 and then to 2, $p_1(t)$ first increases and after some time starts to decrease. Thus, the principal has two candidate interior stopping times satisfying (2), S_1 and S_2. The principal prefers to postpone stopping to S_2, as during (S_1, S_2) the flow benefits are higher than the flow costs.

The forward-looking principal can guarantee himself a payoff of 0 if he does not start investing at $t = 0$. Thus, he will choose to start investing at $t = 0$ only

specified from $t = 0$ perspective.

[9] To observe this, note that the probability of the completing both the first and second stages within a very short time Δt is negligibly small; thus, during some Δt, the principal receives the project completion payoff v iff the first stage has already been completed.

[10] Here I WLOG express the flow benefits and flow costs of investing for the principal in different units of measurement.

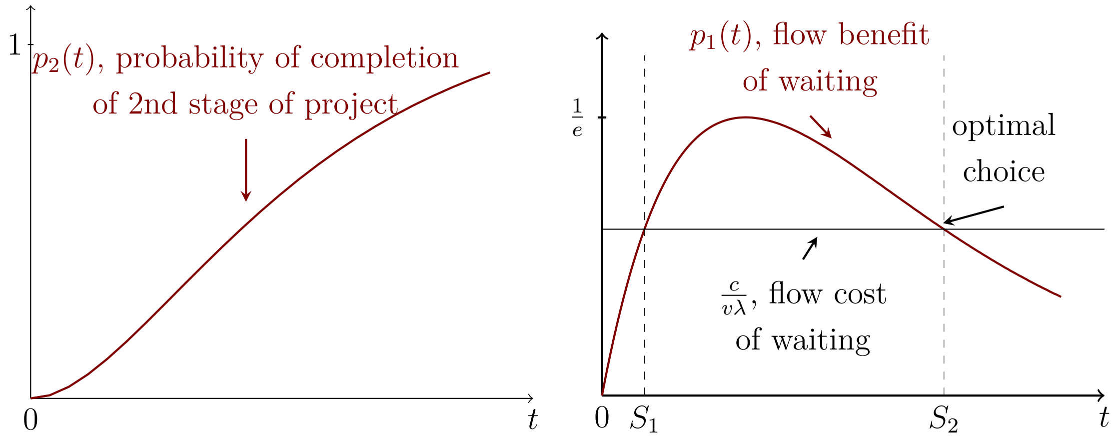

Figure 1: Principal's choice under no information:
left plot: postponing stopping increases the chance of getting a project payoff v;
right plot: principal trades off costs and benefits and optimally stops at S_2.

if his flow gains accumulated up to $T \wedge S_2$ are larger than his flow losses:

$$\int_0^{T\wedge S_2} \left(v \cdot p_1\left(s\right)\lambda - c\right) ds \geq 0. \tag{3}$$

Geometrically, the left-hand side of (3) corresponds to the difference between the shaded areas in Figure 2 that correspond to the accumulated gains and losses. The principal starts investing at $t = 0$ if, given T and λ, the normalized cost-benefit ratio κ is low enough, so that the shaded area of the accumulated gains is at least as large as that of the accumulated losses. I denote such a cutoff value of κ by $\kappa^{NI}\left(T,\lambda\right)$ and summarize the principal's choice without information in Lemma 1.

Lemma 1. *Assume no information regarding the progress of the project arrives over time. Denote the time at which the principal stops investing by S^{NI}. If $\kappa > \kappa^{NI}\left(T,\lambda\right)$, then the principal does not start investing in the project, i.e. $S^{NI} = 0$. If $\kappa \leq \kappa^{NI}\left(T,\lambda\right)$, then the principal's choice of stopping time is given by*

$$S^{NI} = \begin{cases} S_2, & \text{if } \frac{1}{\lambda} \leq T \text{ and } \kappa \geq e^{-\lambda T}\lambda T \\ T, & \text{otherwise}, \end{cases} \tag{4}$$

the closed-form expressions for S_2 and $\kappa^{NI}\left(T,\lambda\right)$ are presented in the proof in Appendix D.

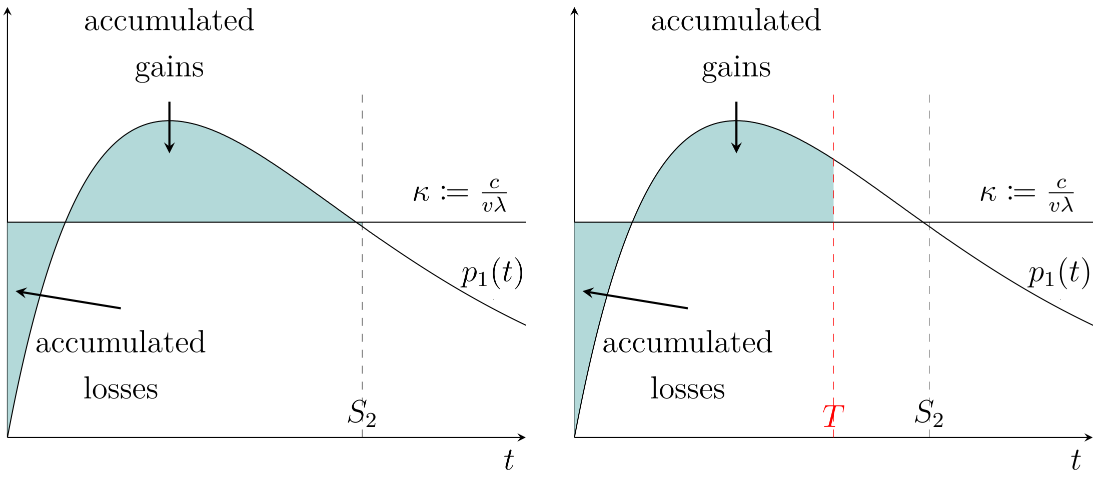

Figure 2: Principal's choice to start investing at $t = 0$ or not:
left plot: $T > S_2$; the project deadline is distant and decision-irrelevant;
right plot: $T \leq S_2$; the project deadline is close, which leads to lower expected benefits of investing.
In both plots the expected accumulated gains are higher than the losses, so the principal starts to invest at $t = 0$.

4.2 Full-information benchmark

Here, I consider the case in which the information policy is given by σ^{FI}: $M = \{m_0, m_1, m_2\}$ and the message m_n is sent for all t such that $x_t = n$, $n \in \{0, 1, 2\}$. Thus, the principal fully observes the progress of the project at each t. I characterize the cutoff level of the cost-benefit ratio below which the principal is willing to start investing. Further, I show that the principal chooses to stop when no stages of the project are completed and the project completion deadline T is sufficiently close.

At each t, the principal uses the information on the number of stages completed to decide either to stop investing or to postpone the stopping. The news on completion of the second stage of the project makes the principal stop immediately, as this way he immediately receives the project payoff v and stops incurring the costs of further investment. If only the first stage of the project is completed, the principal faces the following trade-off. The instantaneous probability that the second stage will be completed during Δt is given by $\lambda \Delta t$, which is constant over the time. Thus, the expected benefit of postponing the stopping for Δt is given by $v \cdot \lambda\Delta t$. Meanwhile, the expected cost of postponing the stopping is given by $c \cdot \Delta t$. As a result, if $\kappa \leq 1$, then the principal who knows that the first stage of the project has already been completed invests until either the completion of the second stage or until the project deadline T is reached.

Consider now the case in which the principal knows that the first stage has not yet been completed. The principal's trade-off with respect to the stopping decision is now more involved. Postponing the stopping for Δt leads to the completion of the first stage of the project with the instantaneous probability $\lambda\Delta t$. Completion of the first stage of the project at some t implies that the principal receives the continuation value of the game, conditional on having the first stage completed. I denote the continuation value of the principal at time t under full information and conditional on the completion of first stage of the project by $V_1^{FI}(t)$. This is given by[11]

$$V_1^{FI}(t) = \left(v - \frac{c}{\lambda}\right)\left(1 - e^{-\lambda(T-t)}\right). \tag{5}$$

The principal's expected benefit from postponing the stopping for Δt is given by $V_1^{FI}(t) \cdot \lambda\Delta t$ and the cost of postponing the stopping is, as before, given by $c \cdot \Delta t$. The continuation value, $V_1^{FI}(t)$, shrinks over time and approaches 0 as the project deadline T approaches. This is because the shorter the time left before the project deadline, the less likely it is that the second stage of the project will be completed before T. If at some t, and given that no stages are completed yet, the expected net benefit of investing reaches 0, it is optimal for the principal to stop at that t.[12] I denote this date by S_0^{FI} and plot it in Figure 3.

[11] See the derivation in the proof of Lemma 2 in the Appendix.

[12] If at t the expected benefit of investing becomes lower than the cost, then, after t, the net expected benefit remains negative. Thus, it is optimal for the principal to stop investing precisely at t.

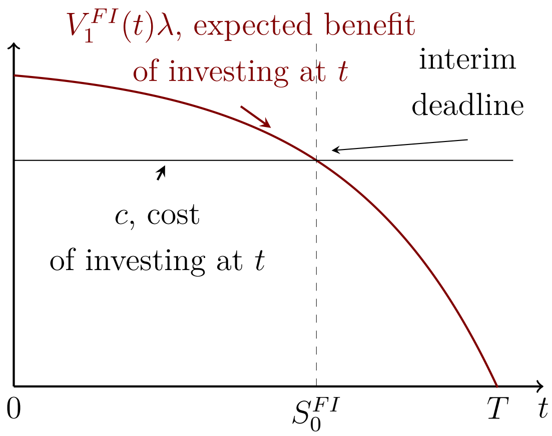

Figure 3: The principal optimally sets an interim deadline $t = S_0^{FI}$ under full information: given that the first stage of the project has not been completed by S_0^{FI}, it is optimal to stop investing at S_0^{FI}.

As the principal has an incentive to stop at S_0^{FI} only if he knows that the first stage or the milestone of the project has not been reached, the economic interpretation of S_0^{FI} is that it is *the interim deadline* that the principal sets for the project. If the milestone has not been reached by the interim deadline, then it is sufficiently unlikely that the project will be completed before the project deadline T. Thus, it is optimal for the principal to "give up" on the project and stop investing at $t = S_0^{FI}$. If the milestone is reached by the interim deadline, then the principal has an incentive not to stop investing until either the second stage is completed or T is hit.

Finally, given the plan to stop either at the interim deadline, or at the completion of the second stage of the project, it is individually rational to start investing only if the principal's expected payoff from the $t = 0$ perspective is non-negative. I denote the upper bound for the normalized cost-benefit ratio such that the principal starts investing at $t = 0$ by $\kappa^{FI}(T, \lambda)$ and summarize the principal's choice under full information in Lemma 2.

Lemma 2. *Assume that the progress of the project is fully observable at each moment in time. If $\kappa > \kappa^{FI}(T, \lambda)$, then the principal does not start investing in the project. If $\kappa \leq \kappa^{FI}(T, \lambda)$, the principal invests either until the moment at which the second stage of the project is completed, $t = \tau_2$, or until the interim* *deadline, $t = S_0^{FI}$, at which he stops if the first stage has not yet been completed. Formally, the time at which the principal stops investing is a random variable τ given by:*

$$\tau = \begin{cases} \tau_2 \wedge T, & \textit{if } x_{S_0^{FI}} \neq 0 \\ S_0^{FI}, & \textit{otherwise} \end{cases},$$

where $S_0^{FI} = T + \frac{1}{\lambda} \log\left(\frac{1-2\kappa}{1-\kappa}\right)$ and the expression for $\kappa^{FI}(T, \lambda)$ is presented in the proof in Appendix D.

Assume now that the agent chooses which information to provide to the principal. As for $\kappa > \kappa^{FI}(T, \lambda)$ the principal is not willing to start investing even under full information, there is no way in which the agent can strategically conceal the information to her benefit. In Section 5, I assume $\kappa \leq \kappa^{FI}(T, \lambda)$ and analyze how the agent can strategically provide information on the progress of the project and extract the principal's surplus.

5 Agent's choice of information provision

5.1 Investment schedules

To characterize the agent's choice of information policy, I rely on efficiency arguments that make use of investment schedules. An *investment schedule* specifies in probabilistic terms the total length of investment depending on the evolution of state process x_t. Formally, an investment schedule is given by a stopping time τ with respect to filtration $F = (\mathcal{F}_t)_{t \geq 0}$, where filtration F is generated by the stochastic process x_t.[13] Informally, τ is a random variable with support $[0, T]$ induced by the rule that specifies when to stop depending on the history of previous realizations of the number of completed stages x_t.[14] Thus, $\mathrm{P}(x_\tau = 2)$ captures the belief about two stages of the project completed by the random time of stopping in the future and $\mathrm{E}[\tau]$ captures the expected length of investment under no information about the realizations of the state process.

[13]Further technical details are provided in Appendix A.

[14]Examples of such rules include "stop 1 minute after the second stage of the project is completed" and "stop at $t = S$ if only the first stage of the project is completed by $t = S$".

Given an investment schedule τ, the long-run payoff of the agent and the principal are given, respectively, by

$$W(\tau) \coloneqq \mathrm{E}[\tau]\, c,$$
$$V(\tau) \coloneqq \mathrm{P}(x_\tau = 2)\, v - \mathrm{E}[\tau]\, c.$$

An *investment schedule τ is efficient* if, given the agent's expected payoff $W(\tau)$ and the principal's expected payoff $V(\tau)$ induced by the schedule, there is no other schedule τ' such that $W(\tau') > W(\tau)$ and $V(\tau') \geq V(\tau)$. That is, a schedule is efficient if there is no way to provide the agent with a strict improvement without harming the principal. An *investment schedule τ is feasible* if it promises the principal at least his reservation value.

To get an insight into efficient investment schedules, consider the agent's long-run payoff given an investment schedule, $W(\tau)$. This can be restated equivalently as follows:

$$W(\tau) = [W(\tau) + V(\tau)] - V(\tau) = \underbrace{\mathrm{P}(x_\tau = 2)\, v}_{\text{total surplus}} - \underbrace{[\mathrm{P}(x_\tau = 2)\, v - \mathrm{E}[\tau]\, c]}_{\text{principal's surplus}}. \tag{6}$$

Consider a schedule τ such that the stopping occurs after the completion of the second stage of the project, unless the project deadline T was hit, i.e. the schedule satisfies the condition $\tau \geq \tau_2 \wedge T$, where τ_2 is the random time at which the second stage is completed. Such a schedule leads to

$$\mathrm{P}(x_\tau = 2) = \mathrm{P}(x_T = 2). \tag{7}$$

Given a schedule τ satisfying (7), the total surplus generated achieves its upper bound and is given by $\mathrm{P}(x_T = 2)\, v$, which depends on the exogenously given project deadline T. As the agent's payoff is given by (6), if such a schedule τ is feasible, then it is also efficient. However, never stopping before the project is completed is not individually rational for the principal when the cost of funding is sufficiently high, the completion payoff is sufficiently low, or the expected time until a project stage completion is sufficiently high. In Lemma 3, I present the cutoff value of cost-benefit ratio κ denoted by $\tilde{\kappa}(T, \lambda)$, which distinguishes the case in which the investment schedule that satisfies (7) is feasible from the case in which it is not feasible. Based on this partition, when $\kappa \in (0, \tilde{\kappa}(T, \lambda)]$, I call the project *ex ante promising* for the principal.

Lemma 3. *For each (T, λ) there exists $\tilde{\kappa}(T, \lambda)$, $\kappa^{NI}(T, \lambda) < \tilde{\kappa}(T, \lambda) < \kappa^{FI}(T, \lambda)$, such that if $\kappa \leq \tilde{\kappa}(T, \lambda)$ ($\kappa > \tilde{\kappa}(T, \lambda)$) then an investment schedule τ in which stopping after $\tau_2 \wedge T$ happens with probability one is individually rational (not individually rational) for the principal.*

In the next Section, I study the case in which stopping only after the completion of the second stage of the project is individually rational for the principal.

5.2 Postponed disclosure of project completion

In this Section, I study the optimal information policy when the project is ex ante attractive for the principal. The investment schedule τ that is *optimal* for the agent is the one that provides the agent the highest payoff among all the efficient and feasible schedules. To characterize the optimal investment schedule, a useful object is the principal's surplus at some interim date t given an investment schedule τ. The principal's surplus stems from the future benefits promised by the investment schedule and corresponds to the continuation value promised by the investment schedule τ at some date t.

The continuation value promised by an investment schedule depends on the beliefs of the principal. The absence of stopping by some time t and, thus, the fact that the game continues at t serves as a source of inference for the principal. First, he forms a belief regarding the number of completed stages of the project by t, conditional on the game still continuing at t, $\mathrm{P}(x_t = n | t < \tau)$. Second, he forms a belief regarding the number of completed stages of the project at the random moment of stopping in the future, $\mathrm{P}(x_\tau = n | t < \tau)$.

Given the absence of stopping by t, the principal's expected payoff promised by the schedule is given by $\mathrm{P}(x_\tau = 2 | t < \tau) v - \mathrm{E}[\tau - t | t < \tau] c$. The principal's expected payoff from stopping at t is given by $\mathrm{P}(x_t = 2 | t < \tau)$. The principal's *continuation value at t* given the investment schedule τ is the difference between these two expected payoffs, I denote it by $V_t(\tau)$:

$$V_t(\tau) := [\mathrm{P}(x_\tau = 2 | t < \tau) - \mathrm{P}(x_t = 2 | t < \tau)] v - \mathrm{E}[\tau - t | t < \tau] c. \tag{8}$$

Clearly, an investment schedule promises zero surplus to the principal if and only if the principal's continuation value at date t given τ is precisely at 0, i.e. the principal is indifferent between continuing and stopping at t given the schedule.

The next Lemma formally presents the optimal investment schedules for the case of a promising project.

Lemma 4. *Assume $\kappa \in (0, \tilde{\kappa}(T, \lambda)]$. Iff an investment schedule τ satisfies conditions*

1. *with probability one, stopping occurs after $\tau_2 \wedge T$, unless the principal invests until T in the no-information benchmark;*
2. *$V(\tau) = \max\left(V^{NI}, 0\right)$, where V^{NI} is the principal's expected payoff in the no-information benchmark,*

then there is no other feasible investment schedule τ' such that $W(\tau') > W(\tau)$. For the case $\kappa \in (0, \kappa^{NI}(T, \lambda)]$ and $T \leq S_2$, the necessary and sufficient condition for optimality is $\tau = T$.

For an investment schedule to be optimal for the agent, it has to simultaneously be efficient, so that the total surplus is maximized, and guarantee extraction of all of the principal's surplus subject to his individual rationality constraint. Efficiency corresponds to condition 1 and surplus extraction is guaranteed by condition 2. Finally, when investment until T in the no-information benchmark is individually-rational for the principal, the investment schedule optimal for the agent corresponds to stopping at T with certainty. Given Lemma 4, it is straightforward to characterize the optimal information policy for the case of a promising project: an optimal information policy σ is one that implements the optimal investment schedule τ characterized in Lemma 4.

An investment schedule τ can be WLOG implemented using a *direct recommendation mechanism* - a simple policy which has $M = \{0, 1\}$, where $m = 1$ received at date t is a recommendation to continue investing at t for the principal and $m = 0$ received at date t is a recommendation to stop investing at t. To see the connection between an investment schedule τ and a direct recommendation mechanism implementing the schedule τ, recall that τ is induced by a stopping rule that specifies the probability of stopping based on the evolution of the state process. A direct recommendation mechanism which implements τ uses exactly the same stopping rule. The only difference is that instead of the probability of stopping, it specifies the probability of sending $m = 0$, the recommendation to stop, based on the evolution of the state process[15].

The recommendations to continue and to stop need to be incentive-compatible for the principal. Following Bergemann and Morris (2019), I call the action recommendation which is incentive-compatible for the principal an *obedient action recommendation*. As the principal does not commit to a policy at $t = 0$, he rationally updates his beliefs given an investment schedule τ and assesses his continuation value at each t. The principal does not want to stop at t whenever the continuation value is non-negative. Thus, ensuring the obedience of the recommendations not to stop at t generated by the mechanism boils down to ensuring the non-negativity of the continuation value. Further, ensuring the obedience of a recommendation to stop at t boils down to ensuring a negative continuation value under no further information provision. This gives a simple sufficient condition for an investment schedule τ to be implementable using a direct recommendation mechanism.

Lemma 5. *An investment schedule τ is implementable using a direct recommendation mechanism if*

$$V_t(\tau) \geq 0, \forall t \geq 0, \tag{9}$$

and, given a recommendation to stop at t, the principal's continuation value at t in the absence of any future information from the agent is negative for all $t \geq 0$.

I characterize the optimal information policy for the case of a promising project in Proposition 1.

Proposition 1. *Assume $\kappa \in (0, \tilde{\kappa}(T, \lambda)]$. If in the no-information benchmark the principal invests until T, then the agent chooses not to provide any information to the principal. Otherwise, the optimal information policy is a direct recommendation mechanism that has the following properties:*

1. *whenever stopping is recommended by the mechanism, the second stage of the project is already completed;*

[15] It suffices to specify the probability of sending $m = 0$ based on the evolution of the state process as the probability of sending $m = 1$ is given by the probability of the complement.

2. *the recommendation to stop is postponed so that the principal's individual rationality constraint at $t = 0$ is binding, i.e.* $V(\tau) = \max\left(V^{NI}, 0\right)$, *where* V^{NI} *is the principal's expected payoff in the no-information benchmark.*

Condition 1 presents the key feature of the optimal information policy. When the cost-benefit ratio is sufficiently low, $\kappa \leq \tilde{\kappa}(T, \lambda)$, stopping is never recommended until the project is completed. This might be puzzling: the principal values stopping both when the project is completed and when no stages of the project are completed and the project deadline T is close, but, at the equilibrium the agent recommends stopping only in the former case. The intuition behind the agent's choice is simple: a recommendation to stop when no stages of the project are completed and the project deadline T is close does indeed incentivize the agent; however, it also reduces the total surplus generated that can be extracted via the agent's control of information. Meanwhile, the recommendation to stop when the two stages of the project are completed incentivizes the principal without reducing the total surplus generated. When $\kappa \leq \tilde{\kappa}(T, \lambda)$, the project is sufficiently promising; thus, a partially informative policy that discloses only the completion of the second stage provides sufficient incentives to the principal and persuades him to start investing.[16]

While condition 1 in Proposition 1 ensures that the total surplus generated is maximized, condition 2 ensures that as much of the surplus generated is extracted as possible subject to the principal's individual rationality constraint. It is worth discussing the distinction between the cases $\kappa \in (0, \kappa^{NI}(T, \lambda)]$ and $\kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda)]$ with respect to condition 2. In the case $\kappa \in (0, \kappa^{NI}(T, \lambda)]$, the principal is willing to start investing and invests until $t = S_2$ in the no-information benchmark, as his expected payoff is non-negative at $t = 0$, $V^{NI} \geq 0$. Thus, the principal's reservation value is given by V^{NI}, and by postponing the recommendation to stop the agent can do no better than $V(\tau) = V^{NI}$. The situation is different in the case $\kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda)]$. The principal is not willing to start in the no-information benchmark as his expected payoff at $t = 0$ is negative, $V^{NI} < 0$. Thus, the principal's reservation value is given by 0, which he obtains by not starting to invest at $t = 0$. The agent optimally postpones the recommenda-

[16]The "leading on" information policy in Ely and Szydlowski (2020) is similar: the only information that the policy provides is that the task is already completed and, thus, it is time to stop investing.

tion to stop and extract the principal's surplus, so that the individual rationality constraint binds, $V(\tau) = 0$.

Not all optimal investment schedules can be implemented as a direct recommendation mechanism. For instance, consider a mechanism that satisfies both conditions from the Proposition 1 and assume it stays silent for $t \in [0, S_2)$, then at some $\hat{t} \geq S_2$ recommends stopping if the second stage is already completed, but stays completely silent at all the subsequent dates $t \in (\hat{t}, T]$. A no stopping recommendation drawn at $\hat{t}$ reveals that the state is either 0 or 1. Clearly, after sufficient time passes after $\hat{t}$, the principal would attach a high probability to the second stage already being completed and would potentially be tempted to deviate from the recommendation not to stop. The direct recommendation mechanism needs to account for this and ensure that the principal will obey the recommendations at all times. I present a simple example of the mechanism implementing an optimal investment schedule in Proposition 2.

Proposition 2. *Assume $\kappa \in (0, \kappa^{NI}(T, \lambda)]$ and $T > S_2$. The optimal mechanism does not provide a recommendation to stop during $t \in [0, S^*)$. At $t = S^*, S^* > S_2$, if the second stage of the project is already completed, then the mechanism recommends the principal to stop. If the second stage of the project is not yet completed, then the mechanism recommends the principal to stop at the moment of its completion $t = \tau_2, \tau_2 > S^*$. Formally,*

$$\tau = \begin{cases} S^*, & \text{if } x_{S^*} = 2 \\ \tau_2 \wedge T, & \text{otherwise,} \end{cases}$$

where S^* is chosen such that the principal's individual rationality constraint at $t = S_2$ is binding, i.e. $V_{S_2}(\tau) = 0$.

The direct recommendation mechanism for the case of a promising project starting from some $S^* > S_2$ generates recommendations to stop if the second stage is completed. As the recommendation to stop comes immediately at the completion of the second stage for all $t > S^*$, hearing no recommendation to stop reveals that the state is either 0 or 1. Further, as time goes on, the principal attaches a higher and higher probability to the state being 1, which ensures obedience to the recommendation not to stop. Further, the start of information provision S^* is sufficiently postponed to ensure that the principal's individual rationality constraint is binding at S_2, which, for this mechanism, satisfies condition 2 from Proposition 1.

An important feature of the mechanism from Proposition 2 is that, in contrast to the no-information benchmark, the principal does not stop at S_2, the date of stopping in the no-information benchmark, and with probability one continues to invest during $t \in [S_2, S^*)$ even though the mechanism provides absolutely no information for all $t < S^*$. This is driven by the fact that the expected benefit from stopping at some future date $t \in [S^*, T]$ and obtaining the project payoff v with certainty compensates the flow losses of investing during $t \in [S_2, S^*)$.[17]

When the project is promising, the set of optimal direct recommendation mechanisms is rich, which constitutes an important advantage for the agent: she can choose a mechanism that is easier to implement from the real-world perspective, depending on the particular environment. In the mechanism from Proposition 2, the recommendation to stop at some date t depends only on the current state of the world x_t. In an alternative mechanism, the recommendation to stop arrives with a pre-specified delay after the second stage was completed. Thus, the recommendation depends only on the past history and not on the current state of the world. In an optimal delayed disclosure mechanism, the delay becomes smaller as more time passes. I characterize such a mechanism in Appendix E.[18]

Recall that the key idea of the mechanism from Proposition 1 is that the agent postpones the disclosure of the completion of the project to extract more surplus, which makes the principal's individual rationality constraint bind. For $\kappa \in \left(\kappa^{NI}(T,\lambda), \tilde{\kappa}(T,\lambda)\right]$ the principal's payoff in the no-information benchmark is negative and the agent's optimal information policy provides the principal just enough additional value to satisfy the individual rationality constraint. The higher the cost-benefit ratio of the project κ becomes, the higher additional value the agent's information provision needs to generate to satisfy the constraint. The implication of this for the optimal information policy is presented in Lemma 6.

[17] Similarly to the "leading on" information policy in Ely and Szydlowski (2020), the promises of future disclosure of the completion of the project are used as a "carrot" to make the receiver continue investing beyond the point at which he stops in the no-information benchmark.

[18] The rich set of optimal direct recommendation mechanisms in my model encompasses both mechanisms in which the information provision depends only on the current state, similarly to the optimal mechanism in Ely and Szydlowski (2020), and the mechanisms that use delay, similarly to the delayed beep from Ely (2017).

Lemma 6. *Assume $\kappa \in (\kappa^{NI}(T,\lambda), \tilde{\kappa}(T,\lambda)]$. Given the recommendation mechanism implementing an optimal investment schedule τ, for a fixed Poisson rate λ, the expected length of investment $\mathrm{E}[\tau]$ decreases in the cost-benefit ratio κ.*

The intuition is that the higher the cost-benefit ratio of the project becomes, the sooner after the second stage of the project is completed the agent recommends the principal to stop. For the cost-benefit ratio as high as $\tilde{\kappa}(T,\lambda)$, the agent provides the recommendation to stop immediately at the date of completion of the second stage. Further, for $\kappa > \tilde{\kappa}(T,\lambda)$, the optimal information policy from Proposition 1 ceases to be feasible as it cannot satisfy the principal's individual rationality constraint. As I show in the next Section, for $\kappa > \tilde{\kappa}(T,\lambda)$, in addition to immediate disclosure of the project completion, the agent sets an interim deadline to persuade the principal.

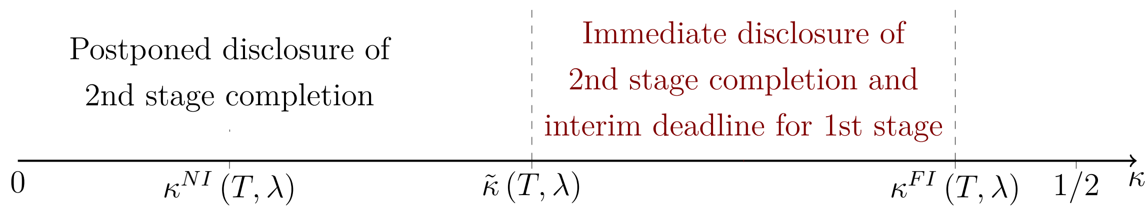

Figure 4: Depending on the cost-benefit ratio of the project, κ, the agent chooses different types of recommendation mechanisms for the principal.

5.3 Immediate disclosure of completion and an interim deadline

When $\kappa > \tilde{\kappa}(T,\lambda)$, the project is not promising for the investor and any investment schedule in which stopping occurs after $\tau_2 \wedge T$ with probability one violates the principal's individual rationality constraint at $t = 0$, and is thus not feasible. In other words, from the ex ante perspective the future reports disclosing only the completion of the project do not motivate the principal to start investing. Thus, an investment schedule that provides an individually rational expected payoff to the principal should assign a positive probability not only to stopping after the completion of the project, but also to stopping in either state 0, when no stages of the project are completed, or state 1, when only the first stage of the project is completed. I present the necessary conditions for an investment schedule to be both efficient and feasible when the project is not promising in Lemma 7.

Lemma 7. *Assume $\kappa \in (\tilde{\kappa}(T,\lambda), \kappa^{FI}(T,\lambda))$. If an investment schedule τ is efficient and yields a nonnegative principal's expected payoff, then it satisfies the conditions*

1. *whenever the second stage of the project is completed, stopping happens immediately and with probability one;*

2. *conditional on no completed stages of the project, stopping happens with a positive probability, and it never happens conditional on one completed stage.*

Stopping when only the first stage of the project is already completed is clearly inefficient. In state 1, the principal prefers to continue investing until the completion of the second stage and this principal's incentive to wait is aligned with the agent's incentive to postpone the stopping. Further, stopping in state 1 does not help overcome the problem of the violated individual rationality constraint under $\kappa > \tilde{\kappa}(T,\lambda)$. Meanwhile, assigning a positive probability to stopping when no stages are completed helps to overcome the feasibility problem, as the principal benefits from stopping at some date t when the first stage of the project is not yet completed and the project deadline T is sufficiently close. Further, the agent clearly prefers to stop after the completion of the second stage rather than in state 0 as the former does not harm the total surplus generated. Thus, a schedule that is not only feasible but also efficient assigns probability 1 to immediate stopping when the second stage is completed. The next lemma presents the investment schedule that is optimal for the agent:

Lemma 8. *Assume $\kappa \in (\tilde{\kappa}(T,\lambda), \kappa^{FI}(T,\lambda))$. Iff the investment schedule τ satisfies conditions*

1. *whenever the second stage of the project is completed, stopping happens immediately and with probability one;*

2. *at $t = S^{INT}$, stopping happens with probability one if the first stage of the project has not yet been completed,*

$$S^{INT} = \frac{1}{\lambda}\left[\gamma + \mathcal{W}_0\left(-\gamma e^{-\gamma}\right)\right], \gamma = e^{\lambda T}\frac{1-2\kappa}{1-\kappa}, \tag{10}$$

where $\mathcal{W}_0(\cdot)$ is the branch 0 of the Lambert W function, then there is no other feasible investment schedule τ' such that $W(\tau') > W(\tau)$.

The optimal schedule from Lemma 8 shows that the date at which stopping happens when no stages of the project are completed is deterministic. This is because the agent's expected payoff is concave in the date at which stopping under no completed stages occurs; thus, an investment schedule which randomizes over multiple such dates cannot yield the agent an improvement. Further, the date of stopping under no completed stages is postponed so much that the principal's individual rationality constraint is binding, which makes the investment schedule optimal for the agent. Finally, the schedule that is optimal for the agent can be WLOG implemented using a direct recommendation mechanism presented in Proposition 3.

Proposition 3. *Assume $\kappa \in (\tilde{\kappa}(T,\lambda), \kappa^{FI}(T,\lambda))$. The optimal information policy is given by a direct recommendation mechanism that generates (a) the recommendation to stop at the moment of completion of the second stage of the project, $t = \tau_2$ and (b) a conditional recommendation to stop at the interim deadline $t = S^{INT}$. At $t = S^{INT}$, stopping is recommended with certainty if the first stage of the project has not yet been completed. S^{INT} is chosen so that the principal's individual rationality constraint at $t = 0$ is binding, i.e. $V(\tau) = 0$.*

Implementation using a direct recommendation mechanism provides a natural interpretation to $t = S^{INT}$, the time of stopping when no stages are completed: S^{INT} is the *optimal interim reporting deadline chosen by the agent.* After observing the stopping recommendation at $t = S^{INT}$ the principal learns that the milestone of the project has not yet been reached and becomes sufficiently pessimistic that the project will be completed by T.

To gain a deeper insight into the optimal interim deadline chosen by the agent, I proceed to discuss the principal's and agent's incentives regarding the timing of the interim deadline. As Lemma 2 suggests, the optimal interim deadline from the principal's perspective is given by $t = S_0^{FI}$. The incentives of the agent with respect to the timing of the interim deadline, *which I denote by S_0*, are as follows. If the agent postpones the interim deadline, then two effects arise. First, the probability that stopping at the interim deadline will happen decreases. Second, the expected loss in total surplus due to stopping at the interim deadline rather than at $\tau_2 \wedge T$ decreases. The agent clearly has an incentive to postpone the interim deadline and uses her control of the information environment to postpone the deadline as much as possible so that the principal's individual rationality constraint binds. Figure 5 demonstrates the expected payoff of the principal as a function of the interim deadline S_0. It is maximized at the principal-preferred interim deadline S_0^{FI}. The agent-preferred interim deadline $S_0 = S^{INT}$ yields the principal's expected payoff of 0.

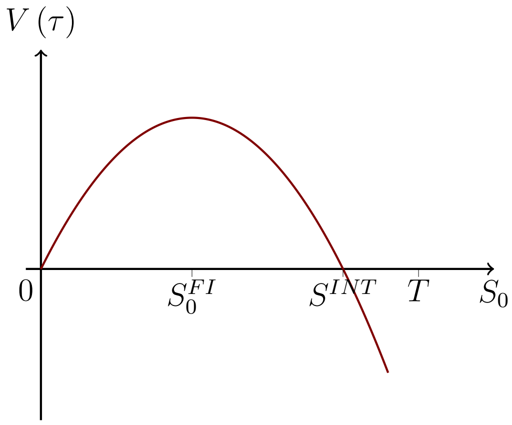

Figure 5: Principal's expected payoff, $V(\tau)$, as a function of an interim reporting deadline chosen by the agent, S_0.

I proceed by considering the comparative statics of the interim deadline. Both the agent-preferred and the principal-preferred interim deadline, S^{INT} and S_0^{FI}, respectively, increase in the exogenous deadline T. This is because less time pressure relaxes the principal's individual rationality constraint and allows the agent to postpone the deadline further in order to extract the principal's surplus. Further, the difference between the interim deadlines $S^{INT} - S_0^{FI}$ is increasing and convex in T.

As the cost-benefit ratio increases up to κ^{FI}, the agent-preferred deadline converges to the principal-preferred deadline. An increase in the cost-benefit ratio of the project makes the principal's $t = 0$ individual rationality constraint tighter.[19]

[19]This is because the increase in κ makes the principal's instantaneous benefit from waiting decrease, and the normalized instantaneous cost of waiting becomes higher.

As a result, for a higher κ, in the absence of completion of the first stage, the principal is willing to wait for a shorter time before stopping. Thus, both the interim deadline preferred by the principal S_0^{FI} and the interim deadline chosen by the agent S^{INT} are lower for a higher κ. Further, for a higher κ the agent has to choose an information policy relatively closer to the full-information benchmark to ensure that the $t = 0$ individual rationality constraint is satisfied. Hence, the agent-chosen interim deadline S^{INT} approaches S_0^{FI}, the interim deadline preferred by the principal. The comparative statics of S_0^{FI} and S^{INT} with respect to the cost-benefit ratio of the project κ are presented in Figure 6.

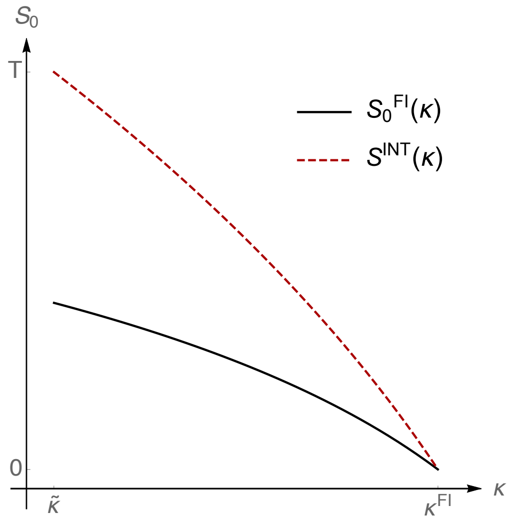

Figure 6: Interim deadline chosen by the agent S^{INT} **(dashed)** and preferred by the principal S_0^{FI} **(thick)**, as functions of the cost-benefit ratio of the project κ.

Finally, a notable feature of the optimal information policy when the project is ex ante not attractive is its uniqueness. The only optimal instrument through which the agent fine tunes the incentive provision to the principal is the choice of interim deadline, and there is a unique optimal way to set the deadline to make the principal's individual rationality constraint bind.

6 Conclusion

I consider a dynamic Bayesian persuasion model in which the agent commits to providing information to the principal with an incentive to postpone the principal's irreversible stopping of funding. I characterize the optimal information policy when the project is ex ante attractive for the principal and when it is not. I show that the interplay of a hard exogenous project completion deadline and high cost-benefit ratio leads to the emergence of an interim reporting deadline. My model has a number of limitations. A natural direction for further research is the study of implications of the agent's limited commitment. Another interesting direction includes considering a more general form of preference alignment between the agent and the principal, e.g. assuming that the agent receives not only the flat wage but also part of the profit from the project completion.

References

Ball, I. (2019). Dynamic information provision: Rewarding the past and guiding the future. *Available at SSRN 3103127*. 2

Bergemann, D. and Hege, U. (1998). Venture capital financing, moral hazard, and learning. *Journal of Banking & Finance*, 22(6-8):703–735. 2

Bergemann, D. and Morris, S. (2019). Information design: A unified perspective. *Journal of Economic Literature*, 57(1):44–95. 5.2

Curello, G. and Sinander, L. (2020). Screening for breakthroughs. *arXiv preprint arXiv:2011.10090*. 2

Ekmekci, M., Gorno, L., Maestri, L., Sun, J., and Wei, D. (2020). Learning from manipulable signals. *arXiv preprint arXiv:2007.08762*. 5

Ely, J. C. (2017). Beeps. *American Economic Review*, 107(1):31–53. 2, 18

Ely, J. C. and Szydlowski, M. (2020). Moving the goalposts. *Journal of Political Economy*, 128(2):468–506. 2, 16, 17, 18

Green, B. and Taylor, C. R. (2016). Breakthroughs, deadlines, and self-reported progress: Contracting for multistage projects. *American Economic Review*, 106(12):3660–99. 2

Henry, E. and Ottaviani, M. (2019). Research and the approval process: The organization of persuasion. *American Economic Review*, 109(3):911–55. 2

Kaya, A. (2020). Paying with information. *Available at SSRN 3661779.* 5

Madsen, E. (2020). Designing deadlines. *American Economic Review.* 2

Orlov, D., Skrzypacz, A., and Zryumov, P. (2020). Persuading the principal to wait. *Journal of Political Economy*, 128(7):2542–2578. 2

Renault, J., Solan, E., and Vieille, N. (2017). Optimal dynamic information provision. *Games and Economic Behavior*, 104:329–349. 2

Wolf, C. (2017). Informative milestones in experimentation. *University of Mannheim, Working Paper.* 2

Appendix

A The state process

The state process is given by $x_t, \forall t \in \mathbb{R}_+$. x_t is the number of stages of the project completed by t. It follows a stationary Poisson point process with arrival rate λ and state 2 being absorbing. x_t is defined on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, $t \in \mathbb{R}_+$. Its natural filtration is denoted by $F = (\mathcal{F}_t)_{t \geq 0}$.[20]

B Notational conventions

Throughout Appendices C and D, the following notational conventions are used:

1. I denote the random time at which the nth stage of the project is completed by τ_n. Formally, $\tau_n \in \mathbb{R}_+$ is a continuously distributed random variable that represents the first hitting time of $x_t = n$.

2. Consider some stopping time τ. Whenever "τ" stands as a term in an inequality, it stands for a realization of the stopping time τ and it should be read as "for each $\omega \in \Omega$ and corresponding $\tau(\omega)$".

Example 1. "$\tau_2 \wedge T \geq \tau$" should be read as "$\tau_2(\omega) \wedge T \geq \tau(\omega)$, for all $\omega \in \Omega$".

Example 2. "for all $t \in [S, \tau)$" should be read as "for all $t \in [S, \tau(\omega))$, for all $\omega \in \Omega$".

[20] The state of the world $\omega \in \Omega$ pins down the time at which the first and second stages are completed, $\tau_1(\omega)$ and $\tau_2(\omega)$.

3. The continuation value of the agent at time t, given τ, and conditional on $t < \tau$:

$$W_t(\tau) := \mathrm{E}[\tau - t | t < \tau]\, c.$$

4. The total (continuation) surplus at time t, given τ, and conditional on $t < \tau$:

$$SV_t(\tau) := W_t(\tau) + V_t(\tau).$$

5. Shorthand for posterior beliefs:

$$\begin{aligned} q_n(t) &:= \mathrm{P}(x_t = n | t < \tau), \\ r_n(t) &:= \mathrm{P}(x_\tau = n | t < \tau). \end{aligned}$$

C The principal's continuation value

Here I present the alternative formulation of the principal's continuation value (8). This helps me to study some of its properties for further use in Appendix D. The continuation value of the principal at time t and given the investment schedule τ is given by (8). Postponing the stopping for Δt brings a benefit in the form of project completion payoff v iff the second stage of the project is completed within Δt. As x_t follows the Poisson process, the probability of two increments in a very short time Δt is negligibly small. Thus, during some Δt, the principal gets the project completion payoff v iff the first stage of the project has already been completed at some earlier time. Thus, postponing the stopping for Δt brings the principal v with probability $\lambda q_1(t)\Delta t$. The second stage is not completed within Δt with the complementary probability of $1 - \lambda q_1(t)\Delta t$. The principal's continuation value is thus given by

$$\begin{aligned} V_t(\tau) &= (v\lambda q_1(t) - c)\Delta t + (1 - \lambda q_1(t)\Delta t) V_{t+\Delta t}(\tau) \\ &= v\lambda(q_1(t) - \kappa)\Delta t + (1 - \lambda q_1(t)\Delta t) V_{t+\Delta t}(\tau). \end{aligned}$$

Differentiating both sides w.r.t. Δt and considering $\lim_{\Delta t \to 0}(.)$ yields

$$0 = v\lambda(q_1(t) - \kappa) - \lambda q_1(t) V_t(\tau) + \dot{V}_t(\tau),$$

which, after rearranging becomes

$$\dot{V}_t(\tau) = \lambda q_1(t) V_t(\tau) + v\lambda(\kappa - q_1(t)). \tag{11}$$

D Proofs

Proof of Lemma 1. The beliefs regarding the number of stages of the project completed by time t, x_t, evolve according to the Poisson process. The principal's unconditional beliefs are given by $p_0(0) = 1$ and for any t such that the game still continues,

$$\begin{aligned} \dot{p}_0(t) &= -\lambda p_0(t), \\ \dot{p}_1(t) &= \lambda(p_0(t) - p_1(t)), \\ \dot{p}_2(t) &= \lambda p_1(t), \end{aligned} \tag{12}$$

where $p_0(t) = e^{-\lambda t}$ and $p_1(t) = \lambda t e^{-\lambda t}$, $p_2(t) = 1 - p_0(t) - p_1(t)$. The principal's problem is given by

$$\max_{S \in [0,T]} v \cdot p_2(S) - c \cdot S. \tag{13}$$

I start with analyzing the choice of S for the interior solution case, $S \in (0, T)$. F.O.C. for (13) is given by

$$v \cdot \dot{p}_2(S) = c, \tag{14}$$

or, equivalently, $p_1(S) = \kappa$. There are two values satisfying (14): S_1 and S_2, $S_1 < S_2$. At each $t \in (S_1, S_2)$ the principal receives a net positive payoff flow. Thus, stopping at S_1 is not optimal and the only candidate for optimal stopping is S_2.[21] Further, one can obtain the closed form expression for the interior stopping time S_2 from (14):

$$S_2 = -\frac{1}{\lambda}\mathcal{W}_{-1}(-\kappa), \tag{15}$$

where $\mathcal{W}_{-1}(x)$ denotes the negative branch of the Lambert W function. S_2 is well-defined for any $\kappa < e^{-1}$.

Thus, the solution to (13) could potentially be 0, S_2, or T. I proceed with a useful lemma.

Lemma 9. *The following is true regarding the principal's continuation value in the no-information benchmark, $V^{NI}(t)$: if $V^{NI}(t) \geq 0$, for some $t \in [0, S_2 \wedge T]$, then $V^{NI}(s) \geq 0$, for all $s \in [t, S_2 \wedge T]$.*

[21] S_1 is a local minimum of the objective.

Proof. The principal's continuation value in the no-information benchmark is given by

$$V^{NI}(t) = [p_2(T \wedge S_2) - p_2(t)]\, v - (T \wedge S_2 - t)\, c. \tag{16}$$

Further,

$$\dot{V}^{NI}(t) = v\lambda\left(\kappa - e^{-\lambda t}\lambda t\right) = v\lambda\left(\kappa - p_1(t)\right).$$

$p_1(t) \leq \kappa$ for all $t \in [0, S_1]$ and $p_1(t) \geq \kappa$ for all $t \in [S_1, S_2 \wedge T]$. Thus, $V^{NI}(t)$ increases for $t \in [0, S_1]$, decreases for $t \in [S_1, T \wedge S_2]$, and $V^{NI}(T \wedge S_2) = 0$, which establishes the result. □

Lemma 9 implies that if $V^{NI}(0) \geq 0$ and the principal chooses to opt in at $t = 0$, then $V^{NI}(t) \geq 0$, $t \in [0, S_2 \wedge T]$, i.e. he invests until $t = T \wedge S_2$. This implies that the solution to (13) is either $T \wedge S_2$ or 0.

Finally, at $t = 0$ the principal chooses to start investing or not. The condition for the principal to start investing at $t = 0$ is given by

$$V^{NI}(0) \geq 0. \tag{17}$$

To specify the set of parameters for which (17) is satisfied, I obtain the cutoff value of κ given T and λ. Such a parameterization is intuitive: κ above the cutoff level corresponds to a project with sufficiently high normalized cost-benefit ratio and implies that the investor does not opt in. I denote this cutoff by $\kappa^{NI}(T, \lambda)$. This solves (17) holding with equality. Two cases are possible.

Case 1: $T \leq S_2 \iff T \leq -\frac{1}{\lambda}\mathcal{W}_{-1}(-\kappa)$. This inequality is satisfied when either $\frac{1}{\lambda} \geq T$ or $\begin{cases} \frac{1}{\lambda} & \geq T \\ \kappa & \leq e^{-\lambda T}\lambda T. \end{cases}$ Given $T \leq S_2$, (17) holding with equality becomes

$$p_2(T)\, v - Tc = 0.$$

Solving it for κ yields $\kappa = e^{-\lambda T}\left(\frac{e^{\lambda T}-1}{\lambda T} - 1\right)$.

Case 2: $T > S_2$. This inequality is satisfied when $\frac{1}{\lambda} < T$ and $\kappa \geq e^{-\lambda T}\lambda T$. Given $T > S_2$, (17) holding with equality becomes

$$vp_2(S_2) - cS_2 = 0 \iff v\left(1 - p_0(S_2) - p_1(S_2)\right) = cS_2,$$

where (recall that $\dot{p}_2(S_2) = \frac{c}{v}$)

$$p_0(S_2) = \frac{1}{\lambda^2 S_2}\dot{p}_2(S_2) = \frac{c}{\lambda^2 S_2 v} = \frac{\kappa}{\lambda S_2}$$

and

$$p_1\left(S_2\right)=\frac{1}{\lambda}\dot{p}_2\left(S_2\right)=\frac{c}{\lambda v}=\kappa.$$

Consequently,

$$vp_2\left(S_2\right)-cS_2=v-v\cdot\kappa\left(1+\lambda S_2+\frac{1}{\lambda S_2}\right).$$

Let $y\coloneqq\lambda S_2$. Note that, by definition, $y>1$. Then $\kappa=ye^{-y}$, and so

$$\left(vp_2\left(S_2\right)-cS_2\right)/v=1-e^{-y}\left(1+y+y^2\right).$$

It follows that $V^{NI}\left(0\right)$ is nonnegative whenever $\lambda S_2\geq y_0\doteq1.79328$, which is equivalent to

$$\kappa\leq\kappa_0\doteq0.298426.$$

Finally, putting the two cases together yields

$$\kappa^{NI}\left(T,\lambda\right)=\begin{cases}\kappa_0\doteq0.298426, & \text{if }\frac{1}{\lambda}\leq T\text{ and }\kappa\geq e^{-\lambda T}\lambda T\\ e^{-\lambda T}\left(\frac{e^{\lambda T}-1}{\lambda T}-1\right), & \text{otherwise.}\end{cases}\tag{18}$$

□

Proof of Lemma 2. The principal chooses $a_t\in\{0,1\}$ sequentially given the observed realizations of $x_t\in\{0,1,2\}$. Whenever the principal observes $t=\tau_2$, he immediately chooses $a_t=0$ and gets v.

Consider the case $x_t=1,t<T$, i.e. the first stage of the project has already been completed. As x_t follows a Poisson process, in expectation it would take $\frac{1}{\lambda}$ units of time for the second stage to be completed and its completion brings the principal the value of v. Thus, the necessary and sufficient condition for the principal to invest at t when $x_t=1,t<T$ is given by

$$v-c\cdot\frac{1}{\lambda}\geq0\iff\kappa\leq1$$

Assume that $\kappa\leq1$ holds and $x_t=1$; thus, the principal chooses to invest at t. In that case, the principal invests until $\tau_2\wedge T$. To see this, recall that the only news that the principal can receive given $x_t=1,t<T$ is the completion of the second stage of the project, τ_2, which leads to immediate stopping. At each $t<\tau_2\wedge T$ such that $x_t=1$, choosing $a_t=0$ yields an instantaneous expected payoff of 0, while choosing $a_t=1$ yields an instantaneous expected payoff of $\lambda v\Delta t-c\Delta t$. Thus, $\kappa\leq1$ suffices for the principal to invest until $\tau_2\wedge T$.

Consider now the case of $x_t = 0, t < T$, i.e. no stages of the project have yet been completed. Postponing the stopping for Δt brings the instantaneous expected payoff of $V_1^{FI}(t)\,\lambda\Delta t - c\Delta t$, where $V_1^{FI}(t)$ is the principal's continuation value at time t under full information, conditional on the completion of the first stage of the project. I proceed with obtaining the expression for $V_1^{FI}(t)$. By definition, the principal gets v whenever the second stage is completed not later than T. The principal invests until $\tau_2 \wedge T$, and knows that at t the first stage of the project is already completed; thus, $V_1^{FI}(t)$ is given by

$$V_1^{FI}(t) = v\,\mathrm{P}(\tau_2 \leq T | x_t = 1) - c\,\mathrm{E}[\tau_2 \wedge T - t | x_t = 1].$$

$\tau_2 | x_t = 1$ corresponds to the time between two consecutive Poisson arrivals, and thus has exponential distribution. First, consider $\mathrm{P}(\tau_2 \leq T | x_t = 1)$:

$$\mathrm{P}(\tau_2 \leq T | x_t = 1) = 1 - e^{-\lambda(T-t)}.$$

Next, consider $\mathrm{E}[\tau_2 \wedge T - t | x_t = 1]$:

$$\begin{aligned}
&\mathrm{E}[\tau_2 \wedge T | x_t = 1] - t \\
=&\mathrm{P}(\tau_2 \leq T | x_t = 1) \int_t^T z \cdot \frac{\lambda e^{-\lambda(z-t)}}{\mathrm{P}(\tau_2 \leq T | x_t = 1)} dz + \mathrm{P}(\tau_2 > T | x_t = 1)\,T - t \\
=&\frac{1}{\lambda}\left(1 - e^{-\lambda(T-t)}\right) + t - e^{-\lambda(T-t)}T + \mathrm{P}(\tau_2 > T | x_t = 1)\,T - t \\
=&\frac{1}{\lambda}\left(1 - e^{-\lambda(T-t)}\right).
\end{aligned} \tag{19}$$

Thus,

$$\begin{aligned}
V_1^{FI}(t) &= v\left(1 - e^{-\lambda(T-t)}\right) - c\frac{1}{\lambda}\left(1 - e^{-\lambda(T-t)}\right) \\
&= \left(v - \frac{c}{\lambda}\right)\left(1 - e^{-\lambda(T-t)}\right).
\end{aligned} \tag{20}$$

From (20) one observes that $V_1^{FI}(t)$ decreases in t. If the net instantaneous benefit given by $V_1^{FI}(t)\,\lambda\Delta t - c\Delta t$ gets as low as 0 at some t, then the principal chooses to stop investing at this t. I denote the time at which the net instantaneous benefit reaches 0 by S_0^{FI}. S_0^{FI} can be obtained from $\left(\lambda V_1^{FI}\left(S_0^{FI}\right) - c\right)\Delta t = 0$. Thus,

$$S_0^{FI} = T + \frac{1}{\lambda}\log\left(\frac{1-2\kappa}{1-\kappa}\right). \tag{21}$$

The principal is willing to start investing iff at $t = 0$ the expected payoff from investing at $t = 0$ covers the costs of investing, i.e. $\left(\lambda V_1^{FI}(0) - c\right)\Delta t \geq 0$. From (21), this corresponds to $S_0^{FI} \geq 0$. I denote the upper bound on the cost-benefit ratio κ such that the principal chooses to start investing in $t = 0$ under full information by $\kappa^{FI}(T, \lambda)$, I solve $S_0^{FI} = 0$ for κ and obtain

$$\kappa^{FI}(T, \lambda) = \frac{1 - e^{-\lambda T}}{2 - e^{-\lambda T}}. \tag{22}$$

In summary, under full information, if $\kappa \leq \kappa^{FI}(T, \lambda)$, then the principal starts investing at $t = 0$. Further, he stops at S_0^{FI} if the first stage of the project has not been completed by that time. Otherwise, he proceeds to invest until $\tau_2 \wedge T$.

□

Proof of Lemma 3. Consider the recommendation mechanism immediately disclosing the completion of the second stage of the project; it is given by $\tau = \tau_2 \wedge T$. There exists such $\tilde{\kappa}(T, \lambda)$ that solves the principal's binding $t = 0$ individual rationality constraint when $\tau = \tau_2 \wedge T$:

$$V(\tau_2) = 0, \tag{23}$$

where

$$\begin{aligned} V(\tau_2) &= p_2(T)\, v - \mathrm{E}_0[\tau_2 \wedge T]\, c \\ &= v\left(1 - e^{-\lambda T} - \lambda T e^{-\lambda T}\right) - c\frac{1}{\lambda}\left(2 - 2e^{-\lambda T} - \lambda T e^{-\lambda T}\right). \end{aligned} \tag{24}$$

The solution to equation (23) is given by

$$\tilde{\kappa}(T, \lambda) = \frac{1 - e^{\lambda T} + \lambda T}{2 - 2e^{\lambda T} + \lambda T}. \tag{25}$$

Further, $\kappa > \tilde{\kappa}(T, \lambda) \Rightarrow V(\tau_2) < 0$ and $\kappa \leq \tilde{\kappa}(T, \lambda) \Rightarrow V(\tau_2) \geq 0$. □

Proof of Lemma 4. As $\kappa \in (0, \tilde{\kappa}(T, \lambda)]$, by Lemma 3 an investment schedule τ that assigns probability one to $\tau \geq \tau_2 \wedge T$ is feasible. Consider the agent's payoff:

$$W(\tau) = \underbrace{\mathrm{P}(x_\tau = 2)\, v}_{\text{total surplus}} \; - \underbrace{\left(\mathrm{P}(x_\tau = 2)\, v - \mathrm{E}[\tau]\, c\right)}_{\text{principal's surplus}}.$$

Given $\tau \geq \tau_2 \wedge T$, the total surplus generated is given by $\mathrm{P}(x_T = 2)\, v$, which is at its upper bound, determined by the exogenously given project deadline T and such τ is feasible. Thus, $\tau \geq \tau_2 \wedge T$ is necessary and sufficient for the total surplus to be maximized.

Consider the case $\kappa \in (0, \kappa^{NI}(T, \lambda)]$. From Lemma 1, the principal's reservation value is given by his payoff in the no-information benchmark, $V^{NI} = v(p_2(T) - p_2(S_2)) + cS_2 > 0$. Thus, $V(\tau) = V^{NI}$ is necessary and sufficient for all surplus to be extracted given the principal's individual rationality constraint. For an investment schedule to be optimal for the principal, it is necessary and sufficient that given τ total surplus is maximized and the principal's individual rationality constraint binds. Thus, $\tau \geq \tau_2 \wedge T$ and $V(\tau) = V^{NI}$ is necessary and sufficient for optimality.

Consider now the case of $\kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda)]$. From Lemma 1, $V^{NI} < 0$. Thus, the principal's reservation value is given by 0. Thus, $V(\tau) = 0$ is necessary and sufficient for all surplus to be extracted given the principal's individual rationality constraint. By arguments similar to the parametric case above, $\tau \geq \tau_2 \wedge T$ and $V(\tau) = 0$ is necessary and sufficient for optimality.

□

Proof of Proposition 1. Consider a direct recommendation mechanism satisfying conditions 1 and 2 from Proposition 1.

A. Optimality. It implements an investment schedule satisfying the conditions from Lemma 4. Thus, the investment schedule implemented is optimal for the agent.

B. Existence. For the case $\kappa \in (0, \kappa^{NI}(T, \lambda)]$, the existence of a mechanism satisfying conditions 1 and 2 is established in Proposition 2. For the case $\kappa \in (\kappa^{NI}(T, \lambda), \tilde{\kappa}(T, \lambda)]$, an example of τ satisfying conditions 1 and 2 is

$$\tau = \begin{cases} S^*, & \text{if } x_{S^*} = 2 \\ \tau_2 \wedge T, & \text{otherwise,} \end{cases}$$

where S^* is chosen such that the principal's individual rationality constraint at $t = 0$ is binding, i.e. $V(\tau) = 0$. The proof of existence of this mechanism can be obtained by considering the proof of Proposition 2 and imposing $S_2 = 0$ in it everywhere; thus, I omit this proof for the sake of space.

□

Proof of Lemma 5. If (9) holds, then at any t such that $m = 1$ is generated by the mechanism, the principal's continuation value (8) is non-negative; thus, it is optimal for the principal to choose $a_t = 1$. The optimality of choosing $a_t = 0$ after observing $m = 0$ is imposed in the Lemma directly, and, thus holds by construction. □

Proof of Proposition 2. I start with proving the existence of mechanism τ. For all $t \in [S_2, S^*)$, stopping never occurs, at $t = S^*$ it occurs if $x_{S^*} = 2$, and for all $t \in (S^*, T]$ it occurs at $t = \tau_2 \wedge T$. For $t \in [S^*, \tau)$, the absence of stopping induces posteriors $q_n(t)$. Further, for $t \in [S^*, \tau)$ the principal discounts future benefits from postponing stopping using the probability of the state being 2. Hence, the continuation value at $t = S_2$ can be written as

$$V_{S_2}(\tau) = v\lambda \left(\int_{S_2}^{S^*} p_1(z) - \kappa dz + \int_{S^*}^{T} (q_1(z) - \kappa)(1 - \mathrm{P}(x_z = 2))\, dz \right). \tag{26}$$

S^* satisfying τ solves equation $V_{S_2}(\tau) = 0$. Using (26), the equation can be written as

$$\int_{S_2}^{S^*} \kappa - p_1(z)\, dz = \int_{S^*}^{T} (q_1(z) - \kappa)(1 - \mathrm{P}(x_z = 2))\, dz.$$

Let $g(S) := \int_{S_2}^{S} \kappa - p_1(z)\, dz$ and $k(S) := \int_{S}^{T} (q_1(z) - \kappa)(1 - \mathrm{P}(x_z = 2))\, dz$, $S \in [S_2, T]$. $q_1(t) \geq \kappa$, for all $t \in [S^*, T]$. Thus, $g(S_2) = 0, k(S_2) > 0$. Further, $p_1(t) < \kappa$, for all $t \in (S_2, T]$. Hence, $g(T) > 0, k(T) = 0$. Finally, $p_1(t) \leq \kappa$, for all $t \in [S_2, T]$ implies that $g'(S) \geq 0$, for all $s \in [S_2, T]$, and $q_1(t) \geq \kappa$, for all $t \in [S^*, T]$ implies that $k'(S) \leq 0$, for all $s \in [S^*, T]$. Thus, by the intermediate value theorem, there exists S^* solving $V_{S_2}(\tau) = 0$.

I proceed with proving the obedience of the mechanism τ. The recommendation to stop at $t \geq S^*$ reveals the completion of the second stage of the project, which makes immediate stopping optimal for the principal. Consider now the obedience of the recommendation not to stop at all $t \leq S_2$. The principal's expected payoff for all $t \in [0, S_2)$ can be written as

$$V_t(\tau) = v(p_2(S_2) - p_2(t)) - c(S_2 - t) + V_{S_2}(\tau).$$

Given the binding individual rationality constraint, it becomes

$$V_t(\tau) = v(p_2(S_2) - p_2(t)) - c(S_2 - t), \text{ for all } t \in [0, S_2).$$

Finally, note that $V_t(\tau)$ above is equivalent to $V^{NI}(t)$ given by (16). Lemma 1 implies that given $\kappa \leq \kappa^{NI}(T,\lambda)$, $V^{NI}(0) = V(\tau) \geq 0$. Further, Lemma 9 implies that $V(\tau) \geq 0 \Rightarrow V_t(\tau) \geq 0, \forall t \in [0, S_2)$. Consider now the obedience of the recommendation not to stop for $t \in [S_2, S^*]$. As (11) suggests, the value function satisfies the following equation

$$\dot{V}_t(\tau) = \lambda p_1(t) V_t(\tau) + v\lambda(\kappa - p_1(t)). \tag{27}$$

I argue that $V_{S_2}(\tau) \geq 0 \Rightarrow V_t(\tau) \geq 0$, $\forall t \in [S_2, S^*]$. Assume that this is not true, then $\exists \hat{t}$ such that $\hat{t} := \inf\{t \in [S_2, S^*) : V_t(\tau) < 0\}$. As $V_t(\tau)$ is continuous in t, it follows that $V_{\hat{t}}(\tau) = 0$, and by the mean value theorem there must be $\bar{t} \in \left(S_2, \hat{t}\right)$ such that $\dot{V}_{\bar{t}}(\tau) \leq 0$. But this is in contradiction with the fact that $V_{\bar{t}}(\tau) \geq 0$ and (27).

Finally, I consider the obedience of the recommendation not to stop for $t \in [S^*, T]$. The absence of stopping at $t \geq S^*$ reveals that $x_t \neq 2$. Thus, $q_1(t) = \frac{p_1(t)}{p_0(t)+p_1(t)} = \frac{\lambda t}{1+\lambda t}$, $\forall t \in [S^*, \tau)$, and, thus, $\dot{q}_1(t) > 0$. Further, $q_1(S^*) > \kappa$. The continuation value $\forall t \in [S^*, \tau)$ is given by

$$V_t(\tau) = \mathrm{E}\left[\textstyle\int_t^\tau v\lambda(q_1(z) - \kappa)\,dz \,|\, t < \tau\right].$$

Thus, $V_t(\tau) \geq 0$, $\forall t \in [S^*, \tau)$.

$\square$

Proof of Lemma 6. The agent's equilibrium payoff under the mechanism from Proposition 1 is given by

$$\mathrm{E}[\tau]\, c = p_2(T)\, v.$$

Rewriting it equivalently, $\mathrm{E}[\tau] = \frac{1}{\lambda}\frac{1}{\kappa}p_2(T) \Rightarrow \partial\,\mathrm{E}[\tau]/\partial\kappa < 0$.

$\square$

Proof of Lemma 7. Lemma 3 implies that if a schedule τ assigns zero probability to stopping in states 0 and 1 then $V(\tau) < 0$ and the individual rationality constraint is violated. Thus, the necessary condition for a schedule τ to be feasible under $\kappa \in (\tilde{\kappa}(T,\lambda), \kappa^{FI}(T,\lambda))$ is that it assigns a positive probability to stopping not only in state 2, but also to stopping in either state 0 or state 1. Consider a schedule τ that assigns a positive probability to stopping in state 1. Consider an alternative schedule τ' which is induced by reallocating the probability mass of stopping in state 1 to stopping at $\tau_2 \wedge T$. Lemma 2 suggests that in state 1 the principal strictly benefits from postponing the stopping until the second stage of the project is completed. Thus, $V(\tau') > V(\tau)$. Further, under τ' the principal invests strictly longer, in expectation. Thus, $W(\tau') > W(\tau)$. Thus, for a schedule to be efficient it should not assign a positive probability to stopping in state 1.

Next, consider a schedule τ which assigns a positive probability to stopping in states 0 and 2. Assume that the stopping in state 0 happens at date S, which can be either deterministic or stochastic: if $x_S = 0$ then $\tau = S$, otherwise, $\tau \geq \tau_2 \wedge T$ and there exists $\omega \in \Omega$ such that $\tau(\omega) > \tau_2(\omega)$, i.e. with a positive probability, stopping in state 2 happens strictly after the date of transition to state 2. Assume that $V(\tau) = 0$. Consider the following investment schedule $\tilde{\tau}$: if $x_{\tilde{S}} = 0$ then $\tilde{\tau} = \tilde{S}$, $\mathrm{E}[\tilde{S}] > \mathrm{E}[S]$, otherwise, $\tilde{\tau} = \tau_2 \wedge T$, and $V(\tilde{\tau}) = 0$. Further, from (6), the agent's objective is given by

$$\begin{aligned} W(\tilde{\tau}) - W(\tau) &= (SV(\tilde{\tau}) - V(\tilde{\tau})) - (SV(\tau) - V(\tau)) \\ &= SV(\tilde{\tau}) - SV(\tau). \end{aligned}$$

The change from $\tau \geq \tau_2 \wedge T$ to $\tau = \tau_2 \wedge T$ induces no loss in total surplus as the measure of $\omega \in \Omega$ satisfying the event $\{\tau_2 \leq T\}$ is equal for both schedules. Further, the change from conditional stopping at S to conditional stopping at $\tilde{S}$ induces an increase in total surplus as $\mathrm{P}(x_{\tilde{S}} = 0) < \mathrm{P}(x_S = 0)$ and thus, in the latter case, conditional stopping happens less frequently. Hence, $SV(\tilde{\tau}) \geq SV(\tau)$. Thus, for a schedule that assigns positive probability to stopping in states 0 and 2 to be efficient, it is necessary that stopping in state 2 happens at τ_2 with probability 1.

$\square$

Proof of Lemma 8. Given Lemma 7, the space of efficient and feasible investment schedules under $\kappa \in (\tilde{\kappa}(T, \lambda), \kappa^{FI}(T, \lambda)]$ simplifies to schedules such that stopping in state 2 happens at τ_2, and also stopping in state 0 happens with positive probability. Thus, to characterize the optimal schedule under $\kappa \in (\tilde{\kappa}(T, \lambda), \kappa^{FI}(T, \lambda)]$, I need to characterize the assignment of the probability mass of stopping in state 0 that is optimal for the agent.

I directly consider the choice of the distribution of stopping time in state 0 that maximizes the agent's payoff subject to the principal's individual rationality constraint. Denote the random time at which stopping in state 0 occurs by $\rho \in \Delta([0,T])$; it is drawn at $t=0$ and publicly observable. It has distribution F_ρ on $[0,T]$ and is independent from the state process x_t, i.e. observing the realization of ρ does not provide any information on the state process other than that $x_\rho = 0$, for any realization of ρ. The formal relation between ρ and stopping time τ w.r.t. filtration generated by the state process x_t is given by

$$\mathrm{P}\left(\tau \leq t \wedge x_\rho = 0\right) = \mathrm{P}\left(\rho \leq t\right)\mathrm{P}\left(x_t = 0\right) = \mathrm{P}\left(\rho \leq t\right)e^{-\lambda t}.$$

I start with characterizing the welfare implications of stopping in state 0 for the agent's and principal's welfare. A few useful objects are $SV_{t|0}\left(\tau_2\right)$ and $V_{t|0}\left(\tau_2\right)$. $SV_{t|0}\left(\tau_2\right)$ is the time t continuation total surplus given that $x_t = 0$ at t and completion of the second stage of the project is immediately disclosed whenever it happens, $\tau = \tau_2 \wedge T$:

$$\begin{aligned} SV_{t|0}\left(\tau_2\right) &= v\,\mathrm{P}\left(\tau_2 \leq T | x_t = 0\right) \\ &= v\left[1 - e^{-\lambda(T-t)} - \lambda\left(T-t\right)e^{-\lambda(T-t)}\right]. \end{aligned} \tag{28}$$

$V_{t|0}\left(\tau_2\right)$ is the principal's time t continuation value given that $x_t = 0$ and completion of the second stage of the project is immediately disclosed, $\tau = \tau_2 \wedge T$:

$$V_{t|0}\left(\tau_2\right) = v\,\mathrm{P}\left(\tau_2 \leq T | x_t = 0\right) - c\,\mathrm{E}\left[\tau_2 \wedge T - t | x_t = 0\right],$$

where $v\,\mathrm{P}\left(\tau_2 \leq T | x_t = 0\right)$ is given by (28) and

$$\begin{aligned} &\mathrm{E}\left[\tau_2 \wedge T - t | x_t = 0\right] \\ =&\mathrm{P}\left(\tau_2 \leq T | x_t = 0\right)\int_t^T z \cdot \frac{\lambda^2\left(z-t\right)e^{-\lambda(z-t)}}{\mathrm{P}\left(\tau_2 \leq T | x_t = 0\right)}dz + \mathrm{P}\left(\tau_2 > T | x_t = 0\right)T - t \\ =&\frac{2}{\lambda} - \frac{2}{\lambda}e^{-\lambda(T-t)} - e^{-\lambda(T-t)}\left(T-t\right). \end{aligned} \tag{29}$$

I proceed with a useful lemma.

Lemma 10. *Given an investment schedule*

$$\tau = \begin{cases} \rho, & \text{if } x_\rho = 0 \\ \tau_2 \wedge T, & \text{otherwise,} \end{cases} \tag{30}$$

where ρ has the distribution F_ρ on $[0,T]$, independent of the state process x_t and drawn at $t=0$, the total surplus at date t can be written as

$$SV_t(\tau) = SV_t(\tau_2) - \mathrm{E}_{F_\rho}\left[\mathrm{P}(x_\rho = 0 | t < \tau_2 \wedge \rho) SV_{\rho|0}(\tau_2)\right],$$

and the principal's expected payoff at date t can be written as

$$V_t(\tau) = V_t(\tau_2) - \mathrm{E}_{F_\rho}\left[\mathrm{P}(x_\rho = 0 | t < \tau_2 \wedge \rho) V_{\rho|0}(\rho;\tau_2)\right],$$

for all $t \in [0, \rho \wedge \tau_2)$ for each realization of ρ.

Proof. By construction, $SV_t(\tau)$ corresponds to the expected value of the project completion payoff under stopping policy τ conditional on stopping not having happened by t, i.e. $t < \tau$. Given (30), the principal gets v either if the second stage is completed before ρ or if the first stage is completed before ρ and the second stage is completed before T. Note that when $t < \rho$, $t < \tau$ implies that the state is either 0 or 1, and, when $t \geq \rho$, $t < \tau$ implies that the state is 1. Thus,

$$SV_t(\tau) = \mathrm{E}_{F_\rho}\left[v\left(\mathrm{P}\left(\{x_\rho = 1\} \cap \{\tau_2 \leq T\} | t < \tau_2 \wedge \rho\right) + \mathrm{P}(x_\rho = 2 | t < \tau_2 \wedge \rho)\right)\right].$$

Further, for each realization of ρ,

$$\mathrm{P}\left(\{x_\rho = 1\} \cap \{\tau_2 \leq T\} | t < \tau_2 \wedge \rho\right) = \mathrm{P}(x_\rho = 1 | t < \tau_2 \wedge \rho)\,\mathrm{P}(\tau_2 \leq T | x_\rho = 1).$$

Thus,

$$SV_t(\tau) = \mathrm{E}_{F_\rho}\left[v\left(\mathrm{P}(x_\rho = 1 | t < \tau_2 \wedge \rho)\,\mathrm{P}(\tau_2 \leq T | x_\rho = 1) + \mathrm{P}(x_\rho = 2 | t < \tau_2 \wedge \rho)\right)\right]. \tag{31}$$

$SV_{\rho|0}(\tau_2)$ corresponds to the expected value of the project completion payoff when $x_\rho = 0$. In that case, v is obtained when the completion of the second stage happens not later than T. Thus, $SV_{\rho|0}(\tau_2) = \mathrm{E}_{F_\rho}\left[v\,\mathrm{P}(\tau_2 \leq T | x_\rho = 0)\right]$. Therefore,

$$\begin{aligned}
&\mathrm{E}_{F_\rho}\left[SV_t(\tau_2) - \mathrm{P}(x_\rho = 0 | t < \tau_2 \wedge \rho) SV_{\rho|0}(\tau_2)\right] \\
&= \mathrm{E}_{F_\rho}\left[\mathrm{P}(x_T = 2 | t < \tau_2 \wedge \rho)\, v - \mathrm{P}(x_\rho = 0 | t < \tau_2 \wedge \rho)\, v\,\mathrm{P}(\tau_2 \leq T | x_\rho = 0)\right] \\
&= \mathrm{E}_{F_\rho}\left[v\left(\mathrm{P}(x_T = 2 | t < \tau_2 \wedge \rho) - \mathrm{P}(x_\rho = 0 | t < \tau_2 \wedge \rho)\,\mathrm{P}(\tau_2 \leq T | x_\rho = 0)\right)\right].
\end{aligned} \tag{32}$$

Thus, given (31) and (32), to complete the proof of the first result of the Lemma 10, it suffices to show that for each realization of ρ,

$$\begin{aligned}&\mathrm{P}\left(x_T=2|t<\tau_2\wedge\rho\right)-\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\mathrm{P}\left(\tau_2\leq T|x_\rho=0\right)\\=&\mathrm{P}\left(x_\rho=2|t<\tau_2\wedge\rho\right)+\mathrm{P}\left(x_\rho=1|t<\tau_2\wedge\rho\right)\mathrm{P}\left(\tau_2\leq T|x_\rho=1\right)\end{aligned}$$

Using the full probability formula,

$$\begin{aligned}\mathrm{P}\left(x_T=2|t<\tau_2\wedge\rho\right)=&\\&\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\mathrm{P}\left(\tau_2\leq T|x_\rho=0\right)\\&+\mathrm{P}\left(x_\rho=1|t<\tau_2\wedge\rho\right)\mathrm{P}\left(\tau_2\leq T|x_\rho=1\right)\\&+\mathrm{P}\left(x_\rho=2|t<\tau_2\wedge\rho\right)\mathrm{P}\left(\tau_2\leq T|x_\rho=2\right).\end{aligned}$$

Hence,

$$SV_t\left(\tau\right)=SV_t\left(\tau_2\right)-\mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)SV_{\rho|0}\left(\tau_2\right)\right],\ \text{for all } t\in[0,\rho\wedge\tau_2). \tag{33}$$

I proceed with proving the second result of Lemma 10. First, applying (33) to $V_t\left(\tau\right)$ yields

$$\begin{aligned}V_t\left(\tau\right)&=SV_t\left(\tau\right)-\mathrm{E}_{F_\rho}\left[c\,\mathrm{E}\left[\tau|t<\tau_2\wedge\rho\right]\right]\\&=SV_t\left(\tau_2\right)-\mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)SV_{\rho|0}\left(\tau_2\right)-c\,\mathrm{E}\left[\tau|t<\tau_2\wedge\rho\right]\right].\end{aligned} \tag{34}$$

For each realization of ρ:

$$\begin{aligned}&\mathrm{E}\left[\tau|t<\tau_2\wedge\rho\right]\\=&\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\mathrm{E}\left[\tau|x_\rho=0\right]\\&+\mathrm{P}\left(x_\rho=1|t<\tau_2\wedge\rho\right)\mathrm{E}\left[\tau|x_\rho=1\right]+\mathrm{P}\left(x_\rho=2|t<\tau_2\wedge\rho\right)\mathrm{E}\left[\tau|x_\rho=2\right]\\=&\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\rho\\&+\mathrm{P}\left(x_\rho=1|t<\tau_2\wedge\rho\right)\mathrm{E}\left[\tau_2\wedge T|x_\rho=1\right]+\mathrm{P}\left(x_\rho=2|t<\tau_2\wedge\rho\right)\mathrm{E}\left[\tau_2\wedge T|x_\rho=2\right]\\=&\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\rho+\mathrm{E}\left[\tau_2\wedge T|t<\tau_2\wedge\rho\right]-\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\mathrm{E}\left[\tau_2\wedge T|x_\rho=0\right]\\=&\mathrm{E}\left[\tau_2\wedge T|t<\tau_2\wedge\rho\right]-\mathrm{P}\left(x_\rho=0|t<\tau_2\wedge\rho\right)\left(\mathrm{E}\left[\tau_2\wedge T|x_\rho=0\right]-\rho\right),\end{aligned} \tag{35}$$

where the second equality uses the full probability formula.

Plugging (35) into (34) yields

$$
\begin{aligned}
&SV_t(\tau_2) - \mathrm{E}_{F_\rho}\left[c\,\mathrm{E}\left[\tau_2 \wedge T | t < \tau_2 \wedge \rho\right]\right] \\
&- \mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0 | t < \tau_2 \wedge \rho\right)\left(SV_{\rho|0}(\tau_2) - c\,\mathrm{E}\left[\tau_2 \wedge T - \rho | x_\rho = 0\right]\right)\right] \\
=&V_t(\tau_2) - \mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0 | t < \tau_2 \wedge \rho\right) V_{\rho|0}(\tau_2)\right],
\end{aligned}
$$

for all $t \in [0, \rho \wedge \tau_2)$ for each realization of ρ.

□

I proceed to formulating the optimization problem. The agent's objective can be represented as

$$
c\,\mathrm{E}[\tau] = SV(\tau) - V(\tau).
$$

Using Lemma 10,

$$
\begin{aligned}
&SV(\tau) - V(\tau) \\
=&SV(\tau_2) - V(\tau_2) - \mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right)\left(V_{\rho|0}(\tau_2) - SV_{\rho|0}(\tau_2)\right)\right] \\
=&SV(\tau_2) - V(\tau_2) - c\,\mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right)\mathrm{E}\left[\tau_2 \wedge T - \rho | x_\rho = 0\right]\right].
\end{aligned} \tag{36}
$$

The individual rationality constraint for the principal can be expressed as

$$
V(\tau) \geq 0 \iff V(\tau_2) \geq \mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right) V_{\rho|0}(\tau_2)\right]. \tag{37}
$$

Finally, (36) yields the objective and (37) yields the individual rationality constraint for the optimal schedule choice problem[22]

$$
\begin{aligned}
&\min_{\rho \in \Delta([0,T])} \left[\mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right)\mathrm{E}\left[\tau_2 \wedge T - \rho | x_\rho = 0\right]\right]\right] \\
&\text{s.t. } \mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right)\left(c\,\mathrm{E}\left[\tau_2 \wedge T - \rho | x_\rho = 0\right] - SV_{\rho|0}(\tau_2)\right)\right] \geq -V(\tau_2).
\end{aligned} \tag{38}
$$

The Lagrangian function for the problem is

$$
\begin{aligned}
\mathcal{L} =& \mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right)\mathrm{E}\left[\tau_2 \wedge T - \rho | x_\rho = 0\right]\right] \\
&- \mu\left(\mathrm{E}_{F_\rho}\left[\mathrm{P}\left(x_\rho = 0\right)\left(c\,\mathrm{E}\left[\tau_2 \wedge T - \rho | x_\rho = 0\right] - SV_{\rho|0}(\tau_2)\right)\right] + V(\tau_2)\right),
\end{aligned}
$$

[22]The natural interpretation of the problem is the choice of stochastic stopping in state 0 to minimize the investment saved by the principal due to stopping at the interim deadline when the project stagnates.

where $\mathrm{P}(x_t = 0) = e^{-\lambda t}$, $\mathrm{E}[\tau_2 \wedge T - t | x_t = 0]$ is given by (29), $SV_{t|0}(\tau)$ is given by (28).

I obtain the F.O.C., which needs to hold for each value of ρ that has a positive probability in F_ρ:

$$e^{-\lambda T}\left(c\left(2e^{-\lambda(T-t)} - 1\right)(\mu - 1) - \mu\lambda v\left(e^{-\lambda(T-t)} - 1\right)\right) = 0. \tag{39}$$

The derivative of the left-hand side of (39) w.r.t. t is given by $e^{-\lambda t}\lambda(2c + \mu(\lambda v - 2c))$. As $\kappa^{FI}(T, \lambda) < \frac{1}{2}$, the derivative is positive. Thus, there exists at most one t that satisfies the FOC (39). Thus, the optimal stopping time in state 0, ρ, is deterministic. I denote it with S^{INT}, the interim deadline.

I proceed with characterizing the optimal S^{INT}:

$$\begin{aligned}
&\min_{S \in [0,T]} \left[\mathrm{P}(x_S = 0)\,\mathrm{E}[\tau_2 \wedge T - S | x_S = 0]\right] \\
&\text{s.t. } \mathrm{P}(x_S = 0)\left(c\,\mathrm{E}[\tau_2 \wedge T - S | x_S = 0] - SV_{S|0}(\tau_2)\right) \geq -V(\tau_2).
\end{aligned} \tag{40}$$

The system of F.O.C. is given by

$$\begin{cases}
\begin{aligned}&e^{-\lambda T}c\left(2e^{-\lambda(T-S)} - 1\right)(\mu - 1) \\ &- e^{-\lambda T}\mu\lambda v\left(e^{-\lambda(T-S)} - 1\right)\end{aligned} & \begin{aligned}&\geq 0 \text{ if } S = 0 \\ &= 0 \text{ if } S \in (0, T) \\ &\leq 0 \text{ if } S = T\end{aligned} \\
\\
\begin{aligned}&\frac{c}{\lambda}e^{-\lambda T}\left(2\left(e^{-\lambda(T-S)} - 1\right) - \lambda(T - S)\right) \\ &- ve^{-\lambda T}\left(\left(e^{-\lambda(T-S)} - 1\right) - \lambda(T - S)\right) + V(\tau_2) \geq 0\end{aligned} & = 0 \text{ if } \mu > 0.
\end{cases}$$

Assume $\mu = 0$. In this case, the first F.O.C. wrt S yields $-ce^{-\lambda T}\left(2e^{-\lambda(T-S)} - 1\right)$. The expression is negative for all $S \in (0, T)$. Thus, $\mu > 0$, and optimal S solves the binding constraint. Thus, I proceed with inspecting the corresponding equation given by

$$\begin{aligned}
&\frac{c}{\lambda}e^{-\lambda T}\left(2\left(e^{-\lambda(T-S)} - 1\right) - \lambda(T - S)\right) \\
&- ve^{-\lambda T}\left(\left(e^{-\lambda(T-S)} - 1\right) - \lambda(T - S)\right) \\
= &- V(\tau_2),
\end{aligned} \tag{41}$$

where $V(\tau_2)$ is given by (24).

The solution to (41) is given by

$$S = \frac{1}{\lambda}\left[\gamma + \mathcal{W}\left(-\gamma e^{-\gamma}\right)\right], \tag{42}$$

where $\gamma = e^{\lambda T}\frac{1-2\kappa}{1-\kappa}$ and $\mathcal{W}(.)$ denotes the Lambert W function.

Denote the 0 and -1 branches of the Lambert W function by $\mathcal{W}_0(.)$ and $\mathcal{W}_{-1}(.)$. $\kappa \in \left(0, \frac{1}{2}\right)$, thus, $\gamma > 0$. (42) depends on γ and for each $\gamma \neq 1$ corresponds to two points as the Lambert W function has two branches. The values of (42) as a function of γ are presented in Figure 7. They are given by

$$S = \begin{cases} \left(\frac{1}{\lambda}\left[\gamma + \mathcal{W}_{-1}\left(-\gamma e^{-\gamma}\right)\right], 0\right), & \text{if } \gamma < 1 \\ \left(0, \frac{1}{\lambda}\left[\gamma + \mathcal{W}_{0}\left(-\gamma e^{-\gamma}\right)\right]\right), & \text{if } \gamma > 1 \\ 0, & \text{if } \gamma = 1. \end{cases}$$

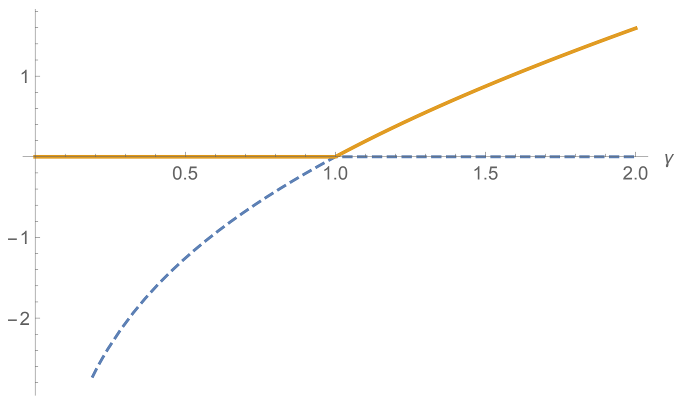

Figure 7: Roots of equation (41) as a function of the parameter γ:
root corresponding to branch 0 of the Lambert W function - **thick**;
root corresponding to branch -1 of the Lambert W function - **dashed**.

γ is decreasing in κ, and $\gamma_{|\kappa=\kappa^{FI}} = 1$. As $\kappa \leq \kappa^{FI}$, which corresponds to $\gamma \geq 1$, the solution to (41) is given by

$$S_A = 0, \qquad S_B = \tfrac{1}{\lambda}\left[\gamma + \mathcal{W}_0\left(-\gamma e^{-\gamma}\right)\right].$$

As the objective of (40) is decreasing in S and $S_B > S_A$, the solution to (40) is given by

$$S^{INT} = \frac{1}{\lambda}\left[\gamma + \mathcal{W}_0\left(-\gamma e^{-\gamma}\right)\right], \gamma = e^{\lambda T}\frac{1-2\kappa}{1-\kappa}. \tag{43}$$

Finally, I can describe the optimal schedule: τ is the stopping time such that stopping occurs either at the moment of completion of the second stage of the project or at S^{INT}, conditional on the absence of the completion of the first stage of the project, i.e.

$$\tau = \begin{cases} S^{INT}, & \text{if } x_{S^{INT}} = 0 \\ \tau_2 \wedge T, & \text{otherwise,} \end{cases} \tag{44}$$

where S^{INT} is given by (43).

□

Proof of Proposition 3. Consider a direct recommendation mechanism satisfying the conditions in Proposition 3.

A. Optimality. It implements an investment schedule satisfying the conditions from Lemma 8. Thus, the investment schedule implemented is optimal for the agent.

B. Existence. The existence of the optimal interim deadline S^{INT} is established in Lemma 8. Thus, here I proceed with demonstrating the obedience of the direct recommendation mechanism.

Lemma 11. *Assume that $\kappa \in \left(\tilde{\kappa}(T,\lambda), \kappa^{FI}(T,\lambda)\right)$ and consider a recommendation mechanism that implements τ given by (44). $V(\tau) \geq 0 \Rightarrow V_t(\tau) \geq 0$, for all $t \in (0,\tau)$.*

Proof. First, note that if the recommendation mechanism τ is given by (44), then, for $t < S^{INT}$ the absence of stopping at some t reveals that $x_t \neq 2$. Thus,

$$q_1(t) = \frac{p_1(t)}{p_1(t) + p_0(t)} = \frac{\lambda t}{1+\lambda t}, \forall t < S^{INT}.$$

Hence, $\dot{q}_1(t) > 0$, for all $t < S^{INT}$. Further, for $t \geq S^{INT}$, the absence of stopping reveals that $x_t = 1$. Thus, $q_1(t) = 1$, for all $t \geq S^{INT}$.

Writing out $V_t(\tau)$ based on (11) yields

$$\dot{V}_t(\tau) = \lambda q_1(t) V_t(\tau) + v\lambda(\kappa - q_1(t)). \tag{45}$$

$q_1(0) = 0$ and $\dot{q}_1(t) > 0$, for all $t < S^{INT}$. I define $\tilde{t}$ as the solution of $\frac{\lambda t}{1+\lambda t} = \kappa$. $q_1(t) < \kappa$, for all $t \in \left[0, \tilde{t} \wedge S^{INT}\right]$. I argue that $V(\tau) \geq 0 \Rightarrow V_t(\tau) \geq 0$, for all $t \in \left(0, \tilde{t} \wedge S^{INT}\right)$. Assume that this is not true, then $\exists \hat{t}$ such that $\hat{t} :=$

$\inf\left\{t\in\left(0,\tilde{t}\wedge S^{INT}\right):V_t(\tau)<0\right\}$. As $V_t(\tau)$ is continuous in t, it follows that $V_{\hat{t}}(\tau)=0$, and by the mean value theorem there must be $\bar{t}\in\left(0,\hat{t}\right)$ such that $\dot{V}_{\bar{t}}(\tau)\leq 0$. But this is in contradiction with the fact that $V_{\bar{t}}(\tau)\geq 0$ and 45.

Consider now $t\in[\tilde{t}\wedge S^{INT},\tau)$. The continuation value can be written as

$$V_t(\tau)=\mathrm{E}\left[\int_t^\tau v\lambda\left(q_1(z)-\kappa\right)dz\,|\,t<\tau\right]. \tag{46}$$

As $\kappa<\frac{1}{2}$ and $q_1(t)=1$, for all $t\in[S^{INT},\tau)$, it holds that $q_1(t)\geq\kappa$, $\forall t\in[\tilde{t}\wedge S^{INT},\tau)$. Thus, it can be seen from (46) that $V_t(\tau)\geq 0$, $\forall t\in[\tilde{t}\wedge S^{INT},\tau)$.

□

The obedience of recommendations not to stop follows from applying Lemma 11 since $V(\tau)=0$ holds by construction. Next, consider the obedience of recommendations to stop. If the recommendation to stop comes at some $t\neq S^{INT}$, then it reveals the completion of the second stage of the project, which makes immediate stopping optimal for the principal. If the recommendation to stop comes at $t=S^{INT}$, then it reveals that $x_{S^{INT}}=0$. As Lemma 2 suggests, if in the full-information benchmark the first stage of the project is not completed by the interim deadline S_0^{FI}, that is, $x_{S_0^{FI}}=0$, then $V_1^{FI}(t)\lambda\Delta t\leq c\Delta t$ for all $t\geq S_0^{FI}$. From (21) and (43), $S^{INT}>S_0^{FI}$, thus, $V_1^{FI}\left(S^{INT}\right)\lambda\Delta t<c\Delta t$ and the principal prefers $a_{S^{INT}}=0$ over $a_{S^{INT}}=1$. As the principal's expected payoff under the recommendation mechanism τ is at most as large as in the full-information benchmark, $V_1^{FI}\left(S^{INT}\right)\geq V_{S^{INT}|1}(\tau)$. Hence, $V_{S^{INT}|1}(\tau)\lambda\Delta t<c\Delta t$ and, given the recommendation to stop, the principal prefers $a_{S^{INT}}=0$ to $a_{S^{INT}}=1$.

□

E Disclosure of project completion with a deterministic delay

Proposition 4. *Assume $\kappa\in(0,\kappa^{NI}(T,\lambda)]$ and $T>S_2$. The optimal mechanism provides no information until $t=S_2$. At each $t\geq S_2$, it generates a recommendation to stop iff the second stage of the project was completed at date $\pi(t)$ in the past, where*

$$\pi(t)=-\frac{1}{\lambda}\left(1+\frac{1}{\lambda}\mathcal{W}_{-1}(-\frac{1}{\kappa}e^{-1-\lambda t}\lambda t)\right),$$

where $\mathcal{W}_{-1}(.)$ denotes the -1 branch of Lambert W function.

The mechanism from Proposition 4 does not recommend stopping until the second stage of the project is completed, and thus maximizes the total surplus. The mechanism makes the principal's individual rationality constraint bind, $V_{S_2}(\tau) = 0$. The absence of a stopping recommendation after $t = S_2$ induces posterior beliefs $q_1(t) = \kappa, \forall t \geq S_2$. Note that the principal's expected instantaneous payoff within Δt is given by

$$v \cdot q_1(t)\,\lambda\Delta t - c \cdot \Delta t = v\lambda\Delta t\,(q_1(t) - \kappa)\,.$$

No information is provided until S_2 and after S_2 the mechanism keeps the principal's expected instantaneous payoff precisely at 0, $\forall t \geq S_2$. As a result, the principal's continuation value is kept at 0 for all $t \in [S_2, \tau)$.

The delay is given by $t - \pi(t)$. At the beginning of the disclosure, $t = S_2$, the delay is S_2. To keep the belief regarding state 1 constant, the delay decreases for all $t \in (S_2, \tau)$.

Proof of Proposition 4. Posterior beliefs at date π induced by the disclosure of the absence of second stage completion are given by

$$\begin{aligned} q_0(\pi) &= \frac{p_0(\pi)}{p_0(\pi) + p_1(\pi)}, \\ q_1(\pi) &= \frac{p_1(\pi)}{p_0(\pi) + p_1(\pi)}. \end{aligned}$$

As no other evidence is provided during $(\pi, t]$, the beliefs evolve according to

$$\begin{aligned} q_0(s) &= \frac{e^{-\lambda s}}{1 + \lambda\pi}, \\ q_1(s) &= \frac{e^{-\lambda s}\lambda(s + \pi)}{1 + \lambda\pi}, \end{aligned}$$

where $s \geq \pi$.

The belief regarding state 1 at current date t is given by

$$q_1(t) = \frac{e^{-\lambda(t-\pi)}\lambda t}{1 + \lambda\pi}. \tag{47}$$

The dynamic of the state is the same as in the no-information benchmark until $t = S_2$. Therefore,

$$q_0(S_2) = p_0(S_2) = \frac{\kappa}{\lambda S_2} \quad \text{and} \quad q_1(S_2) = p_1(S_2) = \kappa.$$

The dynamics for $t \geq S_2$ then is $q_1(t) = \kappa$, $\dot{q}_1(t) = 0$. Solving from (47),

$$\pi = -\frac{1}{\lambda}\left(1 + \frac{1}{\lambda}\mathcal{W}_{-1}(-\frac{1}{\kappa}e^{-1-\lambda t}\lambda t)\right).$$

The recommendation mechanism τ is obedient. $\tau \geq \tau_2 \wedge T$ implies that the recommendation to stop comes only if the second stage of the project has already been completed, and thus immediate stopping is clearly optimal for the principal. The recommendation not to stop is also obedient. $V_t(\tau) \geq 0, \forall t \in [0, S_2)$ is formally demonstrated in the proof of obedience for Proposition 2. I proceed by showing that $V_t(\tau) = 0$, $\forall t \in [S_2, \tau)$. Writing out $V_t(\tau)$ in the recursive form yields

$$\begin{aligned} V_t(\tau) &= (v\lambda q_1(t) - c)\,\Delta t + (1 - \lambda q_1(t)\,\Delta t)\,V_{t+\Delta t}(\tau) \\ &= v\lambda(q_1(t) - \kappa)\,\Delta t + (1 - \lambda q_1(t)\,\Delta t)\,V_{t+\Delta t}(\tau). \end{aligned}$$

As $q_1(t) = \kappa$, $\forall t \in [S_2, \tau)$, it becomes

$$V_t(\tau) = (1 - \lambda q_1(t)\,\Delta t)\,V_{t+\Delta t}(\tau), \forall t \in [S_2, \tau).$$

Differentiating both sides w.r.t. Δt yields

$$0 = -\lambda q_1(t)\,V_{t+\Delta t}(\tau) + \dot{V}_{t+\Delta t}(\tau).$$

This differential equation together with the boundary condition $V_T(\tau) = 0$ has a unique solution $V_t(\tau) = 0$ for all $t \in [S_2, T]$.

□

F The case of no project completion deadline

Importantly, the presence of a hard project deadline T serves as one of the necessary and sufficient conditions for the agent to commit to an interim reporting deadline. Without a hard deadline T, the principal's incentives under full information are different. Recall from Lemma 2 the principal's incentive to continue investing decreases in the length of absence of the first stage completion. In the case $T \to \infty$, the continuation value $V_1^{FI}(t)$ is constant and given by $v(1-\kappa)$. As a result, the principal's incentive to continue investing given the absence of stage completion does not change over time. Thus, if the principal opts in, he never chooses to stop investing before the completion of the second stage occurs. As a result, setting an interim deadline stops serving as an agent's tool to incentivize the principal's investment. The agent's information policy in the case of no project deadline is given in Lemma 12.

Lemma 12. *Assume that $T \to \infty$. In that case, if $\kappa < \frac{1}{2}$, then the agent uses the recommendation mechanism presented in Proposition 1.*

Proof of Lemma 12. Under full information and the absence of an exogenous deadline, the principal assigns value v_x to each state $x \in \{0, 1, 2\}$. Clearly, $v_2 = v$ as the principal stops immediately and gets v. In state 1, at each t the principal gets $v\Delta t$ with probability $\lambda\Delta t$ and pays $c\Delta t$. As a result, the principal's continuation value is constant. Assume that $\kappa < 1$, as otherwise $c \geq \lambda v$ and the principal chooses not to invest in state 1. As the principal's continuation value in state 1 does not change over time,

$$0 = \lambda \cdot (v_2 - v_1) - c,$$

and so

$$v_1 = v - \frac{c}{\lambda} = v(1 - \kappa).$$

Thus, the principal wants to invest in state 0 if $c \leq \lambda v_1$, i.e. $\kappa \leq \frac{1}{2}$.

Finally, as the information regarding τ_1 is not decision-relevant for the principal, for $\kappa < \frac{1}{2}$, the agent chooses the recommendation mechanism that discloses only the completion of the second stage of the project and optimally postpones the disclosure to make the principal's individual rationality constraint bind.

□

Abstrakt

Tento článek zkoumá optimální design samohlášení o průběhu řešení projektu agentem usilujícím o financování, který se zodpovídá principálovi se zájmem na splnění projektu před exogenně daným deadlinem. Projekt má dvě fáze: dokončení první fáze představuje významný milník a dokončení druhé fáze představuje splnění projektu. Ukazuji, že pokud je projekt dostatečně slibný před započetím práce, pak se agent zavazuje oznámit principálovi pouze dobré zprávy o ukončení projektu. Pokud se projekt předem nejeví dostatečně slibně, pak agent přesvědčuje principála k financování projektu tím, že se zavazuje oznamovat jak dobré tak špatné zprávy o splnění nebo nesplnění milníku v první fázi.

Klíčová slova: dynamické Bayesovské přesvědčování, informační pobídky, průběžný deadline, vícefázový projekt

Working Paper Series
ISSN 2788-0443

Individual researchers, as well as the on-line version of the CERGE-EI Working Papers (including their dissemination) were supported from institutional support RVO 67985998 from Economics Institute of the CAS, v. v. i.

Specific research support and/or other grants the researchers/publications benefited from are acknowledged at the beginning of the Paper.

Published by
Charles University, Center for Economic Research and Graduate Education (CERGE)
and
Economics Institute of the CAS, v. v. i. (EI)
CERGE-EI, Politických vězňů 7, 111 21 Prague 1, tel.: +420 224 005 153, Czech Republic.
Phone: + 420 224 005 153
Email: office@cerge-ei.cz
Web: https://www.cerge-ei.cz/

Editor: Byeongju Jeong

The paper is available online at https://www.cerge-ei.cz/working-papers/.

ISBN 978-80-7343-541-7 (Univerzita Karlova, Centrum pro ekonomický výzkum a doktorské studium)
ISBN 978-80-7344-649-9 (Národohospodářský ústav AV ČR, v. v. i.)