Efficiency Defense and Administrative Fuzziness in Merger Regulation

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Abstract

Inclusion of an efficiency defense brings about an asymmetric information problem between an antitrust agency and merging firms concerning efficiencies due to mergers. Effort level and merger type determine the probability of producing the evidence that efficiencies satisfy a consumer welfare standard. The agency minimizes mistakes in its decisions. The model explains the presence of a fuzzy approval rule, i.e. approval probabilities between zero and one. If type I and type II mistakes are perfect substitutes, then only under strict restrictions on exogenous parameters fuzziness is welfare enhancing. If the agency can commit to certain policies or mistakes are non-perfect substitutes, then a fuzzy rule is preferred under wider range of parameters.

Zahrnutí efektivity do obhajoby fúze vytvárí problém asymetrické informace mezi antitrustovou agenturou a firmami, kterým jde o zvýšení efektivity po fúzi. Vyvinuté úsilí a typ fúze urcují pravdepodobnost nalezení dukazu, že efektivita splnuje standard spotrebitelského blahobytu. Agentura minimalizuje chybu svého rozhodnutí. Model vysvetluje prítomnost mlhavého schvalovacího pravidla, tj. pravdepodobnost schválení mezi nulou a jednickou. Pokud chybu I. a II typu jsou perfektní substituty, pouze pri silném omezení exogenních parametru mlhavost pravidla zvyšuje blahobyt. Pokud se agentura muže zavázat k rozhodnutím nebo pokud nejsou chyby perfektní substituty, mlhavé pravidlo je preferováno pro širokou množinu parametru.

JEL classification: K21, L44, L51
Keywords: merger regulation, efficiency defense, commitment

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1 Introduction

Recently, many countries have allowed for an efficiency defense in horizontal merger regulation[1]. The efficiency defense refers to a case in which an antitrust agency is ready to approve a merger if a substantial increase in market concentration is 'compensated' by efficiencies caused by the merger[2]. To be considered the efficiencies should be verifiable, merger specific, and passed on to consumers, i.e. to decrease prices[3]. However, the inclusion of the efficiency defense into merger regulation brings about an asymmetric information problem between an antitrust agency and merging firms concerning the actual cost efficiencies due to the merger.

The main difference between merger regulation and other antitrust laws is that merger regulation looks into the future while antitrust regulation - into the past. Merger regulation aims at preventing future abuses of market power rather than at finding and punishing misbehavior that has already occurred as happened in such antitrust cases as Microsoft and Baby Bells. In the latter cases a firm is ready to invest a substantial amount of money to avoid fines or criminal charges, while in the case of mergers, firms are willing to invest resources only up to the point where the expected profit from the merger equals the costs of getting merged.

There are administrative and adversarial systems regarding merger regulation. The administrative system exists when the agency by itself estimates actual cost efficiencies caused by the merger and, based on this estimation, makes a decision to approve or disapprove a merger (in the EU). The adversarial system exists when the burden of proof to verify required cost efficien-

[1]US, Canada, UK, France, and Sweden consider efficiency defense in their decisions. Current EU Merger Regulation doesn't allow for efficiency defense. However, a draft of a new Merger Regulation contains efficiency defense similar to the one currently used in the US (Official Journal of the European Communities 2002/C331/03).

[2]US Horizontal Merger Guidelines consider a market as "concentrated" if pre-merger Herfidahl-Hirschman Index (HHI) is above 1000.

[3]Yao and Dahdouh (1993).

cies lies with the merging firms (in the USA and Canada). The adversarial system provides an opportunity for both 'good' and 'bad' mergers to take some action in order to improve their chances of getting approval. In this paper I will analyze the adversarial system with the possibility of signaling.

There is an extensive literature on mergers for different market structures[4]. It has been shown that if there are no cost reductions due to a merger, firms find it profitable to exercise their market power through price increases. However, a merger could lead to significant synergies (efficiencies). If synergies between merging firms are substantial, then it could lead to a decrease in price[5]. Actual efficiencies, however, are not observable by the agency. It is assumed that merging firms know the true value. An antitrust agency applies a consumer surplus standard[6]; it wants to approve mergers that decrease prices, while reject the ones that increase prices.

Merging firms can strategically reveal information or even cheat in order to get approval: "no class of evidence is free of the possibility of fabrication" (the U.S. Federal Rules of Evidence, rule 801(a), advisory committee's note). All mergers have a positive probability to produce sufficient evidence at a resource expense[7]. A merging firm chooses an effort level to produce verifiable evidence that efficiencies are sufficient to reduce prices. The agency can observe the effort level (Nobel Prize winners in expert witness testimonies, quality of lawyers and consulting firms). Therefore, an observable effort level to produce evidence is valuable information and neglecting it is not optimal behavior on behalf of the agency. Some competition practitioners report that they do not care how the evidence was produced nor who produced and

[4]Salant et al. (1983), Perry and Porter (1985), Deneckere and Davidson (1985), Zang and Kamien (1990), Shapiro and Farrell (1990), Horn and Persson (2001).

[5]See Williamson (1968), Werden (1996), Shapiro and Farrell (1990), Roller et al. (2000), Besanko and Spulder (1993).

[6]For a discussion about consumer vs. total surplus approaches in merger regulation, see Neven and Roller (2000), Shapiro and Farrell (2001), Besanko and Spulder (1993).

[7]See Sanchirico (2001), where the evidence production process is described. It is assumed that the evidence can be produced or not (0 or 1) rather than a continuous outcome between zero and one.

presented it - a Nobel Prize winner or an ordinary economist[8]. Their only concern is the presented evidence itself. Why not treat the firms' efforts to produce the evidence as a signal?

In this paper a signaling game between the agency and merging firms is constructed. In the model the effort level and type of merger determine the probability to produce verifiable evidence of sufficient efficiencies. Therefore, there is a double signal: the agency not only observes whether the evidence is produced, but also observes the way it has been produced[9]. After observing the produced evidence and effort level, the agency approves the proposed merger with some probability[10].

It is assumed that an agency's objective is to minimize mistakes in its decisions to approve or reject mergers, i.e. approval of 'bad' mergers and rejection of 'good' mergers (type II and type I errors, respectively)[11]. Often it seems that the agency mainly tries to avoid type II errors, i.e. approval of 'bad' mergers[12]. However, if the agency only pays attention to type II errors, then the optimal choice would be to reject all mergers, including all 'good' ones. Therefore, the model should capture the weight the agency assigns to both types of errors.

In many laws and regulations there is fuzziness (uncertainty) in the decision making process. Rarely are there fixed and clearly specified thresholds that guarantee a certain decision. To a certain extent, laws and regulations leave

[8]Mandel (1999) discussed the recent increase in demand for economic analysis and expert witness testimony.

[9]In Spence (1974) the signal (education) doesn't influence workers' productivity. In my model the effort level (one of the signals) is necessary to produce evidence (the other signal); the higher the effort level, the higher the probability to produce evidence.

[10]The agency can choose between behavioral and structural remedies to restore effective competition in relevant markets and protect consumers from price increases. Structural remedies discussed by Medvedev (2004).

[11]Such an objective function is often assumed to characterize behavior of regulatory agencies or bureaucrats, Roller et al. (2000).

[12]In a different context it is believed that the U.S. Food and Drug Administration leans on minimization of type II errors when deciding on the approval of new drugs.

it up to bureaucrats to decide each particular case. As a fuzzy rule we refer to the probability of approval between zero and one. Existence of some type of fuzziness in nearly every law and regulation raises the question of whether fuzziness could be beneficial for society. The answer to this question is closely related to the possibility of commitment to certain policies on behalf of the agency. The model presented gives new insights on different types of commitment that are available to the antitrust agency.

To my knowledge only a few studies have looked at some aspects of these problems. Lagerlof and Heidhues (2001), for instance, investigated an optimal merger control regime under asymmetric information about merger-specific efficiencies. The authors didn't, however, consider the possibility of manipulation of results or cheating on behalf of 'bad' mergers. They showed that in an equilibrium 'bad' mergers would never invest in evidence production and it is never optimal for the agency to choose a mixed strategy. However, my paper leads to opposite results. Both merger types could produce evidence, and hence under some conditions the agency might prefer mixed strategies to pure strategies. Besanko and Spulber (1993) examined optimal mixing between consumer and total welfare standards on behalf of the agency. If the agency were to randomize between these two standards, then merging firms would face uncertainty in the 'rules of the game'. A paper by Potters and van Winden (1992) examined lobbying under asymmetric information between a regulatory agency and a firm. In their model, the presence of a signal just shows that a firm can afford it. In the legal literature, Sanchirico (2001) presented a model of evidence production under asymmetric information when the burden of proof lies with the party of interest. His model stresses the importance of early deterrence strategies on behalf of the court or government agency in order to rely on the presented evidence.

To my knowledge there is no work done on signalling in merger regulation. The paper is one of the first ones to investigate such problems as commitment and fuzziness in antitrust literature. At the same time a new modification of a signalling game is presented in the paper. The structure of the paper is as follows. In the first section I describe the basic model. I then analyze a case of perfect substitutability between different types of mistakes (marginal disutilities from both types of mistakes are equal and constant), and proceed with a non-perfect substitutability case. Then results are discussed, followed by a conclusion.

2 Model

Every merger has actual efficiencies which are not observable by the agency. However, given a market and a firm's characteristics, the agency calculates and asks to show required efficiencies that would guarantee a non-positive price change. Thus, every merger is characterized by the difference between actual and required cost efficiencies, *difference* $\in (-\infty, +\infty)$. Let's consider the case of two merger types $i = \{Good, Bad\}$. They have identical characteristics except one - efficiencies due to the merger. A 'good' merger has efficiencies higher than the required ones, while a 'bad' merger has efficiencies lower than required. For the simplification of further calculations I assume that 'good' and 'bad' mergers are equally distanced from the required efficiencies but only in opposite directions. For any given merger, higher efficiencies bring higher profits, $\Pi_{good} > \Pi_{bad}$ [13].

Both merger types want to get approval and, therefore, are ready to invest some resources to get it. Effort level and type determine the probability to produce evidence, $\beta_i^{Yes}(e)$. The evidence is not produced with probability $\beta_i^{No}(e) = [1 - \beta_i^{Yes}(e)]$. Higher effort leads to higher probability to produce the evidence: $\frac{\partial \beta_i^{Yes}(e)}{\partial e} > 0$, $\frac{\partial^2 \beta_i^{Yes}(e)}{\partial e^2} < 0$ (as $e \to \infty, \beta \to 1$). If a merging firm chooses Zero effort level, then no evidence can be produced: $\beta_i^{Yes}(0) = 0$.

[13] This is a condition for signalling in the 'right direction', i.e. "good" firms have a larger stake in persuading a regulator than do 'bad' firms (Potters and van Winden 1992). This condition holds in great generality: if a 'good' merger is characterized by lower costs, optimal behavior cannot yield it a lower profit.

The same effort level gives 'good' mergers higher probability to produce the evidence: $\beta^{Yes}_{good}(e) > \beta^{Yes}_{bad}(e)$ for $\forall\ e \neq 0$. Assume that merging firms can choose from Zero, Low, or High effort levels to produce verifiable evidence that efficiencies are sufficient. One of the assumptions is that an increase in probability to produce evidence due to higher effort level is greater for 'good' mergers than for 'bad' mergers[14], $\beta^Y_g(e^H) - \beta^Y_g(e^L) > \beta^Y_b(e^H) - \beta^Y_b(e^L)$ [15]. Costs of evidence production are determined by the effort level, $C(e)$, and are independent of merger type. Effort is increasingly costly: $C'(e) > 0$, $C''(e) \geq 0$, $C(0) = 0$. It is assumed that effort level is perfectly correlated with success probability: the most expensive lawyer produces the best evidence.

To define equilibrium the following notation is used:
$\overline{p} = p_{es} = (p_0, p_{HY}, p_{HN}, p_{LN}, p_{LY})$ - agency's strategy - this is the vector of agency's approval probabilities after observing effort level $e \in E$: $\{e^H, e^L, 0\}$ and produced evidence s∈S: $\{Yes\ evidence, No\ evidence\}$. We do not distinguish between p_{0Y} and p_{0N} cases because 0-effort never yields success, so $p_0 = p_{0Y} = p_{0N}$.
$\sigma_i(e) = (\sigma^H_i, \sigma^L_i, \sigma^0_i)$ - firm's strategy - this is the probability that a firm i chooses effort level e, so that $\sum_e \sigma^e_i = 1$.
$\beta^s_i(e)$ - probability of appearing at the decision node (e, s) by type i merger after putting in effort level e, where $0 \leq \beta^s_i(e) < 1$.
α - agency's belief about the initial proportion of 'good' mergers and, consequently, $(1 - \alpha)$ of 'bad' mergers.

A 'good' merging firm chooses an effort level by comparing expected profit:
$\mathrm{E}\Pi_g(e^H) = \ \beta^{Yes}_g(e^H)\ p_{HY}\ \Pi_g\ +\ (1 - \beta^{Yes}_g(e^H))\ p_{HN}\ \Pi_g\ -\ C(e^H)$
$\mathrm{E}\Pi_g(e^L) = \ \beta^{Yes}_g(e^L)\ p_{LY}\ \Pi_g\ +\ (1 - \beta^{Yes}_g(e^L))\ p_{LN}\ \Pi_g\ -\ C(e^L)$

[14] In signalling literature there is the idea of a single-crossing property, i.e. an increase in the marginal probability to produce the evidence is higher for 'good' mergers than for 'bad' mergers $\frac{\partial \beta^Y_g(e)}{\partial e} > \frac{\partial \beta^Y_b(e)}{\partial e}$ (or $\frac{\beta^Y_g(e^H)}{\beta^Y_g(e^L)} > \frac{\beta^Y_b(e^H)}{\beta^Y_b(e^L)}$). Later in the paper I will distinguish cases with and without this property.

[15] This assumption eliminates cases when the probability $\beta^Y_b(e)$ is close to one. Since β is bounded, it cannot increase at a higher rate for a 'good' than for a 'bad' merger throughout (β converges to 1 also for a 'bad' merger).

$\text{E}\Pi_g(0) \ = p_0\ \Pi_g\ -\ C(0)$

The same comparison holds for a 'bad' merger: $\text{E}\Pi_b(e^H)$, $\text{E}\Pi_b(e^L)$ and $\text{E}\Pi_b(0)$. Therefore, firm's maximization problem is:

$\max_{\sigma_i}\ E\Pi_i\ =\ \sigma_i^H E\Pi_i(e^H)\ +\ \sigma_i^L E\Pi_i(e^L)\ +\ \sigma_i^0 E\Pi_i(0)$, where $\sum_e \sigma_i^e = 1$.

Borrowing notations from Shapiro and Farrell (1990), changes in social welfare caused by 'good' and 'bad' mergers are: $\Delta W_g = W_{good} - W_N > 0$ and $\Delta W_b = W_N - W_{bad} > 0$, where W_N is social welfare without a merger. The agency minimizes the value of its expected total mistake across all decision nodes rather than a mistake in a separate decision node.

$\min_p\ \ ETM\ =$ (expected type I and type II errors) $=$
$Welfare\ change\ due\ to\ 'good'\ mergers * Prob(reject\ |\ \ 'good'\ merger) +$
$Welfare\ chenge\ due\ to\ 'bad'\ mergers * Prob(approve\ |\ \ 'bad'\ merger)$

The timing of the game is the following. Given an agency's belief about the initial proportion of 'good' and 'bad' mergers, α and $(1-\alpha)$, respectively, the agency draws up a rule by assigning approval probabilities conditional on the produced evidence and effort level with the goal of minimizing the expected total mistake. Knowing the rule, a merging firm chooses an effort level to maximize its expected profit. Finally, after observing the produced evidence and effort level, the agency acts sequentially rational and approves the proposed merger with the probability according to the rule.

3 Perfect substitutability of mistakes

Consider first a case when Type I and Type II mistakes are 'perfect substitutes'. In other words marginal disutilities from both types of mistakes are equal and constant. The agency cares about the distribution of 'good' and 'bad' mergers across all nodes rather than in each node separately. The agency is minimizing its expected total mistake, i.e. the summation of Type I and II errors across all possible decision nodes:

$\min_p\ ETM\ =$ (expected type I + expected type II errors) =
$\sum_e \sum_s\ \sigma_g^e\ \alpha\ \beta_g^s(e)\ \Delta W_g\ (1-p_{es})\ +\ \sigma_b^e\ (1-\alpha)\ \beta_b^s(e)\ \Delta W_b\ p_{es}$

Given this utility function of the agency we can define an equilibrium in the model.

Equilibrium is defined by the pair of strategies (σ, p) which satisfy the following conditions:
1) $\sigma_i(e)$, where $e \in E$, $i = \{Good, Bad\}$, such that a firm i maximizes expected profit:
$\max_{\sigma_i}\ E\Pi_i = \sum_e\ [\sum_s\ \sigma_i^e \beta_i^{Yes}(e) p_{es}\ \Pi_i] - C(e)$
2) $\overline{p} = (p_0, p_{HY}, p_{HN}, p_{LN}, p_{LY})$ minimizes the agency's expected total mistake:
$\min_p\ ETM\ = \sum_e \sum_s\ \sigma_g^e\ \alpha\ \beta_g^s(e)\ \Delta W_g\ (1-p_{es})\ +\ \sigma_b^e\ (1-\alpha)\ \beta_b^s(e)\ \Delta W_b\ p_{es}$
3) the agency acts sequentially rational in each information set and updates its beliefs using the Bayessian rule.

If the agency acts sequentially rational in every decision node, then the agency makes decisions based on its beliefs about the proportion of 'good' and 'bad' mergers it faces. Let's look at a minimization problem at any decision node (e, s):

$\min_p\ \ \sigma_g^e\ \alpha\ \beta_g^s(e)\ \Delta W_g\ (1-p_{es})\ +\ \sigma_b^e\ (1-\alpha)\ \beta_b^s(e)\ \Delta W_b\ p_{es}$
$= \sigma_g^e\ \alpha\ \beta_g^s(e)\ \Delta W_g\ - p_{es}\ [\sigma_g^e\ \alpha\ \beta_g^s(e)\ \Delta W_g\ -\ \sigma_b^e\ (1-\alpha)\ \beta_b^s(e)\ \Delta W_b]$

This function reaches minimum when p_{es} is equal either to 0 or 1.
If $\sigma_g^e\ \alpha\ \beta_g^s(e)\ \Delta W_g\ >\ \sigma_b^e\ (1-\alpha)\ \beta_b^s(e)\ \Delta W_b$, then $p_{es} = 1$;
if the sign is " $<$ ", then $p_{es} = 0$;
if the sign is " $=$ ", then the agency is indifferent and $p_{es} \in [0; 1]$.
Let's assume that $\Delta W_g = \Delta W_b$ and normalize it to 1 [16]. Hence, if $\sigma_g^e\ \alpha\ \beta_g^s(e) > \sigma_b^e\ (1-\alpha)\ \beta_b^s(e)$, then $p_{es} = 1$, i.e. if the proportion of 'good' mergers in the

[16] I assigned equal weights to welfare changes due to 'bad' and 'good' mergers for the simplification of further calculations. Changing welfare values would change threshold values but won't change conclusions later on in the paper.

decision node (e, s) is greater than the proportion of 'bad' mergers, then the agency is better off approving all mergers in this decision node.

3.1 Equilibria under pure strategies only

First, let's look at the case of pure strategies on behalf of the agency, i.e. the agency can only assign probability 0 or 1. In order to eliminate some trivial cases when effort level is not affordable by one or both merger types let's introduce one assumption.

Assumption 1: If approved with probability one when the evidence is produced, then both merger types can afford any effort level besides zero, i.e. $\beta_i^{Yes}(e) * \Pi_i \geq C(e)$ for $\forall e \neq 0$.

The agency's strategy is represented by the vector of approval probabilities conditional on observed signals (effort level and evidence), i.e. $\overline{p} =$ "$p_0\ p_{HY}\ p_{HN}\ p_{LN}\ p_{LY}$". If an agency's strategy is $\overline{p} =$ "$1\ p_{HY}\ p_{HN}\ p_{LN}\ p_{LY}$", to approve all mergers irrespective of whether the evidence was presented or not, merging firms would always choose Zero effort level to minimize the costs because they would be approved anyway. Hence, $(1 - \alpha)$ 'bad' mergers would be approved.

If the agency's strategy is "0 . . . .", then the possible strategies are:

1. "00000"	5. "00100"	9. "01000"	13. "01100"
2. "00001"	6. "00101"	10. "01001"	14. "01101"
3. "00010"	7. "00110"	11. "01010"	15. "01110"
4. "00011"	8. "00111"	12. "01011"	16. "01111"

Strategies 4, 8, 12, 13, 16 imply that all mergers will be approved because given the strategy of the agency both types would choose the same effort level (either high or low) and the agency approves all mergers with that effort level. Since in my model the agency doesn't care about the firms' costs of evidence production, and given Assumption 1, these strategies are equivalent to "1 p_{HY} p_{HN} p_{LN} p_{LY}" strategy, in which the agency ex ante decides to approve any merger. Although it could be the case that High effort level is so costly that even knowing the agency would undoubtedly approve a merger after observing High effort, a merging firm would prefer Low effort level though it would be approved only if the evidence is found. Then strategy 14="01101" could be optimal.

Strategies 3, 5, 6, 7, 8, 11, 15 imply the approval of a merger when there is no evidence, while rejection of a merger when there is evidence, which makes no sense. If such policies are implemented, then even if the evidence is produced it would be hidden. This is made clearer if we introduce one more step in the decision of merging firms: after producing evidence the firm decides whether to present it or not. Therefore, only "00000","00001", "01000", "01001", "01101", and "11111" strategies are relevant in the analysis.

For simplicity we introduce the following notation:
$\beta_g^{Yes}(e^H) = a \qquad \beta_g^{Yes}(e^L) = c \qquad \beta_b^{Yes}(e^H) = b \qquad \beta_b^{Yes}(e^L) = k$

The agency's objective is to minimize the value of its expected total mistake by choosing a strategy vector $\overline{p}$. In order to derive optimal policies in the model one proposition is needed:

Proposition 1: If $\frac{a}{c} > \frac{b}{k}$, then strategy "00001" is never optimal for the agency.

This proposition states that if $\frac{a}{c} > \frac{b}{k}$ and given Assumption 1, strategy "00001" is always dominated either by strategy "01000" or by strategy "00000". Now we can derive equilibrium strategies given initial belief about α, proportion of 'good' mergers[17]:

Equilibrium 1 (E1): If $\frac{a}{c} > \frac{b}{k}$, then equilibrium is:

[17] I will disregard "knife-edge" cases, such as $a\Pi_g - C(e^H) = c\Pi_g - C(e^L)$, $b\Pi_b - C(e^H) = k\Pi_b - C(e^L)$, $\frac{a}{c} = \frac{b}{k}$ because these equalities come entirely from values of exogenous parameters rather than from rational choice of probabilities by the agency to enforce mixing on behalf of firms. Probability of such events (equalities) is zero.

a) $\alpha \in [0, \alpha_1)$, then $\overline{p} = (0,0,0,0,0), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = \alpha$
b) $\alpha \in [\alpha_1, \alpha_2)$, then $\overline{p} = (0,1,0,0,0), \sigma_g = (1,0,0), \sigma_b = (1,0,0)$,
$ETM = \alpha(1-a) + (1-\alpha)b$
c) $\alpha \in [\alpha_2, 1]$, then $\overline{p} = (1,1,1,1,1), \sigma_g = (0,0,1), \sigma_b = (0,0,1)$,
$ETM = (1-\alpha)$
where $\alpha_1 = \frac{b}{a+b}$ and $\alpha_2 = \frac{1-b}{2-a-b}$

Every strategy can be represented by a line (see Graph 1 in the Appendix). The lowest line (strategy) reflects the optimal strategy for given values of α . When α is small the agency believes that there are too few 'good' mergers and rejects all mergers. Both merger types choose zero effort level to minimize costs. With intermediate values of α the agency approves mergers only if evidence is produced after High effort level but rejects them otherwise. This happens because according to Proposition 1 the agency prefers to enforce a High effort level to produce the evidence. Both merger types choose a High effort level, which is affordable according to Assumption 1. When α is big, then the agency approves all mergers irrespective whether the evidence is produced or not and both merger types choose Zero effort level to minimize costs.

Equilibrium 2 (E2): If $\frac{a}{c} < \frac{b}{k}$, then equilibrium is:
a) $\alpha \in [0, \alpha_1)$, then $\overline{p} = (0,0,0,0,0), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = \alpha$
b) $\alpha \in [\alpha_1, \alpha_2)$, then $\overline{p} = (0,0,0,0,1), \sigma_g = (0,1,0), \sigma_b = (0,1,0)$,
$ETM = \alpha(1-c) + (1-\alpha)k$
c) $\alpha \in [\alpha_2, \alpha_3)$, then p=(0,1,0,0,0), $\sigma_g = (1,0,0), \sigma_b = (1,0,0)$,
$ETM = \alpha(1-a) + (1-\alpha)b$
d) $\alpha \in [\alpha_3, 1]$, then $\overline{p} = (1,1,1,1,1), \sigma_g = (0,0,1), \sigma_b = (0,0,1)$,
$ETM = (1-\alpha)$
where $\alpha_1 = \frac{k}{c+k}$, $\alpha_2 = \frac{b-k}{a-c+b-k}$, $\alpha_3 = \frac{1-b}{2-a-b}$

If $\frac{a}{c} < \frac{b}{k}$, then for certain values of α there is one more optimal strategy "00001" (the agency approves mergers only if evidence is produced after Low effort level but rejects them otherwise). Both merger types choose Low effort level, which is affordable according to Assumption 1. This additional equilibrium strategy arises because the shift from Low to High effort level increases the probability to produce the evidence for 'good' mergers more than for 'bad' ones, $a-c>b-k$, but not as much as when $\frac{a}{c}>\frac{b}{k}$ (see graph 2)[18].

In all cases above the agency acts sequentially rational in every decision node, i.e. the agency makes decisions based on its beliefs about the proportion of 'good' and 'bad' mergers it faces[19].

3.2 Perfect substitutability and no attention to effort

In this section I compare the situation in which the agency does not pay attention to the way the evidence is produced with the situation from the previous section in which there is attention to the effort level. The way to make the comparison is to assume that the agency knows all the probabilities for both merger types under Low and High effort levels; however, it does not use this information in distinguishing merger types. Its decisions are based only on whether the evidence is produced or not. In this case the equilibrium is defined in the following way:

Equilibrium is defined by the pair of strategies (σ, p), which satisfy the following conditions:
1) $\sigma_i(e)$ where $e \in E$, $i=\{Good, Bad\}$, such that a firm i maximizes expected profit:

[18] The single-crossing property $\frac{a}{c}>\frac{b}{k}$ guarantees that $(a-c)>(b-k)$: if $\frac{a}{c}>\frac{b}{k} \Rightarrow \frac{a}{c}-1>\frac{b}{k}-1 \Rightarrow \frac{a-c}{c}>\frac{b-k}{k}$, because $c>k$, then $a-c>b-k$.

[19] It is interesting to analyze whether a value of the Herfidahl-Hirschman Index (HHI) could serve as a proxy for the agency's initial beliefs about the proportion of 'good' mergers (high efficiencies due to a merger) in a market. The US DoJ considers three types of market: not concentrated ($HHI<1000$), concentrated, and highly concentrated ($HHI>1800$). As a result of the model the agency considers 3-4 intervals of α for different optimal strategies (α low, intermediate, and high).

$\max_{\sigma_i} E\Pi_i = \sum_e \; [\; \sum_s \; \sigma_i^e \; \beta_i^s(e) \; p_s \; \Pi_i \;] - C(e)$
2) $\overline{p} = (p_0, p_{yes}, p_{no})$ minimizes the agency's expected total mistake:
$\min_p ETM = \sum_e \sum_s \quad \Delta W_g \; \sigma_g^e \; \alpha \; \beta_g^s(e) \; (1-p_s) + \Delta W_b \; \sigma_b^e \; (1-\alpha)\beta_b^s(e) \; p_s$
3) the agency acts sequentially rational at each information set and updates its beliefs using the Bayessian rule while consciously ignores the information about the effort level.

Given this definition of the equilibrium the agency can choose the following strategies. It can approve or reject all mergers and in these cases both merger types would choose Zero effort level to minimize costs. Also the agency can approve mergers if evidence is produced and reject them otherwise. In the latter case the profitability of different effort levels for merging firms should be considered, i.e. what effort level is more profitable for them given a merger is approved with probability one when the evidence is produced. There are three possible cases: both merger types prefer High or Low effort levels or the case in which High effort level is more profitable for a 'good' merger, while Low effort level is more profitable for a 'bad' merger[20]. Considering these three cases we derive equilibrium strategies given initial beliefs about the proportion of 'good' mergers, α.

In the case when both merger types prefer a High effort level, if a merger is approved with probability one when the evidence is produced:
$a \; \Pi_g - C(e^H) \; > \; c \; \Pi_g - C(e^L)$ and $b \; \Pi_b - C(e^H) \; > \; k \; \Pi_b - C(e^L)$, then Equilibrium 3 **(E3)** is:

a) $\alpha \in [0, \alpha_1)$, then $\overline{p} = (0,0,0), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = \alpha$
b) $\alpha \in [\alpha_1, \alpha_2)$, then $\overline{p} = (0,1,0), \sigma_g = (1,0,0), \sigma_b = (1,0,0), ETM = \alpha(1-a) + (1-\alpha)b$
c) $\alpha \in [\alpha_2, 1]$, then $\overline{p} = (1,1,1), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = (1-\alpha)$

where $\alpha_1 = \frac{b}{a+b}$ and $\alpha_2 = \frac{1-b}{2-a-b}$

[20]It is not possible that a 'bad' merger prefers a High effort level while a 'good' merger prefers a Low level, because $\Pi_g > \Pi_b$ and $(a-c) > (b-k)$.

Given these equilibrium strategies we can compare this outcome with the outcome when the agency pays attention to effort levels.

Proposition 2:
a) if $\frac{a}{c} > \frac{b}{k}$, then E3 gives the same expected total mistake to the agency as E1 for all values of α;
b) if $\frac{a}{c} < \frac{b}{k}$, then E3 gives higher expected total mistake than E2 on the interval $\alpha \in (\frac{k}{c+k}, \frac{b-k}{a-c+b-k})$ and the same mistake for other values of α.

For a visual representation see Graph 1 and 2 in the Appendix. Optimal policy E3 leads to a higher expected total mistake than E2 because the agency is not able to enforce a Low effort level on the interval $\alpha \in (\frac{k}{c+k}, \frac{b-k}{a-c+b-k})$. Whenever the inequality holds, the interval is never empty. If $\frac{a}{c} < \frac{b}{k}$ then the shift from Low to High effort level does not increase the probability to produce the evidence for 'good' mergers enough to outweigh a large number of 'bad' mergers (low values of α).

In the case when both merger types would prefer Low effort level, if a merger is approved with probability one when the evidence is produced,
$a\ \Pi_g - C(e^H) < c\ \Pi_g - C(e^L)$ and $b\ \Pi_b - C(e^H) < k\ \Pi_b - C(e^L)$, then Equilibrium 4 **(E4)** is:

a) $\alpha \in [0, \alpha_1)$, then $\overline{p} = (0,0,0), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = \alpha$
b) $\alpha \in [\alpha_1, \alpha_2)$, then $\overline{p} = (0,1,0), \sigma_g = (0,1,0), \sigma_b = (0,1,0), ETM = \alpha(1-c) + (1-\alpha)k$
c) $\alpha \in [\alpha_2, 1]$, then $\overline{p} = (1,1,1), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = (1-\alpha)$

where $\alpha_1 = \frac{k}{c+k}$ and $\alpha_2 = \frac{1-k}{2-c-k}$

Consequently, the next proposition follows:

Proposition 3:
a) If $\frac{a}{c} > \frac{b}{k}$, then E4 gives a higher expected total mistake than E1 on the interval $\alpha \in (\frac{b}{a+b}, \frac{1-b}{2-a-b})$ and the same mistake for other values of α.
b) If $\frac{a}{c} < \frac{b}{k}$, then E4 gives a higher expected total mistake than E2 on the

interval $\alpha \in (\frac{b-k}{a-c+b-k}, \frac{1-b}{2-a-b})$ and the same mistake for other values of α.

Optimal policy E4 leads to a higher expected total mistake than E1 and E2 for certain values of α because the agency is not able to enforce a High effort level (see Graph 1 and 2 in the Appendix).

In the case when High effort level is more profitable for a 'good' merger, while Low effort level is more profitable for a 'bad' merger, if a merger is approved with probability one when the evidence is produced, $a\ \Pi_g - C(e^H) > c\ \Pi_g - C(e^L)$ and $b\ \Pi_b - C(e^H) < k\ \Pi_b - C(e^L)$, then Equilibrium **(E5)** is:

a) $\alpha \in [0, \alpha_1)$, then $\overline{p} = (0,0,0), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = \alpha$
b) $\alpha \in [\alpha_1, \alpha_2)$, then $\overline{p} = (0,1,0), \sigma_g = (1,0,0), \sigma_b = (0,1,0), ETM = \alpha(1-a) + (1-\alpha)k$
c) $\alpha \in [\alpha_2, 1]$, then $\overline{p} = (1,1,1), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = (1-\alpha)$

where $\alpha_1 = \frac{k}{a+k}$ and $\alpha_2 = \frac{1-k}{2-a-k}$

Thus the next proposition follows:

Proposition 4:
a) E5 gives a smaller expected total mistake than either E1 or E2 on the interval $\alpha \in [\frac{k}{a+k}, \frac{1-k}{2-a-k})$ and the same mistake for other values of α.

For a visual representation see Graph 3 in the Appendix. Equilibrium strategy E5b could be called a "voluntary separation": High effort level is more profitable for a 'good' merger, while Low effort level is more profitable for a 'bad' merger. For the intermediate values of α the agency knows only that 'bad' mergers would prefer a Low effort level and that some of them would succeed in producing evidence. If effort levels are considered in the decision making the agency would assign probability zero to the decision node (e^L, Yes), $p_{LY} = 0$. However, knowing this, a 'bad' merger would choose a High effort level so as not to be distinguished by the agency and according to Assumption 1 it can afford this effort level. The shift in the effort level by a 'bad' merger would lead to the situation in which 'bad' mergers would invest more resources into evidence production. Consequently, more 'bad' mergers would have produced the evidence and, as a result, a bigger mistake would be made by the agency.

Given Proposition 2-4 we can say that a rule to approve all mergers which produce evidence is optimal under certain parameters, yet enforcing certain effort levels could bring a lower expected total mistake. A possibility to to enforce certain effort level in practice is discussed in the Results section. However, under certain conditions, even recognizing potentially harmful mergers, the agency is better off approving rather than blocking them; otherwise, a 'bad' merger would mimic the choice of a 'good' merger. The case of voluntary separation shows that sometimes the use of extra information (the way the evidence is produced) leads to an undesirable outcome (higher expected total mistake). This contradicts the usual perception that the more information used the better.

3.3 Equilibria if mixing is possible

Now let's look at whether by applying a fuzzy approval rule (mixed strategies) the agency can do as well or better as under pure strategies. If Type I and II mistakes are perfect substitutes, then as we saw in Section 3, mixing is possible when there are equal proportions of 'good' and 'bad' merger types in a decision node, i.e. when the agency is indifferent between approving or rejecting a merger. Such a result comes from the assumptions about the perfect substitutability of different types of mistakes and equal weights to welfare changes due to 'good' and 'bad' mergers[21]. Then the proposition follows that it is impossible to have equal proportions of 'good ' and 'bad' mergers in any two nodes simultaneously with the exception of one case.

Proposition 5:

[21] Changing the weights will change only the threshold, which makes the agency indifferent but doesn't change other results.

Given the perfect substitutability of different types of mistakes and assumptions $a > b > k$ and $a > c > k$, mixing in any two decision nodes simultaneously is impossible besides case $\overline{p} = (0, 1, p_{HN}, 0, 0, p_{LY})$.

Now using Proposition 5 and the definition of the equilibrium from Section 3, we can derive the following mixing equilibria (ME) (for all values see the Appendix):

ME1: $\overline{p} = (0, p_{HY}, 0, 0, 0), \sigma_g = (1, 0, 0), \sigma_b = (\sigma_b^H, 0, [1 - \sigma_b^H]), ETM = \alpha$
ME2: $\overline{p} = (0, 0, 0, 0, p_{LY}), \sigma_g = (0, 1, 0), \sigma_b = (0, \sigma_b^L, [1 - \sigma_b^L]), ETM = \alpha$
ME3: $\overline{p} = (0, 1, 1, p_{LN}, 1)$, $\sigma_g = (\sigma_g^H, [1-\sigma_g^H], 0)$, $\sigma_b = (0, 1, 0)$, $ETM = 1-\alpha$
ME4: $\overline{p} = (0, 1, p_{HN}, 0, 0, p_{LY}), \sigma_g = (\sigma_g^H, [1-\sigma_g^H], 0), \sigma_b = (\sigma_b^H, [1-\sigma_b^H], 0)$, $ETM = \alpha(1 - a) + (1 - \alpha)b$

As an example let's look at ME1. When α is small, the agency can assign non-zero probability to the (High efforts, Yes evidence) decision node. The assigned probability makes 'bad' mergers indifferent between High and Zero effort levels and 'bad' mergers can mix, while 'good' mergers choose High efforts to produce the evidence with probability one[22]. Bad mergers mix with a probability that makes an equal proportion of 'good' and 'bad' mergers (50/50) in the decision node (e^H, Yes). If 'good' and 'bad' mergers are in equal proportions, then the agency is indifferent between approving or rejecting them and can choose any probability between 0 and 1. In the equilibrium it chooses p_{HY} such that makes a 'bad' merger indifferent. The value of the expected total mistake stays unchanged, (α), because the agency decreases the number of rejected 'good' mergers (Type I error) from α to $\alpha - \alpha a p_{HY}$ but increases the number of approved 'bad' mergers (Type II error) from 0 to $(1 - \alpha)\sigma_b^H b p_{HY}$.
In all three mixing equilibria above the agency is not better off applying a fuzzy approval rule. But there is one equilibrium that is of prime interest for us.

[22]A 'good' merger reaches first a "point of profitability", i.e. condition when an effort level becomes profitable for a merging firm. It follows from assumptions $\Pi_g > \Pi_b$ and $a > b$ and $c > k$.

ME5: $\overline{p} = (0,1,1,0,p_{LY})$, $\sigma_g = (\sigma_g^H, [1-\sigma_g^H], 0)$, $\sigma_b = (0,1,0)$, $ETM = (1-\alpha)\frac{k}{c}$.

See Graph 4. Such equilibrium exists if the following conditions on exogenous parameters are satisfied: p_{LY}^{ME5} is such that $\Pi_g - C(e^H) = p_{LY}^{ME5} c\Pi_g - C(e^L)$ and $\Pi_b - C(e^H) < p_{LY}^{ME5} k\Pi_b - C(e^L)$. The difference in expected profits, difference in evidence production costs between High and Low effort levels, and difference in success probabilities to produce the evidence, k and c, determine the existence of this equilibrium. If Π_{good} is relatively large compared to Π_{bad} and the difference between High and Low effort costs is large, then the agency can assign such probability p_{LY}^{ME4} that 'good' mergers would mix between High and Low effort levels while all 'bad' ones would choose Low efforts.

Proposition 6: Mixing equilibrium 4 gives lower value of expected total mistake than:
a) E2 on the interval $(\frac{k}{c+k}; 1)$ if $\frac{a}{c} < \frac{b}{k}$;
b) E1 on the interval $(\frac{k-cb}{k-cb+c(1-a)}; 1)$ if $\frac{a}{c} > \frac{b}{k}$.

This is the only mixing equilibrium when a fuzzy approval rule is welfare enhancing, while the agency acts sequentially rational in every decision node.

3.4 Benefits of commitment

The ultimate goal of the agency is to distinguish between 'good' and 'bad' merger types by choosing a vector of probabilities to minimize the value of the total expected mistake. One way to distinguish merger types is to choose such probabilities that 'good' mergers would extract maximum effort level to produce evidence, while 'bad' mergers would find it either unprofitable to produce the evidence or would choose a low effort level. In the model it transforms into the situation where 'good' mergers should choose a High effort level while 'bad' mergers should choose Zero or Low effort level. However, assigning an arbitrary vector of probabilities, which minimizes the agency's mistakes, could lead to the situation when at least one of the players wants to deviate from its ex-ante strategy. This raises a commitment problem.

One way to overcome a potential commitment problem is to assume that lawmakers write down the agency's rule, which minimizes the expected total mistake, and then the agency automatically applies this rule[23]. This is so-called full commitment. It is interesting to compare the case of full commitment with that of partial commitment, where the agency has some freedom and sometimes makes a decision based on its beliefs after observing effort level and evidence. In both cases the agency doesn't consider evidence production costs in its decision, because the agency pays attention to consumer welfare but not total welfare.

In the case of full commitment the agency is just an executor of lawmakers' policy. Lawmakers draw up a policy and the agency automatically applies it no matter what its beliefs. The lawmakers acting as a social planner assign probabilities to each decision node with the goal of minimizing the expected total mistake: 'good' mergers extract a maximum effort level to produce the evidence, while 'bad' mergers find it either unprofitable to produce the evidence at all or choose a Low effort level. There are probabilities that make a 'bad' merger indifferent between two effort levels, the assigned probabilities should be by a fraction smaller than these (by ε, where $\varepsilon \to 0$) to guarantee that 'bad' mergers stay at Low or Zero effort levels while the share of rejected 'good ' mergers is minimized. Let's consider one possible case to estimate the approximate benefits for the agency that can come from full commitment. If the agency can fully commit to certain probabilities then the full commitment **(FCE)** equilibrium strategies for the case $\frac{C(e^H)}{C(e^L)} > \frac{bc}{ak}$ are the following[24]:

[23] Usually the commitment problem is resolved by delegating decisions to some independent body with a different utility function. See for example Cukerman (1994) central bank and inflation rate, Melumad and Mookherjee (1989) government and tax collection.

[24] If $\frac{C(e^H)}{C(e^L)} < \frac{bc}{ak}$ the initial strategy (strategy FCEa) would be $\overline{p} = (0, 0, 0, 0, \tilde{p}_{LY}), \sigma_g = (0, 1, 0), \sigma_b = (0, 0, 1)$, where $\tilde{p}_{LY}$ makes it profitable only for a 'good' merger to choose e^L.

a) $\alpha \in [0, \alpha_1)$,$\overline{p} = (0, p^{FC}_{HY}, 0, 0, 0), \sigma_g = (1, 0, 0), \sigma_b = (0, 0, 1)$,
$ETM = \alpha - \alpha\ a\ p^{FC}_{HY}$

b) $\alpha \in [\alpha_1, \alpha_2]$, then $\overline{p} = (0, 1, 0, 0, p^{FC}_{LY}), \sigma_g = (1, 0, 0), \sigma_b = (0, 1, 0)$,
$ETM = \alpha(1 - a) + (1 - \alpha)\ k\ p^{FC}_{LY}$

c) $\alpha \in (\alpha_2, 1]$, then $\overline{p} = (0, 1, 1, p^{FC}_{LN}, 1)$,$\sigma_g = (1, 0, 0)$, $\sigma_b = (0, 1, 0)$,
$ETM = (1 - \alpha)\ k + (1 - \alpha)\ (1 - k)\ p^{FC}_{LN}$

See the Appendix for α_1, α_2, p^{FC}_{HY}, p^{FC}_{LN}, p^{FC}_{LY}. Probabilities are such that 'good' mergers choose a High effort level, while 'bad' mergers choose Zero or Low effort level. This separation minimizes an agency's total mistake. This is the best the agency can do under such parametrization. Such a policy brings the lowest total mistake for $\alpha \in (0, 1)$ (see Graph 6). However, it is not a sequentially rational equilibrium. The agency would like to deviate ex-post from the assigned probabilities, because in certain decision nodes there will be only 'good' or 'bad' mergers and optimal approval probabilities will be one or zero, respectively.

One partial commitment could be such that there are precise instructions for the agency to follow to reject all mergers when the evidence is not produced, while the agency has freedom to apply a probability between zero and one when the evidence is produced, i.e. to act sequentially rational in some decision nodes. If a commitment $p(e, No) = p_{HN} = p_{LN} = 0$, then under some parameters there is a partial commitment equilibrium **PCE** strategy:

$\overline{p} = (0, 1, 0, 0, p^{PC}_{LY}), \sigma_g = (\sigma^H_g, [1 - \sigma^H_g], 0), \sigma_b = (0, 1, 0)$,
$ETM = \alpha\sigma^H_g(1 - a) + \alpha(1 - \sigma^H_g)(1 - c\ p^{PC}_{LY}) + (1 - \alpha)k\ p^{PC}_{LY}$

where $\sigma^H_g = [1 - \frac{(1-\alpha)}{\alpha}\frac{k}{c}]$, $p^{PC}_{LY} = \frac{1}{c}(a - \frac{C(e^H)-C(e^L)}{\Pi_g})$,
$p^{PC}_{LY} : 0\ <\ b\ \Pi_b - C(e^H) < p^{PC}_{LY}\ k\ \Pi_b - C(e^L)$

The agency can choose probability p_{LY}, which makes a 'good' merging firm indifferent between High and Low effort levels and opens the possibility for mixing. One of the restrictions is that p_{LY} should be such that a Low effort level is more profitable than a High effort level for 'bad' mergers. A 'good' firm can mix in such proportion that makes the agency indifferent between approving or rejecting mergers in the decision node (e^L, Yes). The agency is indifferent when there are equal proportions of 'good' and 'bad' mergers in this decision node. As soon as the agency is indifferent, it can choose a probability p_{LY} that makes a 'good' firm indifferent between High and Low. There is a restriction on the range of α's that this mixing strategy can be applied to: mixing probability $\sigma_g^H \in [0;1]$, when $\alpha \in [\frac{k}{c+k};1]$.

The problem with such an equilibrium is that the agency knows that only 'good' mergers are in the (e^H, No) decision node. There is hence an incentive to deviate and approve all mergers in this decision node. However, if the commitment to reject all mergers that haven't produced evidence is credible, then the agency might have a lower expected total mistake for some values of parameters.

Proposition 7:
If the commitment to reject all mergers without produced evidence is credible, $\frac{a}{c} < \frac{b}{k}$, and $p_{LY}^{PC}:\ b\ \Pi_b - C(e^H) < p_{LY}^{PC}\ k\ \Pi_b - C(e^L)$, then the agency has a lower total mistake on the interval $\alpha \in (\frac{k}{c+k}; \frac{1-a\frac{k}{c}}{2-a-a\frac{k}{c}})$ than in the case of equilibrium strategies ES2.

See Graph 5 in the Appendix. From the practical point of view it is easier to commit to zero or one probability rather than to a probability between zero and one. It is also legally enforceable, because a decision is observable, but a mixing probability is not. From the theoretical point it is interesting to notice that commitment to the probability zero in one decision node leads to the choice of the fuzzy approval rule in another node.

As we see in the case of full commitment and Proposition 4 separation of merger types leads to a lower expected mistake. In equilibrium FCE(a), i.e. when α is relatively small, the agency chooses such probabilities that a 'good' merger opts for a High effort level, while a 'bad' merger decided not to produce the evidence and consequently not to merge. However, the agency can achieve the same result by imposing an application fee on all firms that are willing to merge and to be involved in evidence production. If the agency can commit to probability zero when the evidence is not produced, then by imposing a fixed application fee F it can achieve the separation of merger types. Merger types make the decision to go into evidence production and to choose High effort level whenever the following inequalities hold: $a\Pi_g p_{HY} - C(e^H) > 0$ and $b\Pi_b p_{HY} - C(e^H) > 0$. Since $\Pi_g > \Pi_b$ and $a > b$, a 'good' merger has higher incentives (more financial resources) and higher chances to produce the evidence. The agency can choose such F that makes evidence production unprofitable for a 'bad' merger. Then only 'good' mergers would appear in the decision node (e^H, Yes), and acting sequentially rational the agency will assign probability one to this node. A fixed application fee should be $F = b\Pi_b - C(e^H) + \varepsilon$, where $\varepsilon \to 0$. Expected total mistake in this equilibrium will be $\alpha(1-a)$ which is always lower than in equilibria E1(a), E2(a), PCE(a), and FCE(a). Since the agency doesn't consider evidence production costs in its decision, an application fee is a useful instrument to separate merger types and to lower expected mistake when commitment is possible.

4 Non-perfect substitutability of mistakes

The strict assumption that different types of mistakes are perfect substitutes leads to Proposition 5, which stipulates that if the agency acts sequentially rational in all decision nodes then the fuzzy approval rule is welfare enhancing only under strict restrictions on exogenous parameters. Now we look at whether a change in the substitutability of mistakes would change this result.

In the case of a perfect substitutability of different types of mistakes the agency pays attention only to the expected total mistake. However, the agency repeatedly faces mergers in each decision node and may be the agency prefers to have a balanced composition of different types of mistakes in its approval/rejection history. The idea is that the agency bears extra costs by making too many Type I or Type II errors in a particular decision node. Besides welfare losses to the society, rejecting too many 'good' or approving too many 'bad' mergers after observing the same signals could lead to an increase in court appeals to reverse an agency's rulings or to overlooking future actions of the agency. Then the objective function of the agency might look in the following way. The expected total mistake is equal to expected Type I mistake in power γ and type II mistake in power η (where $\gamma, \eta > 1$) in a decision node summed across all possible decision nodes.

$$ETM = \sum_e \sum_s \ \sigma_g(e)\alpha\beta_g^s(e)[\Delta W_g(1 - p_{es})]^\gamma + \sigma_b(e)(1-\alpha)\beta_b^s(e)[\Delta W_b p_{es}]^\eta$$

Obviously the chosen form is arbitrary, but the purpose of the analysis is just to show that changing the form of the substitutability of different types of mistakes can lead to different results. If the agency acts sequentially rational in every decision node, then the agency makes decisions based on its beliefs about the proportion of 'good' and 'bad' mergers it faces. Let's consider the minimization problem at any decision node (e, s):

$$min_{p_{es}} \ \ \sigma_g^e\alpha\beta_g^s(e)[\Delta W_g(1 - p_{es})]^\gamma + \sigma_b^e(1-\alpha)\beta_b^s(e)[\Delta W_b p_{es}]^\eta$$

Then FOC with respect to p_{es} is the following:

$$\gamma[\Delta W_g]^\gamma \alpha\sigma_g^e\beta_g^s(e)(1-p_{es})^{\gamma-1} = \eta[\Delta W_b]^\eta(1-\alpha)\sigma_b^e\beta_b^s(e)(p_{es})^{\eta-1}$$

and the optimal approval probability is derived from $\frac{(1-p_{es})^{\gamma-1}}{(p_{es})^{\eta-1}} = \frac{\eta[\Delta W_g]^\eta(1-\alpha)\sigma_b^e\beta_b^s(e)}{\gamma[\Delta W_g]^\gamma\alpha\sigma_g^e\beta_g^s(e)}$

In order to analyze analytically the case of non-perfect substitutability between different types of mistakes, we can simplify the analysis by assuming a special case.

4.1 Case of a quadratic disutility

Let's consider a quadratic objective function of the agency, i.e. $\gamma = \eta = 2$. Hence the total expected mistake of the agency is:

$ETM \ = \sum_e \sum_s \ \sigma_g(e)\alpha\beta_g^s(e)[\Delta W_g(1-p_{es})]^2 + \sigma_b(e)(1-\alpha)\beta_b^s(e)[\Delta W_b p_{es}]^2$

Consequently, the equilibrium conditions in Section 3 should modified accordingly to the new objective function of the agency. Let's consider the minimization problem at any decision node (e, s):

$min_{p(e,s)} \ \ \sigma_g^e\alpha\beta_g^s(e)[\Delta W_g(1-p_{es})]^2 + \sigma_b^e(1-\alpha)\beta_b^s(e)[\Delta W_b p_{es}]^2$

Then FOC with respect to p_{es} is the following:

$[\Delta W_g]^2\alpha\sigma_g^e\beta_g^s(e)(1-p_{es}) = [\Delta W_b]^2(1-\alpha)\sigma_b^e\beta_b^s(e)p_{es}$

$p_{es}^* = \frac{[\Delta W_g]^2\alpha\sigma_g^e\beta_g^s(e)}{[\Delta W_g]^2\alpha\sigma_g^e\beta_g^s(e)+[\Delta W_b]^2(1-\alpha)\sigma_b^e\beta_b^s(e)}$

As in the linear case, if we assume $\Delta W_g = \Delta W_b$, then optimal probability to approve a merger after observing produced evidence s and the effort level e is: $p_{es}^* = \frac{\alpha\sigma_g^e\beta_g^s(e)}{\alpha\sigma_g^e\beta_g^s(e)+(1-\alpha)\sigma_b^e\beta_b^s(e)}$. The optimal probabilities vary from 0 to 1. If there are no 'good' mergers in a decision node then the agency's optimal probability is zero. If there are only 'good' mergers in a decision node then the agency approves all of them with probability one. From the formula of optimal probability it is clear that if α increases and there are both merger types in the decision node (e, s), then optimal probability of approval p_{es}^* increases as well[25]. One assumption is needed to proceed with the analysis.

Assumption 2:
If $\alpha < \frac{1}{2}$, then $p(0, No) = 0$.

This assumption guarantees that if the costs of evidence production are so high that both merger types prefer to choose zero effort level and the agency believes that most mergers in this market would probably hurt consumers, $\alpha < \frac{1}{2}$, then all mergers would be rejected. In such a market where $\alpha < \frac{1}{2}$, merging firms must get approval and extract some effort to get it (at least to file a request to allow a merger).

[25]Derivative of optimal approval probability with respect to α is non-negative: $\frac{\partial p_{es}^*}{\partial \alpha} \geq 0$ and optimal probability lies between 0 and 1, i.e. the agency always mixes.

Proposition 8:
If the agency has a utility function as specified above and whenever there are both merger types in a decision node besides $(0, NO)$, then probabilities 0 and 1 are never optimal.

This proposition follows from the agency's optimization problems. Now let's look at all possible equilibria in the model:

EQ1: $\overline{p} = (0, 0, 0, 0, 0), \sigma_g = (0, 0, 1), \sigma_b = (0, 0, 1)$
EQ2: $\overline{p} = (0, p_{HY}, p_{HN}, 0, 0), \sigma_g = (1, 0, 0), \sigma_b = (1, 0, 0)$
EQ3: $\overline{p} = (0, 0, 0, p_{LN}, p_{LY}), \sigma_g = (0, 1, 0), \sigma_b = (0, 1, 0)$
EQ4: $\overline{p} = (0, 1, 1, p_{LN}, p_{LY}),\ \sigma_g = (\sigma_g^H, [1 - \sigma_g^H], 0),\ \sigma_b = (0, 1, 0)$
EQ5: $\overline{p} = (0, p_{HY}, p_{HN}, p_{LN}, p_{LY}), \sigma_g = (\sigma_g^H, [1 - \sigma_g^H], 0), \sigma_b = (\sigma_b^H, [1 - \sigma_b^H], 0)$

For all values see the Appendix. If an agency's strategy is $\overline{p}$ = "0 0 0 0 0" or $\overline{p}$ = "1 1 1 1 1", then merging firms will always choose Zero effort level to minimize the costs because they would be rejected or approved regardless of whether the evidence is produced or not. The agency's expected total mistakes are α or $(1 - \alpha)$, respectively. In equilibria 2 and 3 the agency enforces High and Low effort levels, respectively. Equilibrium 4 arises when the expected payoff from High effort level is equal to the expected payoff from a Low effort level for a 'good' merger type, while 'bad' merger prefers a Low effort level. Probabilities should be such that a 'good' merger mixes, while a 'bad' one chooses Low effort level. It is a system of 3 equations with 3 unknowns σ_g, p_{LN}, p_{LY}:

$$\begin{cases} \Pi_g - C(e^H) = [cp_{LY} + (1-c)p_{LN}]\ \Pi_g - C(e^L) \\ \alpha(1-\sigma_g)(1-c)(1-p_{LN}) = (1-\alpha)(1-k)p_{LN} \\ \alpha(1-\sigma_g)c(1-p_{LY}) = (1-\alpha)kp_{LY} \\ given\ \ that\ \ \Pi_b - C(e^H) < [kp_{LY} + (1-k)p_{LN}]\ \Pi_b - C(e^L) \end{cases}$$

For a solution to the system see the Appendix.

There can be a global mixing equilibrium when both firms mix High and Low effort levels and the agency is mixing at each decision node. Equilibrium 5 might be derived from the following conditions:

$$\begin{cases} (p_{HN} - p_{LN}) + a(p_{HY} - p_{HN}) + c(p_{LN} - p_{LY}) = \frac{C^H - C^L}{\Pi_g} \\ (p_{HN} - p_{LN}) + b(p_{HY} - p_{HN}) + k(p_{LN} - p_{LY}) = \frac{C^H - C^L}{\Pi_b} \\ \alpha\sigma_g a(1 - p_{HY}) = (1 - \alpha)\sigma_b b p_{HY} \\ \alpha\sigma_g(1 - a)(1 - p_{HN}) = ((1 - \alpha)\sigma_b(1 - b)p_{HN} \\ \alpha(1 - \sigma_g)(1 - c)(1 - p_{LN}) = (1 - \alpha)(1 - \sigma_b)(1 - k)p_{LN} \\ \alpha(1 - \sigma_g)c(1 - p_{LY}) = (1 - \alpha)(1 - \sigma_b)k p_{LY} \end{cases}$$

It is a system of 6 equations with 6 unknowns $\sigma_g, \sigma_b, p_{HY}, p_{HN}, p_{LN}, p_{LY}$. For a solution to the system see the Appendix.

Now let's look at possible equilibrium strategies given the agency's initial belief about α. For a visual representation see Graph 7 in the Appendix. The lowest line (strategy) reflects the optimal strategy for the given values of α. Given the complexity of the system I will only broadly describe how the system behaves. When α is small, the optimal probabilities in any decision node are small as well. Hence, the expected profit from a merger is smaller than the costs of getting merged, and merging firms would choose Zero effort level. Hence, given Assumption 2 the agency rejects all mergers when α is small. The agency shifts to another strategy at α_1, where α_1: $[cp_{eY} + (1 - c)p_{eN}]\Pi_g \geq C^L$ and $[kp_{eY} + (1-k)p_{eN}]\Pi_b \geq C^L$, i.e. approval probabilities are high enough to make evidence production profitable for both merger types. Then, depending on exogenous parameters, equilibria EQ2-EQ5 can emerge. With intermediate values of α the agency prefers merging firms to choose either High or Low effort levels depending on exogenous parameters. When α is big, the agency then chooses such probabilities that a 'good' merger type mixes High and Low effort level, while a 'bad' type chooses a Low effort level.

5 Results

The signalling model allows us to look at some effects of the inclusion of efficiency defense in merger regulation. The model explains the presence of a fuzzy approval rule in this regulation. Usually fuzziness in any type of regulation is explained by the inability to include and formally specify all relevant variables into the regulation, or because it is too costly to do so. However, the present analysis shows that fuzziness could be a rational choice of a regulatory agency.

If different types of mistakes are perfect substitutes, then only under strict restrictions on exogenous parameters fuzziness could there be welfare enhancing (ME5). If the agency can commit to certain probabilities (to act not sequentially rational in some decision nodes), then by applying a fuzzy approval rule it can lower the value of the expected mistake. In the case of full commitment the agency is just an executer of lawmakers' policy while in the partial commitment case the agency commits to reject all mergers when the evidence is not produced, but acts sequentially rational in other decision nodes. In this case for some parameters the agency is better off applying a fuzzy approval rule (Proposition 6). In the case of partial commitment the agency commits to zero probabilities after observing certain signals. Such commitment is easy to imagine in reality than commitment to probabilities between zero and one, like in the case of full commitment.

A rule to approve all mergers that produce evidence is optimal under some parameters. However, enforcing certain effort levels could bring a lower expected total mistake (Proposition 2a, 3a, 3b). In practice the agency could select a small number of consulting and law firms (extremely expensive) or create a governmental agency (relatively cheap but inefficient) and accept the evidence only from them. For some parameters the agency is better off approving harmful mergers rather than blocking them (Proposition 4); otherwise, 'bad' mergers would invest more resources into evidence production and, as a result, a bigger mistake is made by the agency. Ignoring some information is a type of commitment. Hence, Proposition 4 gives a value of commitment of not using the information.[26].

[26]If effort level e is a continuous choice variable $[0, +\infty)$, the mistakes are perfect substitutes, the agency can assign only probability 0 or 1 and ignores the effort level, then the optimal effort level for a 'good' merger type will always be higher than for a 'bad' merger

Introduction of application fees for merging firms together with commitment to reject all mergers when the evidence is not produced can help the agency to "separate" two merger types. As a result, it lowers the expected mistakes of the agency. Also this policy is relatively easy to implement.

If different types of mistakes are non-perfect substitutes, then the agency would prefer a fuzzy approval rule under a wider range of exogenous parameters. Although the choice of objective function was arbitrary, it showed that the change in the form of agency's objective function could lead to the emergence of fuzzy approval rules.

In the paper the agency uses mixed strategies in accepting efficiency defense arguments after observing certain signals as a tool to (partially) separate different merger types by changing approval probabilities and, consequently, firms' expected payoffs from a merger. The separation leads to a lower value of the expected mistake by the agency. There is a long-lasting discussion about the feasibility of mixed strategies in real life. Rubinstein (1991) provides an overview of rationale behind mixed strategies. One of them is Harsanyi's (1973) idea of purification. Mixing probabilities represent a distribution of preferences among bureaucrats within the agency. One can argue that really individual bureaucrats do not play mixed strategies, but that they slightly differ and, hence, because cases are randomly allocated to bureaucrats, the result is as if the agency plays a mixed strategy. At the same time, in some equilibria merging firms use mixed strategies between different effort levels. Why would a firm mix in a fixed proportion (say, with 40% High effort level and with 60% Low effort level)? One of the explanations could be that the 40-60 proportion reflects a divide between risk-loving and risk-averse people. Although both efforts give equal expected payoff, risk-averse agents would choose a High effort level and be approved with probability 1 if the evidence is produced, while risk-lovers would choose a Low effort level and

type (the situation of a "voluntary separation"). It happens because both merger types face identical cost function $C(e)$ but $\beta^{Yes}_{good}(e) > \beta^{Yes}_{bad}(e)$ and $\Pi_{good} > \Pi_{bad}$:
$argmax_e \ \beta^{Yes}_{g}(e)\Pi_{good} - C(e) > argmax_e \ \beta^{Yes}_{bad}(e)\Pi_{bad} - C(e)$.

be approved with probability p_{LY}^{PC}.

On the one hand, the idea that the agency is "flipping a coin" while accepting efficiency defense arguments[27] is hard to sell to lawyers, who request a consistency in decisions (or in other words the agency can play only pure strategies), but, on the other hand, if such behavior enhances the agency's welfare, I do not see why it should be excluded from the analysis. The paper shows that a regulatory agency prefers to maintain some randomization while making approval decisions.

An overly intense effort to produce evidence could be a sign that 'good' mergers are trying to increase their chances to get approval and benefit financially from the merger. Hence, a high effort level could be a signal about their efficiencies. 'Bad' merger types, on the other hand, could be putting more effort than is optimal for them to mimic 'good' mergers and to be approved. The prime difference from Spence's (1974) and Potters' and van Winden's (1992) models is that the choice of the effort level effects the probability of success. Overall, the antitrust agency can decrease the value of the total expected mistake in its decisions by paying attention to the effort level the evidence is produced with.

6 Conclusion

Results of the analysis show that fuzziness could be a rational choice of the regulatory agency. In the case of perfect substitutability of different types of mistakes the fuzziness brings lower expected total mistakes only under relatively strict restrictions on exogenous parameters. Another conclusion is that for some parameters the agency is better off approving potentially harmful mergers rather than blocking them; otherwise, 'bad' mergers would

[27]Motta (2004) discusses some inconsistencies in accepting efficiency defense arguments. Sanchirico (1997) discusses uncertainty in the court's final assessments when burden of proof lies on a party of interest.

invest more resources into evidence production. If the agency can commit to certain policies (probabilities), then it could lower expected mistakes by applying a fuzzy approval rule. If different types of mistakes are non-perfect substitutes for the agency then the fuzziness is welfare enhancing under a wider range of parameters.

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Appendix

As a reminder: $\beta_g^{Yes}(e^H) = a \quad \beta_b^{Yes}(e^H) = b \quad \beta_g^{Yes}(e^L) = c \quad \beta_b^{Yes}(e^L) = k$

Proposition 1:

If $\frac{a}{c} > \frac{b}{k}$, then strategy 2 is never optimal for the agency.

Proof: In other words, if $\frac{a}{c} > \frac{b}{k}$, then strategy 2 is always dominated either by strategy 9 or by strategy 1, i.e "00001" $\succ$ "01000" **and** "00001" $\succ$ "00000" is impossible. The payoff of the agency under strategy "00001" is $\alpha(1-c)+(1-\alpha)k$, the payoff under strategy "01000" is $\alpha(1-a)+(1-\alpha)b$, and the payoff under strategy "00000" is α.

$$\begin{cases} \alpha(1-c)+(1-\alpha)k < \alpha \\ \alpha(1-c)+(1-\alpha)k < \alpha(1-a)+(1-\alpha)b \end{cases} \Leftrightarrow \begin{cases} \alpha > \frac{k}{c+k} \\ \alpha < \frac{b-k}{a+b-c-k} \end{cases}$$

$\Leftrightarrow \frac{b-k}{a+b-c-k} > \alpha > \frac{k}{c+k} \Leftrightarrow \frac{b-k}{a+b-c-k} > \frac{k}{c+k} \Leftrightarrow \frac{a}{c} < \frac{b}{k}$. It is a contradiction.

Proposition 5:

Given the perfect substitutability of different types of mistakes and assumptions $a > b > k$ and $a > c > k$, mixing in any two decision nodes simultaneously is impossible.

Proof: Let μ_{HY}, μ_{HN}, μ_{LY}, μ_{LN} be proportions of 'good' mergers in respective decision nodes.

a) $\mu_{HY} \neq \mu_{HN} \Leftrightarrow \frac{\alpha\sigma_g a}{\alpha\sigma_g a+(1-\alpha)\sigma_b b} \neq \frac{\alpha\sigma_g(1-a)}{\alpha\sigma_g(1-a)+(1-\alpha)\sigma_b(1-b)} \Leftrightarrow \alpha^2\sigma_g a(1-a)+\alpha(1-\alpha)\sigma_b a(1-b) \neq \alpha^2\sigma_g a(1-a)+\alpha(1-\alpha)\sigma_b(1-a)b \Leftrightarrow a(1-b) \neq (1-a)b \Leftrightarrow$

$a \neq b$.
Analogously $\mu_{LY} \neq \mu_{LN}$.
b) $\mu_{HY} \neq \mu_{LY}$. Mixing is possible when there are equal proportions of 'good' and 'bad' merger in the decision nodes: $\frac{\alpha\sigma_g a}{\alpha\sigma_g a+(1-\alpha)\sigma_b b} = \frac{\alpha\sigma_g c}{\alpha\sigma_g c+(1-\alpha)\sigma_b k} = \frac{1}{2}$.
From these equalities we derive mixing probabilities on behalf of merger types: $\sigma_b = \frac{1-\frac{1-\alpha}{\alpha}\frac{c}{k}}{\frac{ak-cb}{ak}}$ and $\sigma_g = \frac{bk((1-\alpha)k-\alpha c)}{\alpha(ak-cb)}$. In order both mixing probabilities to be non-negative, condition $ak - cb > 0$ should hold.
Mergers can appear in both decision nodes if they mix High and Low effort levels, i.e. both effort levels give the same payoffs to both types:
$ap_{HY}\Pi_g - C(e^H) = cp_{LY}\Pi_g - C(e^L)$
$bp_{HY}\Pi_b - C(e^H) = kp_{LY}\Pi_b - C(e^L)$
From these conditions we can derive optimal approval probabilities on behalf of the agency: $p_{LY} = \frac{a\frac{C(e^H)-C(e^L)}{\Pi_b} - b\frac{C(e^H)-C(e^L)}{\Pi_g}}{bc-ak}$. In order for this probability to be non-negative condition $ak - cb < 0$, which contradicts the previous condition.
Analogously $\mu_{HN} \neq \mu_{LN}$.

Derivative of an optimal probability of approval p^*_{es} at any decision node (e, s) with respect to α is non-negative: $\frac{\partial p^*(e,s)}{\partial\alpha} \geq 0$.
$\frac{\partial p^*(e,s)}{\partial\alpha} = \frac{\sigma_g\beta_g^s(e)[\alpha\sigma_g\beta_g^s(e)+(1-\alpha)\sigma_b\beta_b^s(e)]-\alpha\sigma_g\beta_g^s(e)[\sigma_g\beta_g^s(e)-\sigma_b\beta_b^s(e)]}{[\alpha\sigma_g\beta_g^s(e)+(1-\alpha)\sigma_b\beta_b^s(e)]^2} =$
$= \frac{\sigma_g\sigma_b\beta_g^s(e)\beta_b^s(e)}{[\alpha\sigma_g\beta_g^s(e)+[(1-\alpha)\sigma_b\beta_b^s(e)]^2} \geq 0$

Mixing equilibria
ME1: $\overline{p} = (0, p_{HY}, 0, 0, 0), \sigma_g = (1, 0, 0), \sigma_b = (\sigma_b^H, 0, [1 - \sigma_b^H]), ETM = \alpha$
where $p_{HY} = \frac{C(e^H)}{b\Pi_b}$, $\sigma_b^H = \frac{\alpha}{(1-\alpha)}\frac{a}{b} \in [0, 1]$ for $\alpha \in [0, \frac{b}{a+b}]$

ME2: $\overline{p} = (0, 0, 0, 0, p_{LY}), \sigma_g = (0, 1, 0), \sigma_b = (0, \sigma_b^L, [1 - \sigma_b^L]), ETM = \alpha$
where $p_{LY} = \frac{C(e^L)}{k\Pi_b}$, $\sigma_b^L = \frac{\alpha}{(1-\alpha)}\frac{c}{k} \in [0, 1]$ for $\alpha \in [0, \frac{k}{c+k}]$

ME3: $\overline{p} = (0, 1, 1, p_{LN}, 1)$, $\sigma_g = (\sigma_g^H, [1-\sigma_g^H], 0)$, $\sigma_b = (0, 1, 0)$, $ETM = 1-\alpha$
where $p_{LN} = \frac{1}{1-c}[1 - c - \frac{C(e^H)-C(e^L)}{\Pi_g}]$, $p_{LN} : \Pi_b - C(e^H) < [p_{LN}(1 - k) + k]\Pi_b - C(e^L)$
$\sigma_g^H = 1 - \frac{1-\alpha}{\alpha}\frac{1-k}{1-c} \in [0, 1]$ for $\alpha \in [\frac{1-k}{2-c-k}, 1]$

ME4: $\overline{p} = (0, 1, p_{HY}, 0, p_{LY})$, $\sigma_g = (\sigma_g^H, [1 - \sigma_g^H], 0)$, $\sigma_b = (\sigma_b^H, [1 - \sigma_b^H], 0)$, $ETM = \alpha(1 - a) + (1 - \alpha)b$.
$\Pi_g(a + (1 - a)p_{HN}^{ME4}) - C(e^H) = \Pi_g c p_{LY}^{ME4}) - C(e^L)$
$\Pi_b(b+(1-b)p_{HN}^{ME4}) - C(e^H) = \Pi_b k p_{LY}^{ME4}) - C(e^L)$ $\sigma_b^H = \frac{\alpha(1-a)c-(1-\alpha)(1-a)k}{(1-\alpha)(1-b)c-(1-\alpha)(1-a)k} \in [0, 1]$

ME5: $\overline{p} = (0, 1, 1, 0, p_{LY})$, $\sigma_g = (\sigma_g^H, [1 - \sigma_g^H], 0)$, $\sigma_b = (0, 1, 0)$, $ETM = (1 - \alpha)\frac{k}{c}$.
$p_{LY}^{ME5} = \frac{1}{c}(1 - \frac{C(e^H)-C(e^L)}{\Pi_g}) \in [0, 1]$ and $\Pi_b - C(e^H) < p_{LY}^{ME5} k \Pi_b - C(e^L)$
$\sigma_g^H = 1 - \frac{1-\alpha}{\alpha}\frac{k}{c} \in [0, 1]\ for\ \alpha \in [\frac{k}{c+k}, 1]$

Proposition 6:

Mixing equilibrium 4 gives lower expected mistake than:
a) E2 on the interval $(\frac{k}{c+k}; 1)$ if $\frac{a}{c} < \frac{b}{k}$;
b) E1 on the interval $(\frac{k-cb}{k-cb+c(1-a)}; 1)$ if $\frac{a}{c} > \frac{b}{k}$.
Proof: a) ME4 has an expected mistake $(1 - \alpha)\frac{k}{c}$ which is always lower than $(1 - \alpha)$, the mistake when all mergers are approved under strategy "11111"
b) if $\frac{a}{c} < \frac{b}{k}$, ME4 has lower mistake than equilibrium E2a and E2b on the interval $(\frac{k}{c+k}; 1)$ because $(1 - \alpha)\frac{k}{c} < \alpha$ and $(1 - \alpha)\frac{k}{c} < \alpha(1 - c) + (1 - \alpha)k$ when $\alpha > \frac{k}{c+k}$. Equilibrium E2c has higher expected mistake than E2b when $\alpha = \frac{k}{c+k}$ and higher then ME4 when $\alpha = \frac{k}{c+k}$.
c) ME4 has a lower mistake than equilibrium E1b when $(1 - \alpha)\frac{k}{c} < \alpha(1 - a) + (1 - \alpha)b$, i.e. when $\alpha = \frac{k-cb}{k-cb+c(1-a)} \geq 0$. This value of α is always larger than $\frac{b}{a+b}$, where E1a and E1b give the same mistake if $\frac{a}{c} > \frac{b}{k}$. Hence, ME4 has a lower expected mistake on the interval $(\frac{k-cb}{k-cb+c(1-a)}; 1)$ if $\frac{a}{c} > \frac{b}{k}$.

Full commitment equilibrium:

a) $\alpha \in [0, \alpha_1), p = (0, p_{HY}^{FC}, 0, 0, 0), \sigma_g = (1, 0, 0), \sigma_b = (0, 0, 1)$,
$ETM = \alpha - \alpha\ a\ p_{HY}^{FC}$
b) $\alpha \in [\alpha_1, \alpha_2), p = (0, 1, 0, 0, p_{LY}^{FC}), \sigma_g = (1, 0, 0), \sigma_b = (0, 1, 0)$,
$ETM = \alpha(1 - a) + (1 - \alpha)\ k\ p_{LY}^{FC}$
c) $\alpha \in [\alpha_2, 1], p = (0, 1, 1, p_{LN}^{FC}, 1), \sigma_g = (1, 0, 0)$, $\sigma_b = (0, 1, 0)$,
$ETM = (1 - \alpha)\ k + (1 - \alpha)\ (1 - k)\ p_{LN}^{FC}$

where $\alpha_1 = \frac{b-k-\frac{C^H-C^L}{\Pi_b}}{a(1-\frac{C^H}{b\Pi_b})+(b-k-\frac{C^H-C^L}{\Pi_b})}$, $\alpha_2 = \frac{1-b}{2-a-b}$,
$p^{FC}_{HY} = \frac{C(e^H)}{b\ \Pi_b} - \varepsilon$, $p^{FC}_{LN} = \frac{1}{1-k}[1-k-\frac{C(e^H)-C(e^L)}{\Pi_b}] + \varepsilon$,
$p^{FC}_{LY} = \frac{1}{k}[b - \frac{C(e^H)-C(e^L)}{\Pi_b}] + \varepsilon$, $p^{FC}_{LY} : a\ \Pi_g - C(e^H) > p^{FC}_{LY}\ c\ \Pi_b - C(e^L)$, $\varepsilon \to 0$.
If $p_{HY} = \frac{C(e^H)}{b\ \Pi_b} - \varepsilon$, $p_{LY} = \frac{C(e^L)}{k\ \Pi_b} - \varepsilon$ and $\frac{C^H}{C^L} > \frac{cb}{ak}$, then
$"(0, p_{HY}, 0, 0, 0)" \succ "(0, 0, 0, 0, p_{LY})"$ by the agency.

Proof: $\alpha a(1-p_{HY}) + \alpha(1-a) < \alpha c(1-p_{LY}) + \alpha(1-c) \Rightarrow$
$\alpha a(1-\frac{C(e^H)}{b\ \Pi_b}) + \alpha(1-a) < \alpha c(1-\frac{C(e^L)}{k\ \Pi_b}) + \alpha(1-c) \Rightarrow \frac{C^H}{C^L} > \frac{cb}{ak}$ (if $\frac{a}{c} > \frac{b}{k}$, then it is always true).

Proposition 7:
If the commitment to disapprove all mergers that haven't produced the evidence is credible, $\frac{a}{c} < \frac{b}{k}$, and $p^{PC}_{LY} :\ b\ \Pi_b - C(e^H) < p^{PC}_{LY}\ k\ \Pi_b - C(e^L)$, then the agency has lower total mistake on the interval $\alpha \in (\frac{k}{c+k}; \frac{1-a\frac{k}{c}}{2-a-a\frac{k}{c}})$ than equilibrium strategies ES2.
Proof: Strategy $\overline{p} = (0100p^{PC}_{LY})$, when $\sigma_g = (\sigma^H_g, [1-\sigma^H_g], 0), \sigma_b = (0,1,0)$, $\sigma^H_g = [1 - \frac{(1-\alpha)}{\alpha}\frac{k}{c}]$ to make the agency indifferent, $p^{PC}_{LY} = \frac{1}{c}(a - \frac{C(e^H)-C(e^L)}{\Pi_g})$, given that $p^{PC}_{LY} : b\ \Pi_b - C(e^H) < p^{PC}_{LY}\ k\ \Pi_b - C(e^L)$, to make a 'good' merger indifferent, gives a lower expected total mistake than:
1. Strategy $\overline{p} = (01000)$, if $\alpha\sigma^H_g(1-a) + \alpha(1-\sigma^H_g)(1-c\ p^{PC}_{LY}) + (1-\alpha)k\ p^{PC}_{LY} < \alpha(1-a) + (1-\alpha)(1-b)$
After plugging in formulas for σ^H_g and p^{PC}_{LY} in this inequality we receive:
$\alpha[1 - \frac{(1-\alpha)}{\alpha}\frac{k}{c}](1-a) + \alpha(1-[1-\frac{(1-\alpha)}{\alpha}\frac{k}{c}])(1 - c\ \frac{1}{c}(a - \frac{C(e^H)-C(e^L)}{\Pi_g})) + (1-\alpha)k\ \frac{1}{c}(a - \frac{C(e^H)-C(e^L)}{\Pi_g}) < \alpha(1-a) + (1-\alpha)(1-b) \quad \Leftrightarrow \quad \frac{a}{k} < \frac{b}{c}$
2. Strategy $\overline{p} = (1,1,1,1,1)$, if $\alpha\sigma^H_g(1-a) + \alpha(1-\sigma^H_g)(1-c\ p^{PC}_{LY}) + (1-\alpha)k\ p^{PC}_{LY} < 1-\alpha$. This inequality holds if $\alpha < \frac{1-a\frac{k}{c}}{2-a-a\frac{k}{c}}$
3. Given that $\sigma^H_g = [1 - \frac{(1-\alpha)}{\alpha}\frac{k}{c}] \in [0,1]$, strategy $\overline{p} = (0100p^{PC}_{LY})$ exists only on the interval $\alpha \in (\frac{k}{c+k}; 1]$. On this interval it gives lower mistake than strategies (00000) and (00001).

Proposition 8:
Proof: Let's look at decision node (e, s). Optimal probability is

$p^*(e,s) = \frac{\alpha\sigma_g\beta_g^s(e)}{\alpha\sigma_g\beta_g^s(e)+(1-\alpha)\sigma_b\beta_b^s(e)}$. Given that $\beta_i(e,s) \in (0,1)$, the optimal probability is always greater than zero, $p^*(e,s) > 0$, because $\alpha\sigma_g\beta_g^s(e) > 0$ when $\alpha > 0$ and $\sigma_g > 0$, i.e. there are some 'good' mergers in the decision node (e,s). Optimal probability is always less than one, $p^*(e,s) < 1$, if $(1-\alpha)\sigma_b\beta_b^s(e) > 0$, i.e. when $\alpha < 1$ and $\sigma_b > 0$, which means that there are some 'bad' mergers in the decision node (e,s). **Q.E.D**

Optimal policies for a quadratic function

$\overline{p} = (0,0,0,0,0), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = \alpha$

$\overline{p} = (1,1,1,1,1), \sigma_g = (0,0,1), \sigma_b = (0,0,1), ETM = (1-\alpha)$

$\overline{p} = (0, p_{HY}, p_{HN}, 0, 0), \sigma_g = (1,0,0), \sigma_b = (1,0,0),$

$ETM = \alpha a(1-p_{HY})^2 + (1-\alpha)bp_{HY}^2 + \alpha(1-a)(1-p_{HN})^2 +$

$+(1-\alpha)(1-b)p_{HN}^2$

where $p_{HY} = \frac{\alpha a}{\alpha a+(1-\alpha)b}$ and $p_{HN} = \frac{\alpha(1-a)}{\alpha(1-a)+(1-\alpha)(1-b)}$

$\overline{p} = (0,0,0,p_{LN},p_{LY}), \sigma_g = (0,1,0), \sigma_b = (0,1,0),$

$ETM = \alpha(1-c)(1-p_{LN})^2 + (1-\alpha)(1-k)p_{LN}^2 +$

$\alpha c(1-p_{LY})^2 + (1-\alpha)kp_{LY}^2$

where $p_{LN} = \frac{\alpha(1-c)}{\alpha(1-c)+(1-\alpha)(1-k)}$ and $p_{LY} = \frac{\alpha c}{\alpha c+(1-\alpha)k}$

If $0 < \Pi_g - C(e^H) = [cp_{LY} + (1-c)p_{LN}]\ \Pi_g - C(e^L)$

$0 < \Pi_b - C(e^H) < [kp_{LY} + (1-k)p_{LN}]\ \Pi_b - C(e^L)$

$\overline{p} = (0,1,1,p_{LN},p_{LY}),\ \sigma_g = (\sigma_g^H, [1-\sigma_g^H], 0),\ \sigma_b = (0,1,0),$

$ETM = \alpha(1-\sigma_g)(1-c)(1-p_{LN})^2 + (1-\alpha)(1-k)p_{LN}^2 +$

$\alpha(1-\sigma_g)c(1-p_{LY})^2 + (1-\alpha)kp_{LY}^2$

where $p_{LN} = \frac{[R_g-c][C-K]-K}{2(1-c)[C-K]} + \sqrt{[\frac{[R_g-c][C-K]-K}{2(1-c)[C-K]}]^2 + \frac{R_gK}{(1-c)[C-K]}} \quad \in (0,1),$

$p_{LY} = \frac{1}{c}[(1-R_g)-(1-c)p_{LN}]$, $\sigma_g = 1 - \frac{1-\alpha}{\alpha}\frac{1-k}{1-c}\frac{p_{LN}}{1-p_{LN}}$

where $R_g = \frac{C^H-C^L}{\Pi_g}$, $C = \frac{c}{(1-c)}$, $K = \frac{k}{(1-k)}$.

Equilibrium exists only when $\sigma_g^H \in [0,1]$, i.e. on the interval:

$\alpha \in [\frac{(1-k)\frac{p_{LN}}{1-p_{LN}}}{(1-k)\frac{p_{LN}}{1-p_{LN}}+(1-c)}, 1]$.

Global mixing equilibrium (Quadratic Eq.5):

Equilibrium 5 might be derived from the following conditions:

$$\begin{cases} (p_{HN}-p_{LN})+a(p_{HY}-p_{HN})+c(p_{LN}-p_{LY})=\frac{C^H-C^L}{\Pi_g} & (1)\\ (p_{HN}-p_{LN})+b(p_{HY}-p_{HN})+k(p_{LN}-p_{LY})=\frac{C^H-C^L}{\Pi_b} & (2)\\ \alpha\sigma_g a(1-p_{HY})=(1-\alpha)\sigma_b b p_{HY} & (3)\\ \alpha\sigma_g(1-a)(1-p_{HN})=(1-\alpha)\sigma_b(1-b)p_{HN} & (4)\\ \alpha(1-\sigma_g)(1-c)(1-p_{LN})=(1-\alpha)(1-\sigma_b)(1-k)p_{LN} & (5)\\ \alpha(1-\sigma_g)c(1-p_{LY})=(1-\alpha)(1-\sigma_b)kp_{LY} & (6) \end{cases}$$

From equation 3, 4, 5, 6, respectively:

$p_{HY}=\frac{1}{\frac{1-\alpha}{\alpha}\frac{b}{a}\frac{\sigma_b^H}{\sigma_g^H}+1}$, $p_{HN}=\frac{1}{\frac{1-\alpha}{\alpha}\frac{1-b}{1-a}\frac{\sigma_b^H}{\sigma_g^H}+1}$, $p_{LN}=\frac{1}{\frac{1-\alpha}{\alpha}\frac{1-k}{1-c}\frac{1-\sigma_b^H}{1-\sigma_g^H}+1}$,
$p_{LY}=\frac{1}{\frac{1-\alpha}{\alpha}\frac{k}{c}\frac{1-\sigma_b^H}{1-\sigma_g^H}+1}$

Denote $x_1=\frac{\sigma_b^H}{\sigma_g^H}$, $x_2=\frac{1-\sigma_b^H}{1-\sigma_g^H}$, $a_1=\frac{1-\alpha}{\alpha}\frac{b}{a}$, $a_2=\frac{1-\alpha}{\alpha}\frac{1-k}{1-c}$, $b_1=\frac{1-\alpha}{\alpha}\frac{1-b}{1-a}$,
$b_2=\frac{1-\alpha}{\alpha}\frac{k}{c}$,
$R_g=\frac{C^H-C^L}{\Pi_g}$, $R_b=\frac{C^H-C^L}{\Pi_b}$

Then $p_{HY}=\frac{1}{a_1x_1+1}$, $p_{HN}=\frac{1}{b_1x_1+1}$,$p_{HY}=\frac{1}{a_2x_2+1}$,$p_{HY}=\frac{1}{b_2x_2+1}$. Then

$$x_1=\frac{1}{a_1}(\frac{1}{p_{HY}}-1)=\frac{1}{b_1}(\frac{1}{p_{HN}}-1)\Rightarrow a_1p_{HY}(1-p_{HN})=b_1p_{HN}(1-p_{HY}) \quad (7)$$
$$x_2=\frac{1}{a_2}(\frac{1}{p_{LN}}-1)=\frac{1}{b_2}(\frac{1}{p_{LY}}-1)\Rightarrow a_2p_{LN}(1-p_{LY})=b_2p_{LY}(1-p_{LN}) \quad (8)$$

From equation (1) and (2)

$\begin{cases} (p_{HN}-p_{LN})+a(p_{HY}-p_{HN})+c(p_{LN}-p_{LY})=R_g\\ (p_{HN}-p_{LN})+b(p_{HY}-p_{HN})+k(p_{LN}-p_{LY})=R_b \end{cases}$ we can derive

$p_{HY}=\frac{1}{a-b}((R_g+(1-c)p_{LN}+cp_{LY})(1-b)-(R_b+(1-k)p_{LN}+kp_{LY})(1-a))=$
$\frac{(1-c)(1-b)-(1-k)(1-a)}{a-b}p_{LN}+\frac{c(a-b)-k(1-a)}{a-b}p_{LY}+\frac{R_g(1-b)-R_b(1-a)}{a-b}\equiv$
$\alpha_1p_{LN}+\beta_1p_{LY}+\gamma_1$

$p_{HN}=\frac{1}{a-b}(a(R_b+(1-k)p_{LN}+kp_{LY})-b(R_g+(1-c)p_{LN}+c$

$p_{LY}))=\frac{a(1-k)-b(1-c)}{a-b}p_{LN}+\frac{ak-bc}{a-b}p_{LY}+\frac{aR_g-bR_b}{a-b}\equiv\alpha_2p_{LN}+\beta_2p_{LY}+\gamma_2$

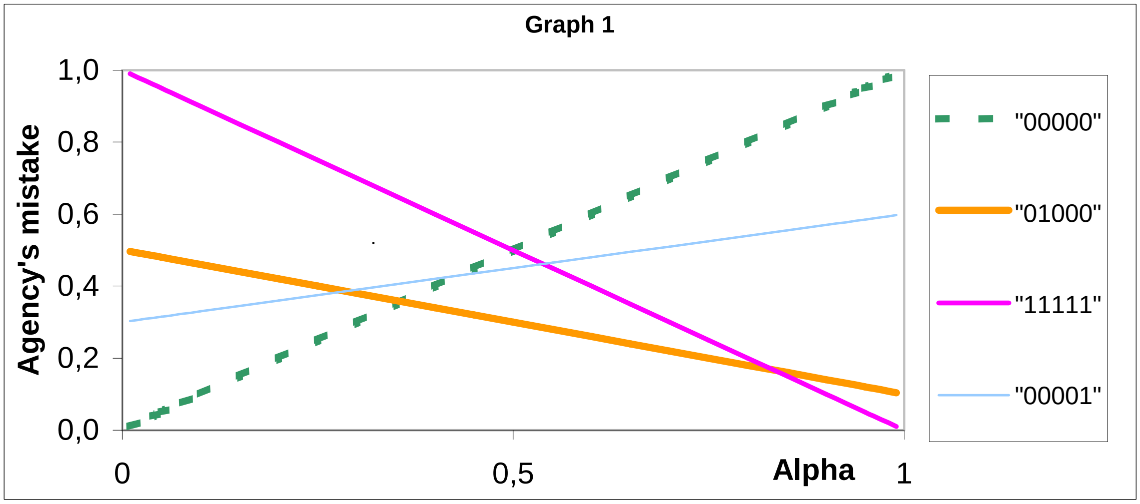
where $\alpha_1=\frac{(1-c)(1-b)-(1-k)(1-a)}{a-b}$,$\beta_1=\frac{c(a-b)-k(1-a)}{a-b}$,$\gamma_1=\frac{R_g(1-b)-R_b(1-a)}{a-b}$,

$\alpha_2 = \frac{a(1-k)-b(1-c)}{a-b}, \beta_2 = \frac{ak-bc}{a-b}, \gamma_2 = \frac{aR_g-bR_b}{a-b}$

Plug it into equation (7):
$a_1(\alpha_1 p_{LN} + \beta_1 p_{LY} + \gamma_1)(1 - \alpha_2 p_{LN} - \beta_2 p_{LY} - \gamma_2) =$
$b_1(\alpha_2 p_{LN} + \beta_2 p_{LY} + \gamma_2)(1 - \alpha_1 p_{LN} - \beta_1 p_{LY} - \gamma_1);$

$p_{LN}^2 * (-a_1\alpha_1\alpha_2 + b_1\alpha_1\alpha_2) + p_{LY}^2(-a_1\beta_1\beta_2 + b_1\beta_1\beta_2) + p_{LN}p_{LY}(-a_1\alpha_1\beta_2 - a_1\beta_1\alpha_2 + b_1\alpha_2\beta_1 + b_1\alpha_1\beta_2) + p_{LN}(a_1\alpha_1(1-\gamma_2) - a_1\gamma_1\alpha_2 - b_1\alpha_2(1-\gamma_1) + b_1\gamma_2\alpha_1) + p_{LY}(a_1\beta_1(1-\gamma_2) - a_1\gamma_1\beta_2 - b_1\beta_2(1-\gamma_1) + b_1\gamma_2\beta_1) + a_1\gamma_1(1-\gamma_2) - b_1\gamma_2(1-\gamma_1) = 0 \quad (9)$

This is a quadratic form. By linear change of variables it can be transformed to a canonical form like $A\tilde{p_{LN}}^2 + B\tilde{p_{LY}}^2 = C$. Also, equation(8) needs to be transformed by the same transformation, which will be another quadratic form. The solution will depend on signs of A, B, C and the coefficients in transformed (8). It can be from zero to four solutions. From equation (8) we can receive: $p_{LN} = \frac{1}{1+\frac{a_2}{b_2}(\frac{1}{p_{LY}}-1)}$ and plug it into equation (9) to obtain an equation with one unknown (p_{LY}):
$(\frac{1}{1+\frac{a_2}{b_2}(\frac{1}{p_{LY}}-1)})^2 * (-a_1\alpha_1\alpha_2 + b_1\alpha_1\alpha_2) + p_{LY}^2(-a_1\beta_1\beta_2 + b_1\beta_1\beta_2) +$
$(\frac{1}{1+\frac{a_2}{b_2}(\frac{1}{p_{LY}}-1)})p_{LY}(-a_1\alpha_1\beta_2 - a_1\beta_1\alpha_2 + b_1\alpha_2\beta_1 + b_1\alpha_1\beta_2) +$
$(\frac{1}{1+\frac{a_2}{b_2}(\frac{1}{p_{LY}}-1)})(a_1\alpha_1(1-\gamma_2) - a_1\gamma_1\alpha_2 - b_1\alpha_2(1-\gamma_1) + b_1\gamma_2\alpha_1) +$
$p_{LY}(a_1\beta_1(1-\gamma_2) - a_1\gamma_1\beta_2 - b_1\beta_2(1-\gamma_1) + b_1\gamma_2\beta_1) + a_1\gamma_1(1-\gamma_2) - b_1\gamma_2(1-\gamma_1) = 0$
The solution is obtained by using MATEMATICA software.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0,4
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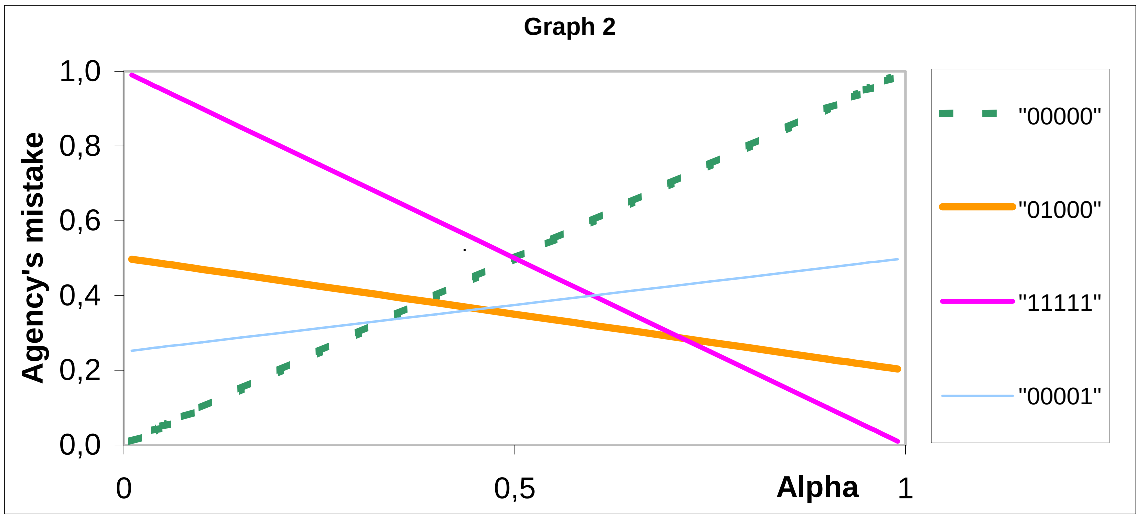
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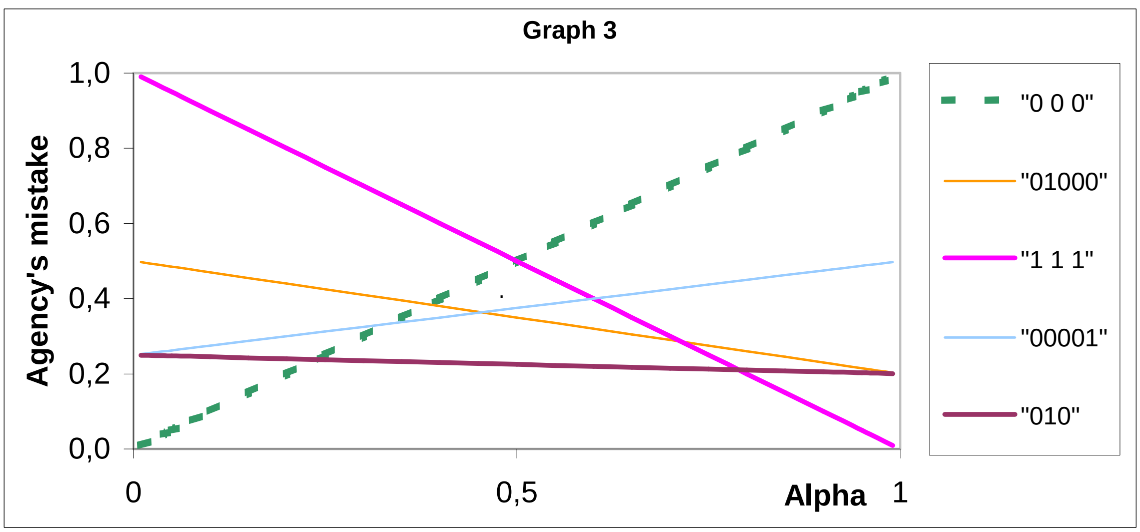
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0,5
Alpha
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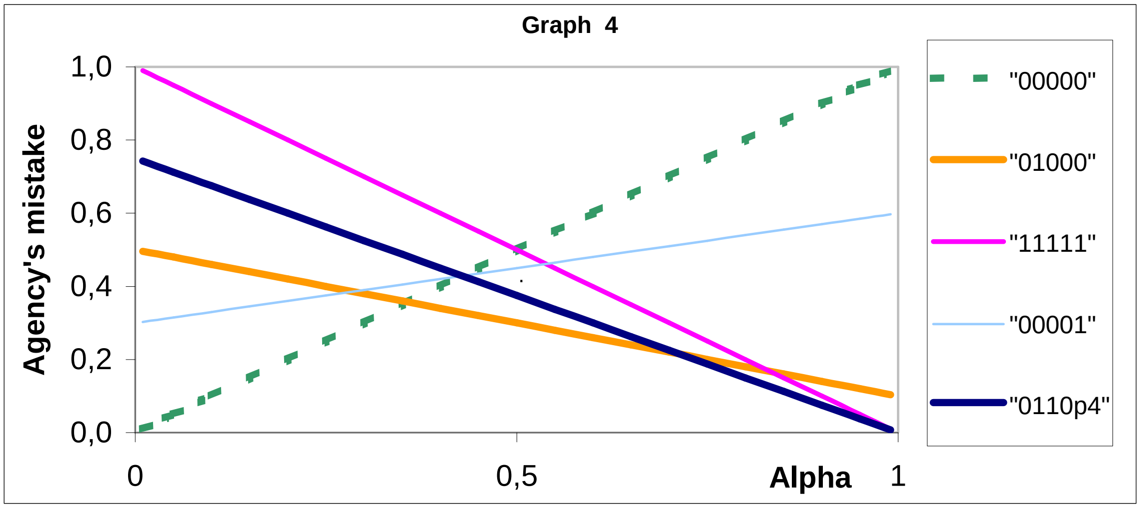
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Agency's mistake
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Alpha
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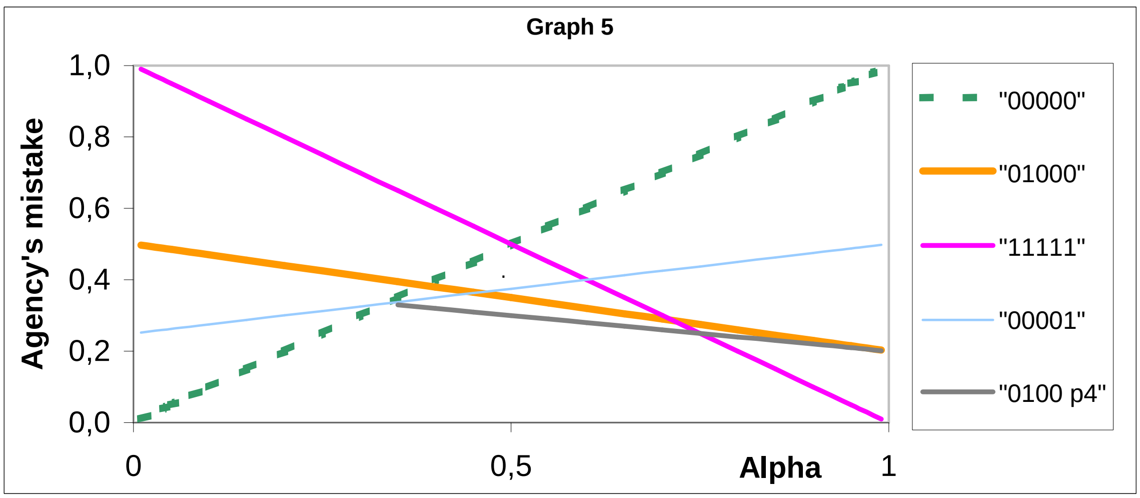
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0,4
0,2
0,0
Agency's mistake
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Alpha
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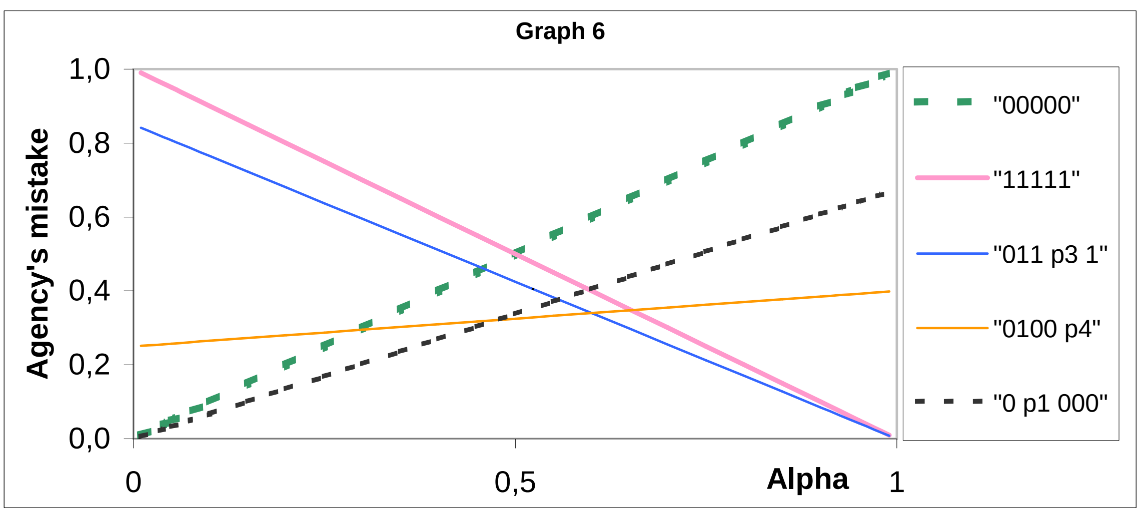
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0,5
Alpha
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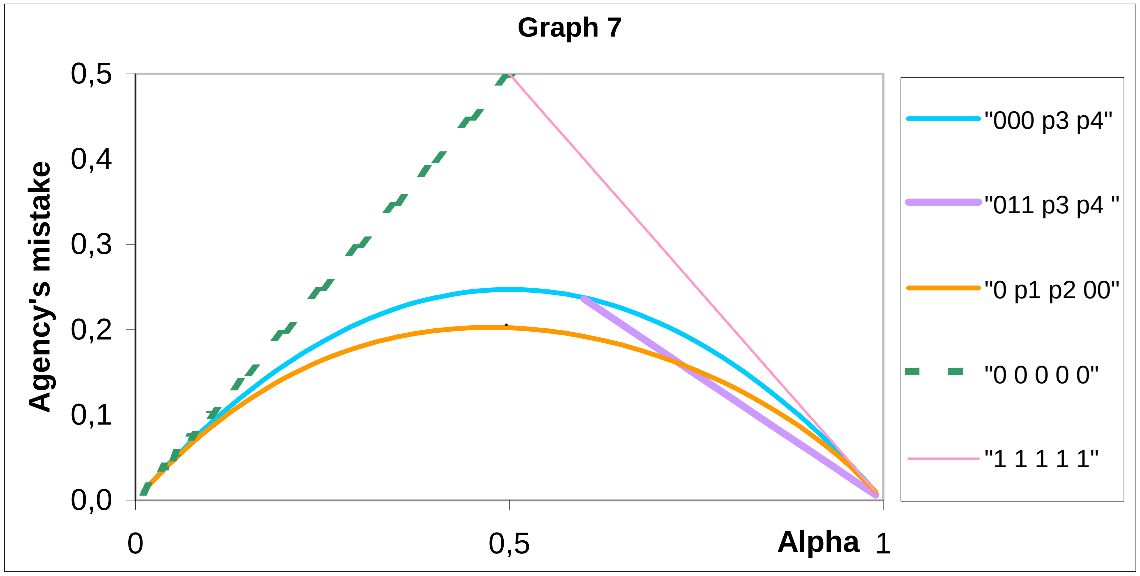
"11111"
"011 p3 1"
"0100 p4"
"0 p1 000"



Graph 7
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0,4
0,3
0,2
0,1
0,0
Agency's mistake
0
0,5
Alpha
1
"000 p3 p4"
"011 p3 p4 "
"0 p1 p2 00"
"0 0 0 0 0"
"1 1 1 1 1"

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