Firm Dynamics and Earnings Risk*

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Abstract

We study the role of firm and worker level shocks for individual labor earnings dynamics. A key feature of the distribution of earnings changes is excess kurtosis, with substantial earnings changes for a significant proportion of workers. Using Danish matched employer-employee data, we show that large worker earnings changes occur along the entire firm revenue growth distribution but more frequently in the tails. In particular, large earnings losses are more likely in shrinking firms due to more employment separations, but also because of large wage losses of stayers. We interpret the evidence through the lens of an equilibrium search model with two-sided heterogeneity. The model reveals that while worker shocks account for the majority of earnings fluctuations, firm shocks generate around a third of endogenous separations and large wage losses for stayers. Finally, the model implies significant endogenous responses of earnings dynamics to policy changes aiming to insure workers directly or indirectly through firms.

Keywords: Earnings Risk, Firm Dynamics, Search, Unemployment, Wages, Policy
JEL Codes: E24, E60, J63, J64, J68, H30

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1 Introduction

Governments invest vast resources to mitigate individuals' exposure to labor market risks, both through direct payments to workers and efforts to preserve matches, the benefits and costs of which depend on the endogenous response of heterogeneous firms and workers.[1] To design such policies, it is hence essential to understand the sources of individual labor earnings dynamics. A quickly growing literature documents that higher order moments are crucial to accurately characterize earnings dynamics (Guvenen, Karahan, Ozkan, and Song 2021). Most individuals experience very small earnings changes in any given year, but there is a sizable tail with very large earnings changes. The welfare costs of being exposed to an earnings process with large tail risks are significant.[2]

In this paper, we investigate the forces underlying worker earnings dynamics and in particular the drivers of large earnings changes, differentiating between unemployment and wage risk and disentangling the importance of firm- and worker-level shocks along both margins. First, we provide evidence on the joint distribution of worker earnings growth and firm revenue growth from Danish matched employer-employee data. Second, we build an equilibrium random search model with rich worker and firm heterogeneity, providing a structural interpretation of the observed firm and worker dynamics. Finally, we use the model to show that equilibrium interactions between heterogeneous workers and firms are crucial to understand the aggregate and distributional effects of policies.

Three key features stand out in the data. First, median earnings growth is remarkably stable along the entire firm revenue growth distribution. Second, in contrast to the median, the tails of the earnings growth distribution are strongly related to the revenue growth of employers. In particular, large earnings losses are more likely in shrinking firms. Third, this pattern is driven both by a higher likelihood of separations and wage losses for stayers.

In the model, earnings dynamics are driven by firm- and worker productivity shocks, unemployment risk through exogenous and endogenous separations, wage setting with two-sided limited commitment, and a job ladder. Firm- and worker level shocks account for downward earnings risk by potentially causing endogenous separations or downward wage renegotiations. In the model, roughly one third of earnings losses through these events are caused by firm level shocks.

[1] See Giupponi, Landais, and Lapeyre (2022) for a review of how different countries have used different approaches to insure individuals against earnings losses.

[2] See Guvenen, Karahan, and Ozkan (2018), Madera (2019), De Nardi, Fella, and Paz-Pardo (2020), Constantinides (2021), and Busch and Ludwig (2023) for a variety of approaches to measure the welfare costs of higher order moments of idiosyncratic earnings risk.

We begin the analysis by providing evidence on the joint distribution of firm revenue growth and worker earnings growth using Danish administrative matched employer-employee data. The point of departure is the distribution of annual worker earnings growth, which has been analyzed extensively in the recent literature. A key property is that the majority of individuals has stable earnings from one year to the next, but that the distribution has fat tails with a significant proportion facing sizable earnings changes, which can be summarized by excess kurtosis.[3] We add to these facts by addressing two main questions in the data. First, how strongly is the likelihood of large earnings changes linked to the revenue growth of the firms workers are employed in? Second, is this relation driven by separations to unemployment or by earnings losses for stayers?

We document that on average there is a positive relation between worker earnings growth and firm revenue growth, and that this relation is primarily driven by the tails. Large earnings changes occur along the entire distribution of firm revenue growth, but they are more concentrated in the tails. This is to a significant extent driven by the lower tail of the earnings growth distribution: it is much more likely to experience very large earnings losses in shrinking rather than in growing firms. The 10th percentile of the earnings growth distribution in the firms that shrink by 40-50% is around -25%, whereas it is only -10% in the strongly growing firms. In contrast to the strong relation between between firm growth and the tails of the earnings growth distribution, median earnings growth is almost constant along the entire firm growth distribution.[4]

The very largest earnings losses are driven by unemployment risk, which strongly differs along the firm revenue growth distribution. Interestingly, however, large earnings changes are not exclusively driven by unemployment, but are also more likely in firms with large revenue changes for workers who remain continuously employed at the same firm. At the bottom of the firm distribution, the 10th percentile of the stayer earnings growth distribution is below -10%, while it is closer to -5% at the top. By contrast, at the top there is a larger probability of very large gains for stayers, with the 90th percentile of the earnings growth distribution increasing in firm revenue growth. We finally show that these dynamics for stayers are to a large extent driven by wages rather than hours.

[3] This feature is emphasized in the seminal contribution of Guvenen, Karahan, Ozkan, and Song (2021) based on U.S. social security data. Similar findings across a range of countries are provided by the Global Repository of Income Dynamics (GRID) described in Guvenen, Pistaferri, and Violante (2022). The first version of the database covers Argentina, Brazil, Canada, Denmark, France, Germany, Italy, Mexico, Norway, Spain, Sweden, the UK, and the U.S. Most relevant for this paper is Leth-Petersen and Sæverud (2022), which documents strong excess kurtosis as the key feature of Danish earnings dynamics.

[4] As part of the GRID project, Bowlus, Gouin-Bonenfant, Liu, Lochner, and Park (2022) documents that average earnings growth strongly increases with firm employment growth and that its dispersion is U-shaped in firm employment growth, using Canadian data.

We develop a random search model with rich worker and firm heterogeneity, to provide a structural interpretation of the evidence and to study the equilibrium consequences of policies aiming to insure workers against earnings fluctuations. Importantly for the question, and in deviation from most equilibrium search models, the model features risk averse workers and productivity shocks to both workers and firms.

Stability in earnings for many, with infrequent large changes, naturally calls for wage-setting with two-sided limited commitment, where workers and firms agree on constant wages unless a participation constraint is violated. In that case, large wage adjustments or endogenous separations are triggered.[5] Additional exogenous separations capture job losses across the entire firm distribution. For upward earnings mobility, the model features a job ladder, with job-to-job transitions and renegotiations after outside offers. Endogenous search effort is important as it generates heterogeneity in unemployment spell length (Hubmer 2018). In interaction with the job creation decision of heterogeneous firms it is also crucial to study the equilibrium effects of policy changes.

We calibrate the model to reproduce important features of the Danish labor market. Key parameters are governing the productivity processes of heterogeneous firms and workers, as well as the search effort and job creation behavior of firms. Important moments disciplining these parameters are the wage distribution, firm size distribution, earnings growth distribution, revenue growth distribution, and labor market flows. The calibrated model captures the relation between worker earnings growth and firm revenue growth well. Most importantly, the model captures the higher incidence of unemployment and large wage losses in shrinking firms. It also generates larger earnings gains for workers starting out in growing firms. Finally, the model is consistent with stable median earnings growth along the firm growth distribution.

The quantified model allows to understand the fundamental drivers of earnings dynamics. In particular, large earnings losses are driven by exogenous separations, endogenous separations, and firm demanded downward wage renegotiations. The model calls for a large share of exogenous separations to generate earnings losses across the entire firm growth distribution. For endogenous separations and wage losses both worker and firm productivity shocks are important. Around a third of endogenous separations and firm demanded renegotiations are triggered by firm productivity shocks, almost 10% result as

[5] Long-term wage contracts between risk neutral firms and risk averse workers with two-sided limited commitment are first studied in Thomas and Worrall (1988), building on the work of Baily (1974) and Azariadis (1975) under commitment and Harris and Holmstrom (1982) with worker-side lack of commitment. An earlier application to the study of earnings dynamics of this wage-setting protocol is Postel-Vinay and Turon (2010).

a combination of both firm and worker productivity shocks, and the remainder results from worker productivity shocks.

With the equilibrium model that captures the determinants of earnings risk and where both workers and firms can adjust their job search and creation behavior, we can study the impact of policies designed to mitigate earnings fluctuations. Indeed, different countries employ available tools to insure workers against earnings fluctuations to very different degrees. In recent years, many European countries have heavily relied on employment protection, whereas the U.S. has focused on direct payments to workers. Progressive taxes are commonly used, but the degree of progressivity varies widely across countries.

We first study two policies targeting unemployment risk. We start with unemployment insurance. Higher unemployment benefits have the immediate benefit of providing insurance against the largest earnings losses. However, in equilibrium earnings dynamics respond in that unemployment spells get longer and reallocation to good firms is slowed down. Higher unemployment benefits strongly reduce search effort of unemployed workers. Also, workers require higher starting wages and this reduces job creation by firms. Hence, unemployment increases in equilibrium and aggregate output decreases. Employed workers suffer through slower reallocation towards good firms and because of higher taxes required to finance more unemployment benefits and other government spending, given lower output.

As an alternative policy to protect against unemployment risk, we consider layoff taxes, requiring firms to pay a penalty in case of an endogenous separation from a worker. This policy indeed can reduce unemployment risk for employed workers in response to productivity shocks. It has more favorable equilibrium effects than higher unemployment benefits. With layoff taxes, job creation by unproductive firms falls, as they are more likely to be affected by the layoff tax, but job creation by productive firms increases, as they face reduced competition from unproductive firms. Employment rises and output even more because of more workers being in better firms.

While the previous two policies are mostly targeted to insure against unemployment risk, a more progressive tax system aims to reduce after-tax earnings fluctuations across the entire distribution. However, this also implies that it has immediate efficiency costs along the entire distribution: Because progressive taxes reduce the return to climbing to the top of the job ladder, they reduce search effort of both unemployed and employed workers. This implies higher unemployment in equilibrium. Output falls even more than employment because of a worse allocation of workers across firms.

1.1 Related Literature

Earnings Dynamics. Departing from a large literature studying earnings dynamics under the assumption of Gaussian innovations,[6] recent literature has emphasized the importance of higher order moments to accurately characterize earnings dynamics. Key contributions are Guvenen, Karahan, Ozkan, and Song (2021), documenting stark deviations from normality such as negative skewness and excess kurtosis of the earnings growth distribution applying non-parametric methods to U.S. social security data, and Arellano, Blundell, and Bonhomme (2017), providing evidence for nonlinear and nonnormal earnings dynamics using parametric methods applied to U.S. data from the Panel Study of Income Dynamics. We expand on these analyses by linking the earnings growth distribution of workers to the revenue growth distribution of firms.

A number of papers share our interest in understanding the relative contributions of unemployment, wages, and hours to earnings dynamics. Using French and Dutch data, respectively, Pora and Wilner (2020) and De Nardi, Fella, Knoef, Paz-Pardo, and Van Ooijen (2021) find larger roles of hours and unemployment in accounting for higher order moments of the earnings change distribution, whereas Halvorsen, Holter, Ozkan, and Storesletten (2023) finds important contributions of hours, wages, and their covariance in Norwegian data.[7] We also find that while unemployment spells are responsible for the very largest earnings drops, for continuously employed workers wage changes play an important role.

On the theoretical side, Hubmer (2018) proposes a partial equilibrium job ladder model to account for higher order moments of earnings risk. McKay and Papp (2012), Graber (2018), Pascal (2020), Ai and Bhandari (2021), and Harmenberg (2021) propose search models to account for the cyclicality of income risk.

Pass-through. The paper also relates to a literature that estimates the pass-through from firm shocks to worker wages. The seminal paper by Guiso, Pistaferri, and Schivardi (2005) finds moderate pass-through from value-added shocks to wages using Italian data, but only if the shocks are permanent.[8] Three papers are closely related to ours in that

[6] See for example Lillard and Weiss (1979), MaCurdy (1981), Abowd and Card (1989).

[7] Concerning the business cycle dynamics of higher order moments, Hoffmann and Malacrino (2019) emphasizes a large role of hours rather than wages using Italian data. Busch, Domeij, Guvenen, and Madera (2022), by contrast, argues that the hours margin is not the main driver of cyclical fluctuations of higher order moments of earnings dynamics, based on data from the U.S., Germany, Sweden, and France.

[8] For similar analyses in other countries, see Lagakos and Ordonez (2011) and Juhn, McCue, Monti, and Pierce (2018) for the U.S., Guertzgen (2014) for Germany, Fagereng, Guiso, and Pistaferri (2017) for Norway, and Friedrich, Laun, Meghir, and Pistaferri (2021) for Sweden. Carlsson, Messina, and Skans (2016) and Souchier (2023) investigate the difference between firm level and sectoral productivity shocks.

they study the relation between firm outcomes and worker earnings using Danish data. Chan, Salgado, and Xu (2023) refines the methodology of Guiso, Pistaferri, and Schivardi (2005) and finds significantly larger pass-through of firm productivity shocks to wages, uncovering significant heterogeneity across margins such as firm productivity and size as well as worker age and tenure. Maibom and Vejlin (2021) considers employment stability in addition to wages. Bertheau, Kudlyak, Larsen, and Bennedsen (2023) designs a survey among Danish firms to understand when firms choose to layoff workers or cut wages, reporting that among firms experiencing a reduction in revenue 68% adjust employment and 29% adjust pay. Relative to these papers, we do not aim to establish a causal link from firm productivity to worker wages purely from the data, but instead provide a comprehensive picture of the entire joint distribution of firm revenue growth and worker earnings growth to inform our theory.

Theories of optimal contracting have implications for the pass-through from firm shocks to wages. In frameworks with labor market frictions, Tsuyuhara (2016) studies a contracting problem with moral hazard in search effort, Ábrahám, Alvarez-Parra, and Forstner (2017) one with moral hazard in production, and Lentz (2015) one with non-contractible search. Relative to this literature, we assume an empirically motivated rather than theoretically optimal wage setting rule, but consider a much richer equilibrium search model. The most closely related paper from this strand of the literature is Balke and Lamadon (2022), which studies pass-through in an equilibrium search model with optimal contracting under firm side commitment.[9] The key distinction of our paper is the focus on the link between large earnings changes and firm outcomes, in addition to modeling differences such as random rather than directed search.

Equilibrium Search Models. Our equilibrium search model with heterogeneous risk-averse workers and heterogeneous firms is most closely related to Bagger, Hejlesen, Sumiya, and Vejlin (2018), Bagger and Lentz (2019), and Gulyas (2023). The main focus of these papers is sorting between heterogeneous workers and firms and the resulting (mis-)allocation and wage distribution, whereas we focus on earnings dynamics. More broadly, the paper also relates to a quickly growing literature studying multi-worker firms in frictional labor markets. Important contributions in the random search tradition include Elsby and Michaels (2013), Moscarini and Postel-Vinay (2013), Bilal, Engbom, Mongey, and Violante (2022), Elsby and Gottfries (2022), McCrary (2022), and Audoly (2023). With directed search, key papers are Acemoglu and Hawkins (2014), Kaas and Kircher (2015), and Schaal (2017).

[9] Azzalini (2023) and Souchier (2023) extend this framework with aggregate risk.

Roadmap. The remainder of the paper proceeds as follows. In Section 2 we present the data and the empirical findings. In Section 3 we describe the equilibrium search model, which we bring to the data in Section 4. We perform the policy analysis in Section 5 and conclude in Section 6.

2 Evidence

2.1 Institutional Background

Denmark is an interesting laboratory to investigate unemployment and wage risk stemming from the interaction of firms and workers because its labor market is very flexible. Through labor market reforms in the 1990s, Denmark adopted a system that has become known as "flexicurity", combining income security through generous unemployment insurance and means-tested transfer programs with very flexible hiring and firing.[10] Hence, firms and workers are the relevant decision makers, who agree upon starting and ending employment relations and wage setting under relatively few constraints.

Employment flexibility. A key feature of Danish flexicurity is that it is easy to fire workers. In an OECD ranking of countries according to their strictness of employment protection, Denmark falls into the category with the weakest restrictions. Denmark does not require substantive conditions for a dismissal for economic reasons, in contrast to two thirds of OECD countries (OECD 2020).

Wage flexibility. There is no statutory minimum wage. While many employees are covered by collective bargaining agreements, for the vast majority of workers compensation is determined at the firm level. According to the Danish Employers' Association around 60% of workers are covered by collective agreements that only specify minimum pay requirements. These minima are meant to be binding only for very inexperienced workers and to impose a floor that cannot be violated (Dahl, Le Maire, and Munch 2013), but most individual wages are set at the firm level. 20% of workers are not covered by agreements specifying anything related to their compensation. For 20% of workers a collective agreement specifies their base wage, but even for those workers bonuses allow for individual deviations from the collective agreement.

A survey of Danish firms in Bertheau, Kudlyak, Larsen, and Bennedsen (2023) confirms that legal restrictions do not play a major role for preventing wage cuts in Denmark. The majority of firms that does not cut wages when facing revenue reductions states as

[10] For a detailed review of the features of "Danish flexicurity" see Kreiner and Svarer (2022).

reasons this could damage morale or employees could quit. The concern that this would be illegal or impossible to do plays a much smaller role.

2.2 Data

The evidence we present on the relationship between individual earnings and wage changes on the one hand and firm outcomes on the other hand is based on Danish administrative data. We combine information from several registries, which are described in detail in Appendix A.1. We restrict ourselves to the time period from 2008 to 2016 because for these years we have the highest quality information on employment spells and earnings.

On the worker side, we observe labor earnings at a monthly frequency. We observe the start and end date of an employment spell within a month and an identifier for the firm at which the worker is employed. If a worker is employed at several firms within a single month we observe all these spells.[11] The earnings measure is of high quality as it is third-party reported by tax authorities. Earnings are not top-coded. There is also an hours measure available, which allows us to construct a measure of hourly wages. Hours are actual hours worked for hourly workers and contractual hours plus any overtime for salaried workers. The data set covers the entire population.

On the firm side, we observe the universe of firms. We can link workers to their employers. In addition to the information on workers employed at the firms, we use information from value added tax data on firm sales.

2.3 Empirical Findings

Worker earnings dynamics. We start with documenting the key features of individual earnings dynamics. For that purpose, we aggregate monthly labor income to total annual earnings and compute earnings growth rates at the individual level. We transform nominal to real earnings using the consumer price index. We impose mild sample restrictions that are standard in the earnings risk literature, to ensure some labor force attachment. First, we restrict ourselves to individuals who are between 25 and 60 years old. Second, minimum annual earnings are set to DKK20,000, which corresponds to roughly €2,700

[11] For total earnings we add up earnings across all employers. For the analysis of the relationship between firm outcomes and worker earnings we mostly restrict the sample to workers only employed at one firm simultaneously; see the details below.

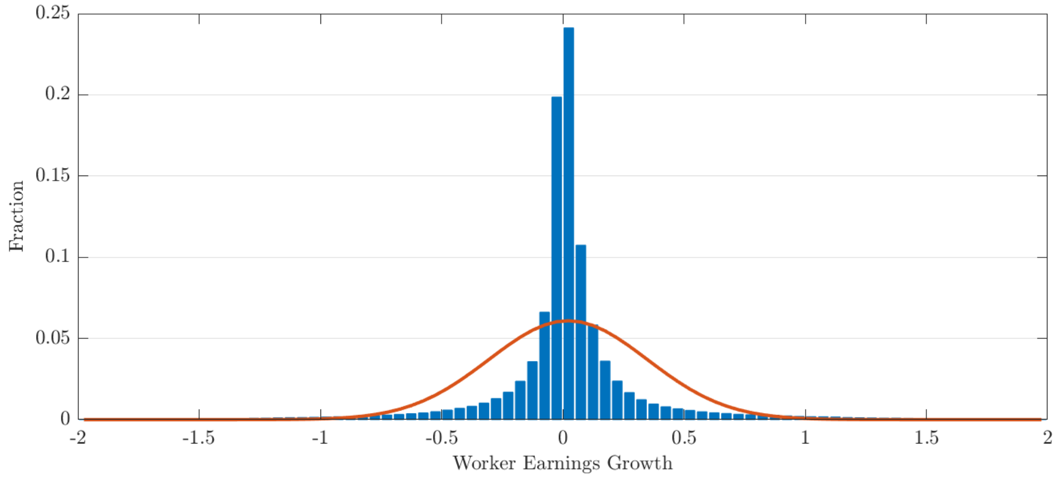

Figure 1: Annual Earnings Growth Distribution

Notes: This figure shows the annual earnings growth distribution for the entire sample. The red line compares a normal distribution with the same mean and standard deviation.

or $2,800. Third, we impose that individuals work for at least 200 hours within a year. We compute earnings growth rates as log-differences.[12]

Figure 1 shows the histogram of the annual worker earnings growth distribution, benchmarked against a normal distribution with the same mean and variance. This is essentially the replication of the well-known Figure 1 from Guvenen, Karahan, Ozkan, and Song (2021), which uses U.S. social security data to investigate higher order moments of the earnings growth distribution. The key feature of the Danish earnings growth distribution is excess kurtosis, with a kurtosis of 12 compared to 3 for a normal distribution. Compared to a normal distribution there is much more mass concentrated around zero, implying that the majority of workers experiences very small earnings changes in any given year. There is, however, also a significant mass of workers with very large earnings changes. The 10th percentile of the distribution is an earnings loss of 23%; the 90th percentile is an earnings gain of 25%.[13]

[12]For the results reported here, we use raw earnings. The results are similar when using residualized earnings after taking out education, gender, location, industry, and occupation. See Appendix A.2 for statistics of the annual earnings growth distribution using raw and residualized earnings.

[13]In contrast to the findings reported in Guvenen, Karahan, Ozkan, and Song (2021), the distribution of earnings changes in the Danish data is essentially symmetric. The skewness of the earnings growth distribution exhibits the same cyclical patterns as reported for U.S. data in Guvenen, Ozkan, and Song (2014), with negative skewness during the Great Recession years and positive skewness post-2010. This is in line with the findings in Leth-Petersen and Sæverud (2022), which reports positive skewness in booms and negative skewness in recessions also in a longer time series of Danish data. Note, however, that the level of the skewness is quite sensitive to the imposed minimum earnings threshold. We thus focus here on the feature that is extremely robust in our data, and across countries as reported in the literature, which is excess kurtosis.

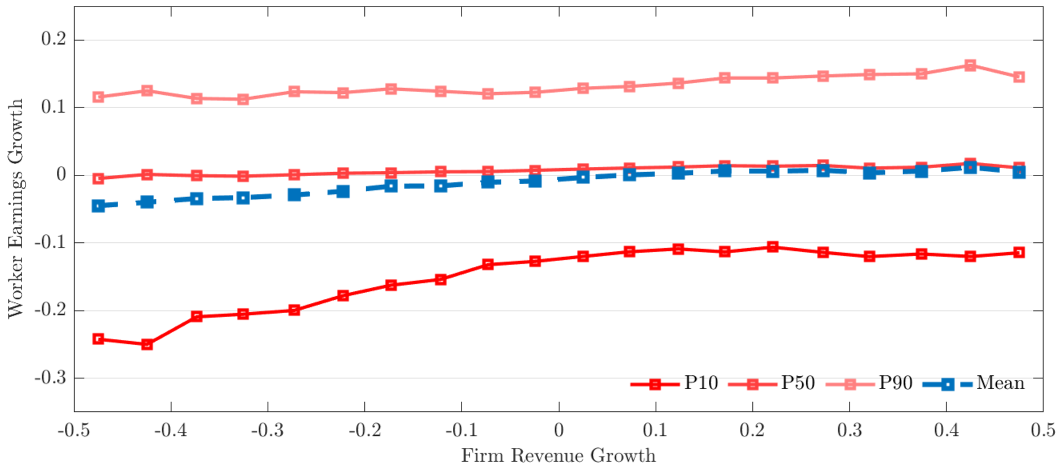

Figure 2: Worker Earnings Growth by Firm Revenue Growth

Notes: This figure shows moments of the annual earnings growth distribution, conditional on the revenue growth of the firm at which workers are employed in the initial year.

In the next step, we investigate how the incidence of such large earnings changes is related to the outcomes of the firms workers are employed in.

Earnings dynamics by firm revenue growth. To analyze the comovement of worker earnings and firm revenues we match workers to firms in year $t-1$. Then, we group firms by their revenue growth from $t-1$ to t. We compute the worker earnings growth distribution from $t-1$ to t by firm group. In $t-1$ we impose the restriction that workers have to be employed for the entire year at one firm only. For the first case that we present next, we do not impose any restriction on whether or where workers work in year t – they may still be at the same firm, work for a different firm, be unemployed, or any combination of those.

Figure 2 shows the mean, median, 10th, and 90th percentile of the worker earnings growth distribution for 20 groups of workers, characterized by the revenue growth of their firms. The range of revenue growth we plot here is from -0.5 to 0.5. These are large changes, but there is significant mass even in the tails. The 10th percentile of the employment weighted firm revenue growth distribution is -34%, and the 90th percentile is 29%.

On average, earnings growth of workers is 5 percentage points higher in the firms that grow the most relative to the firms that are strongly shrinking.[14] Note, however, that the median of the earnings growth distribution is essentially flat across the entire

[14]The average is negative because of the restriction that everybody is full-year employed in the base year, but not necessarily in the second year.

firm revenue growth distribution. The median worker across firm groups is unaffected by whether the firm's revenue is shrinking or growing strongly.

To a large extent, the lower mean of the earnings growth distribution in shrinking firms is driven by the lower tail of the earnings growth distribution. In these firms, it is much more likely that workers experience very large earnings losses relative to growing firms. In firms with positive revenue growth, the 10th percentile of the worker earnings growth distribution is flat at -10%. At the bottom of the revenue growth distribution it is -25%. Hence, workers in shrinking firms are much more likely to experience large earnings losses compared to workers in growing firms.

To a lesser extent, the mean relationship is also driven by differences in the right tail of the earnings growth distribution. The 90th percentile is around 3-4 percentage points higher for workers in strongly growing relative to strongly shrinking firms.[15][16]

Up to now, we have not conditioned on workers' employment status in the second year. In particular, the gradient in the 10th percentile of the earnings growth distribution could be purely driven by different likelihoods of transitioning to unemployment, but also by a larger chance of wage losses for stayers. We now zoom in on the contributions of the extensive and the intensive margin.

Extensive margin. We first investigate the contribution of the extensive margin. Figure 3 shows the likelihood of experiencing unemployment spells of varying lengths in year t. The left panel shows the share of workers with no unemployment in year t. For workers in growing firms this share is fairly stable at around 93%. The more a firm's revenue goes down, the larger becomes the share of workers with unemployment spells. Between growing firms and the strongest shrinking firms the difference is around 5 percentage points.

The right panel shows how this larger probability of experiencing unemployment spells is distributed across different unemployment durations. Workers starting in shrinking firms are more likely to experience unemployment spells both of short and longer duration than those starting in growing firms.

[15]The patterns described here are robust to using an alternative source of revenue information from accounting/tax data rather than value added tax. This alternative measure is only available for a subsample of firms, biased towards larger firms, which is why we focus on the measure from value added taxes. Using the alternative measure for computing revenue growth, the difference in 10th percentile worker earnings growth between the bottom and the top of the revenue growth distribution is slightly larger at around 20% instead of 15%. See Figure A.1 in the appendix.

[16]The key empirical patterns are robust to the state of the business cycle. While the likelihood of very large losses is higher in the recession period, there is also a strong connection between the tails of the earnings growth distribution and firm revenue growth in the expansion. See Figures A.2 and A.3 in the appendix. When using residualized earnings the lower tail of the distribution is closer to the expansion figure since we only use data from 2010 on due to a break in the occupational classification.

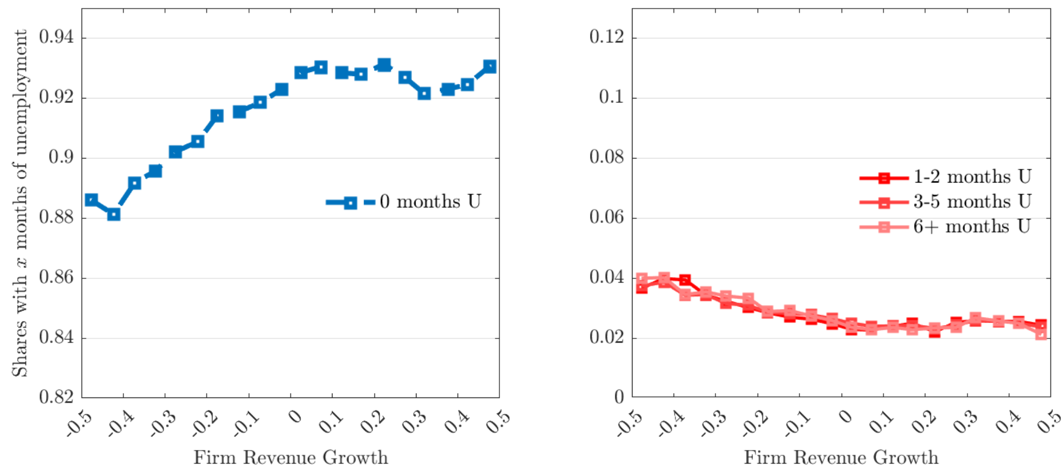

Figure 3: Unemployment Propensities by Firm Revenue Growth

Notes: This figure shows the likelihood of experiencing unemployment spells of varying duration, conditional on the revenue growth of the firm at which workers are employed in the initial year.

Intensive margin. While job losses are important for the very largest earnings losses and are strongly associated with firm revenue growth, we now proceed to show that there is also a tight link between firm revenue growth and the likelihood of large earnings losses for stayers. To that end, we focus on a stayer sample, for which we impose the additional restriction that they are employed in the same firm in the entire years $t-1$ and t, without any unemployment spell or employment at a different firm.

Figure 4 shows that there is a positive relation between revenue growth and earnings growth for the stayer sample as well – on average, earnings growth is around 3 percentage

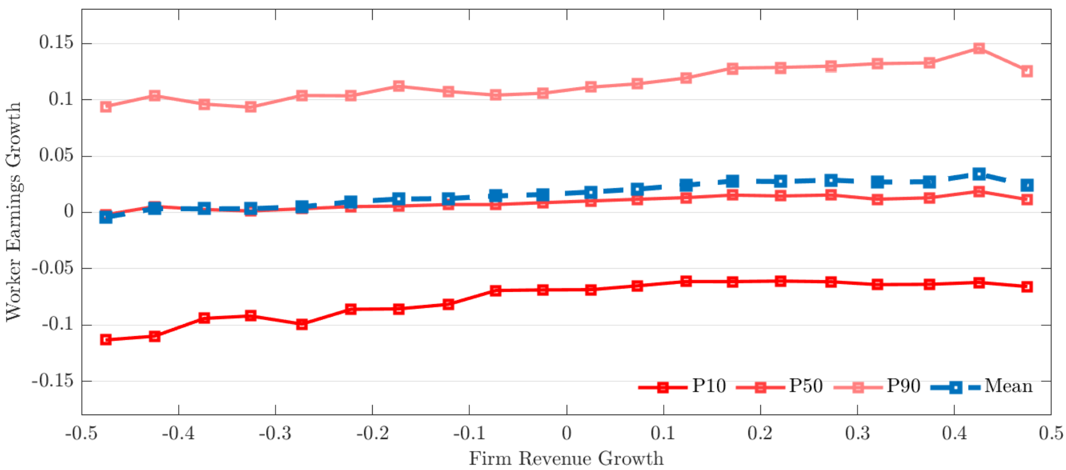

Figure 4: Worker Earnings Growth of Stayers by Firm Revenue Growth

Notes: This figure shows moments of the annual earnings growth distribution for workers staying at the same firm, conditional on the revenue growth of the firm.

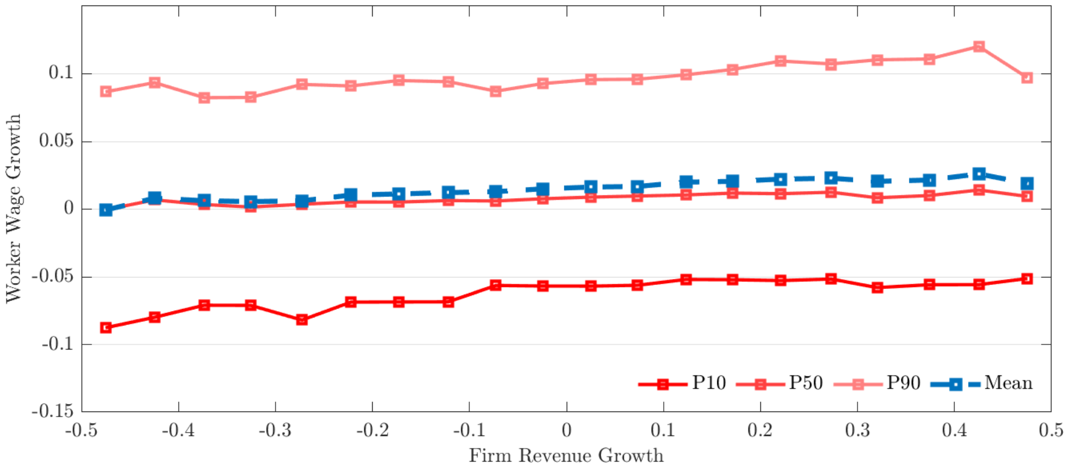

Figure 5: Worker Wage Growth of Stayers by Firm Revenue Growth

Notes: This figure shows moments of the hourly wage growth distribution for workers staying at the same firm, conditional on the revenue growth of the firm.

points higher at the top of the firm revenue growth distribution than at the bottom. Again, this is mostly driven by the tails of the earnings growth distribution. While the median is slightly increasing along the firm distribution, the increase is much stronger for both the 10th and the 90th percentile. It is much more likely to experience a large earnings loss while continuously employed at the same firm in a firm whose revenue goes down, and much more likely to experience a large gain in a firm whose revenue is going up. For both the 10th and the 90th percentile the difference between the top and the bottom firms is almost 5 percentage points.

Wage dynamics by firm revenue growth. In addition to earnings, we also observe hours worked. As our measure of hours, we use an hours variable, where missing and implausible values have already been imputed by Statistics Denmark. With this hours measure we will not capture if employer and employee decide to vary hours worked over time within the contract to obtain the right average hours worked, without adjusting contractual reported hours. However, if there was a change in contractual hours, which is arguably the relevant margin for earnings dynamics, this would be captured.

With the measure of hours, we can compute hourly wages by dividing income by hours. Moments of the hourly wage growth distribution for stayers by firm revenue growth are shown in Figure 5. A significant part of the earnings changes of stayers is driven by wage changes, and there is a relation between wage changes and firm revenue growth as there is for earnings.

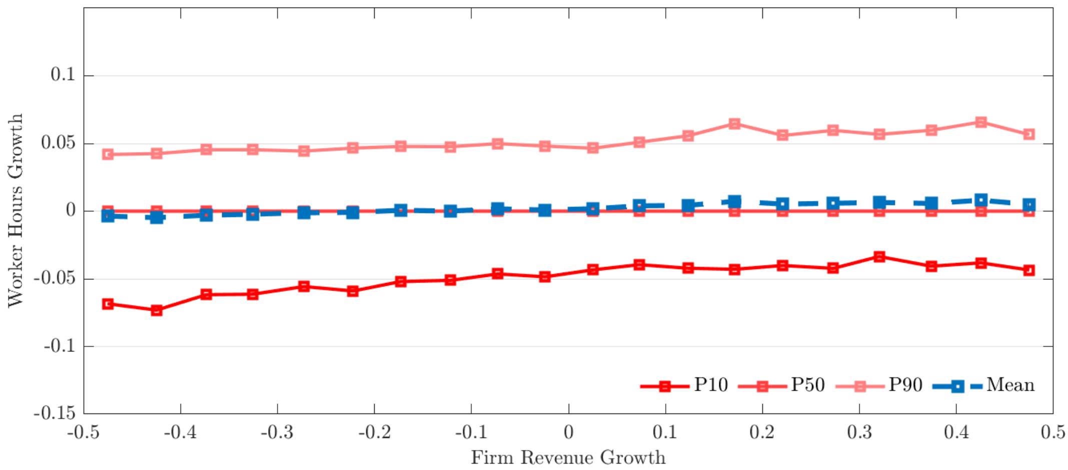

Figure 6: Worker Hours Growth of Stayers by Firm Revenue Growth

Notes: This figure shows moments of the hours growth distribution for workers staying at the same firm, conditional on the revenue growth of the firm.

Hours dynamics by firm revenue growth. Finally, we plot the same type of figure for hours growth in Figure 6. We see a similar pattern, with larger hours drops more likely in shrinking firms and larger hours rises more likely in growing firms. However, overall the variability in hours is lower than the variability in wages, so that movements in the latter are more important for understanding earnings dynamics.[17]

2.4 Taking Stock

Before we move to the quantitative model, we can summarize the key empirical patterns uncovered in this section, with which the model should be consistent, as follows. First, the worker earnings growth distribution is characterized by excess kurtosis, with many small changes and a significant tail of very large earnings changes. Median earnings growth is quite stable close to zero along the entire firm revenue growth distribution. Second, the probability of very large earnings changes is strongly correlated with the revenue growth of the firm where workers are employed. In particular, there is a much higher likelihood of large earnings losses for workers in shrinking firms, but also a higher likelihood of very large gains in growing firms. Third, many of the very largest earnings losses in shrinking firms are driven by a higher likelihood of unemployment spells. Fourth, however, large earnings losses are also much more likely for continuing workers in shrinking firms, as are large gains in growing firms. Fifth, and finally, in spite of the strong relation between firm revenue growth and the likelihood of large earnings changes for workers, large earnings

[17] Within the pass-through literature, Bell, Bloom, and Blundell (2022) shows using UK data that only roughly a quarter of the average effect of firm-level shocks on weekly wages is driven by hours.

changes are observed along the entire firm distribution, including large earnings gains for workers in shrinking firms and large losses for workers in growing firms.

These patterns motivate the setup of the quantitative model that we discuss next. The large share of small worker earnings changes across the entire firm revenue growth distribution calls for a wage setting protocol creating a lot of inaction. At the same time, the fact that large earnings changes occur along the entire firm distribution, but are more concentrated in the tails of the revenue growth distribution, points to the need for both firm and worker level shocks.

3 Model

In this section, we describe the model environment. Throughout, we already emphasize which features are important for enabling the model to be consistent with the key empirical patterns, before bringing the model to the data in the next section.

The model economy consists of heterogeneous workers, who differ in their employment status, productivity, and job search effort, heterogeneous firms, who differ in their productivity, number of employees, and job creation choice, and a government, which raises taxes and provides unemployment insurance. We start the model description by discussing each of these agents.

3.1 Agents

Workers. The economy is populated by a unit mass of infinitely lived workers, who are heterogeneous in their labor productivity x. Worker productivity is stochastic with stationary distribution $\phi_x(x)$. It follows a first order Markov process with conditional transition probabilities denoted by $p(x' \mid x)$. Workers can be either employed (E) or unemployed (U). If unemployed, they receive an unemployment benefit b and search for jobs. If employed, they receive a wage w and search on the job. Arrival rates of job offers are determined in equilibrium and depend on search effort.

The choice of search effort s is modeled similar to Faberman, Mueller, Şahin, and Topa (2022).[18] The probability of meeting a vacant job depends both on individual search effort and equilibrium market tightness θ and is given by

$$\lambda_i(s,\theta) = \Lambda_i(s)\,\lambda(\theta), \quad i \in \{U, E\}, \tag{1}$$

[18]Earlier contributions modeling an endogenous search effort choice include Christensen, Lentz, Mortensen, Neumann, and Werwatz (2005), Hornstein, Krusell, and Violante (2011), and Bagger and Lentz (2019).

where $\Lambda_i(s)$ translates search effort into efficiency units of search and $\lambda(\theta)$ is the equilibrium arrival rate of offers per efficiency unit of search. The cost of exerting search effort is given by $c_w(s)$. This cost is an increasing and convex function of search effort.

Workers are hand-to-mouth; they cannot save or borrow. Hence, consumption is equal to the wage or benefit they receive every period. Households value consumption according to a utility function $u(c)$, which is assumed to be increasing and concave in c such that workers are risk averse. Workers' discount factor is β.

Firms. There is a mass F of firms, who can employ many workers. Firm productivity y is also stochastic with stationary mass $\phi_y(y)$ and first order Markov with transition probabilities $p(y' \mid y)$. Firms create jobs, which can be either vacant or filled. Job creation is a costly process: The job creation cost function $c_f\left(v^{\mathrm{N}}\right)$, where v^{N} denotes the measure of newly created jobs, is increasing and convex in v^N.

Jobs are costless to maintain. A filled job can become vacant again through an endogenous separation after adverse productivity shocks or after a job-to-job transition of the worker. Additionally, both filled and vacant jobs can be exogenously destroyed with probability $\delta(x)$.[19] Through an exogenous destruction shock, the job disappears and the worker becomes unemployed. The modeling of vacant jobs is different from standard search models, in which all vacancies are newly created. We rather model vacant jobs as persistent, resulting from both creation of new vacancies and opening up of positions because workers left through a job-to-job transition or quit. This approach is appealing for at least a couple of reasons. First, it makes the decision of whether to create a match more realistic because upon a meeting it is not just the worker who has an option value of continuing to search for a better match, but also the firm. Second, it is supported empirically: A number of recent papers, including Faberman and Nagypal (2008), Mercan and Schoefer (2020), Elsby, Gottfries, Michaels, and Ratner (2023), and Qiu (2023), document that replacement hiring of quitting workers is quantitatively important.

A job filled with a worker of type x produces output $f(x, y)$. This output is independent of other matches, so that output of firm j is given by the sum of output over all its matches. This modeling choice preserves tractability because it allows for the contracting problem between firm and worker to be independent of other workers in the firm. At the same time, there is still a well-defined firm size distribution because of search frictions and convex job creation costs. In the frictionless limit, all workers would be employed at the most productive firm, but given the cost of job creation and delays of filling jobs

[19]The job destruction probability depends on worker productivity. For a vacant job, we assume the probability to be equal to that of the median productivity worker.

because of labor market frictions, not all workers can be reallocated to the best firm. This is a common assumption in the literature to keep the problem tractable while still capturing a firm size distribution; see among others Moscarini and Postel-Vinay (2013) and Gulyas (2023).

Vacant jobs meet a searching worker with a probability that is determined in equilibrium. Firms maximize expected profits and are risk-neutral. They also discount future profits with discount factor β.

Government. The government pays benefits b to unemployed workers. Furthermore, the government raises income taxes using a progressive income tax function $T(\cdot)$, which is applied to both wages and unemployment benefits.[20]

3.2 Labor Market

Search and Matching. Unemployed and employed searchers meet vacant jobs in the labor market, which we model with random search and matching. In a steady state equilibrium, there is a distribution of unemployed $\mu_x(x)$ and employed workers at the search stage $\psi^s(x, y, w)$. Aggregate efficiency units of search are given by

$$L = \int_x \mu_x(x) \Lambda_U(s(x)) dx + \int_x \int_y \int_w \psi^s(x, y, w) \Lambda_E(s(x, y, w)) dwdydx. \qquad (2)$$

We denote the mass of vacant jobs with $\mu_y(y)$ and the total number of vacant jobs with v. The number of meetings M is given by a standard constant returns to scale matching function $\mathcal{M}(L, v)$.[21] The probability of a vacant job meeting a worker is given by $\lambda_f = M/v$. Conditional on a meeting, the probability that the worker is unemployed is given by

$$p_U = \frac{\int_x \mu_x(x) \Lambda_U(s(x)) dx}{L}. \qquad (3)$$

Given market tightness $\theta = v/L$, the equilibrium arrival rate of offers per efficiency unit of search is given by $\lambda(\theta) = \mathcal{M}(1, \theta)$.

If a vacant job and an unemployed worker meet, given firm and worker productivity there is a maximum wage the firm is willing to pay, $\bar{w}_y(x, y)$, and a minimum wage the worker is willing to accept, $\underline{w}_x(x, y)$. As long as the firm can pay more than the worker requires, the match is created. If a vacant job meets an employed worker, there will be a

[20] For tractability, we assume that taxes are levied based on monthly income. In the data, the tax base is annual income; however, modeling this would require the introduction of another state variable.

[21] See Petrongolo and Pissarides (2001) and the references therein for a discussion of the returns to scale in the matching function.

job-to-job transition if the potential employer can pay a higher wage than the incumbent. Otherwise, the worker stays at the incumbent firm.

Wage Setting. We assume a wage setting protocol with two-sided limited commitment, similar to Thomas and Worrall (1988), Postel-Vinay and Robin (2002), and Cahuc, Postel-Vinay, and Robin (2006). Specifically, we assume that firms and workers agree upon a constant wage at the beginning of the match. This wage is only adjusted in response to either side having a credible threat to otherwise leave the match, as we describe in detail now. This wage setting protocol is empirically motivated because it can deliver stable wages for many workers, while still potentially generating large adjustments in some instances, which is an important feature of the data.[22]

If an unemployed worker meets a vacant job and it is possible to create a match, the worker has some bargaining power to extract a share of the surplus. As we discuss in more detail in the calibration section, this initial bargaining power is required because otherwise workers might be willing to accept implausibly low starting wages. Specifically, we assume that the initial wage negotiated between a firm of type y and a worker of type x is an average between the maximum wage the firm would pay and the minimum the worker would accept:

$$w^{\text{init}}(x, y) = (1 - \alpha) \underline{w}_x (x, y) + \alpha \bar{w}_y (x, y). \tag{4}$$

We will explain how to compute $\underline{w}_x (x, y)$ and $\bar{w}_y (x, y)$ below, after having introduced the value functions.

Employed workers can renegotiate their wages if they receive a relevant outside offer. Suppose a worker employed at a firm with productivity y, which is able to pay at most $\bar{w}_y (x, y)$, meets another firm with productivity $\tilde{y}$ being able to pay $\bar{w}_y (x, \tilde{y})$. Three cases can occur. First, if the maximum the potential poaching firm can pay is below the current wage, this is an irrelevant outside offer and nothing will happen. Second, the current firm is able to pay more than the potential new firm, but the current wage is lower than the wage the new firm could pay. Then, the wage is increased to this level, but the worker stays at the old firm. This can generate large wage gains for stayers, and potentially larger in growing firms because they tend to have high productivity and can counter more outside offers. Third, if the new firm can pay a higher wage than the old firm, there

[22]We assume that search effort is solely the choice of the worker and cannot be contracted upon. As an example with jointly efficient search intensities in a model with risk-neutral workers and firms, see Bagger and Lentz (2019). For an optimal contracting problem with hidden search by the worker, see Lentz (2015).

will be a job-to-job transition and the worker will receive the maximum wage the old firm could have paid as the starting wage at the new firm.[23] Such employment-to-employment (EE) transitions will also generate some of the largest wage gains in the economy, again potentially more so in growing firms since outside offers tend to be more valuable.

Even without an outside offer wages may change while a worker is employed. We assume that either side can demand a renegotiation of the wage if it has a credible threat to leave the match. For the worker that means that the wage can be renegotiated if quitting to unemployment is preferable to staying in the match at the current wage. For the firm it means that it can ask for a renegotiation if it prefers having a vacant job over employing the worker at the current wage. In both cases, wages are reset such that the agent demanding the renegotiation is just indifferent between staying in the match or leaving. Firm demanded renegotiations are a key mechanism to generate wage losses in the model.

Timing. The order of events in a period is as follows. At the beginning of the period there is the production stage. The distribution of matches and wages at the production stage is denoted with $\psi(x, y, w)$. We denote as $\tilde{\psi}(x, y)$ the distribution of matches, integrating out wages. At the production stage, output is produced, wages and benefits are paid out, and consumption takes place. Furthermore, firms decide how many new vacant jobs to create.

After production and the creation of new vacant jobs, exogenous job destruction shocks occur. This implies that a newly created vacant job can be destroyed immediately. If a filled job is destroyed, the worker becomes unemployed. A newly separated worker cannot search immediately but only in the next period, so that a separated worker will be unemployed for at least one period.

Next, productivity shocks realize. This can lead to endogenous separations. If a match is dissolved, the job becomes vacant and the worker transitions to unemployment. Again, unemployed workers have to be unemployed for at least one period and the job also has to be vacant for a period.

A period concludes with the matching stage. Workers search, meetings realize, new matches between unemployed searchers and vacant jobs are created, and job-to-job transitions take place. The relevant distributions at this stage are the distribution of matches and wages at the search stage, $\psi^S(x, y, w)$, the distribution of unemployed workers $\mu_x(x)$,

[23]If the two firms have the same productivity, the tie breaking rule we assume is that there is a job-to-job transition in 50% of the cases.

and the distribution of vacant jobs $\mu_y(y)$. Again, we introduce the distribution of matches at the search stage integrating out wages $\tilde{\psi}^S(x,y)$.

3.3 Value Functions

We can now write the value functions of an unemployed worker, an employed worker, a vacant job, and a filled job. We write all value functions from the perspective of the production stage.

Value of unemployment. We start with the value function of an unemployed worker:

$$
\begin{aligned}
U(x) = \; & \underbrace{u(b - T(b))}_{\text{Flow utility from benefit}} + \int_{x'} \max_{0 \le s(x') \le \bar{s}} \Bigg\{ \underbrace{-c_w(s(x'))}_{\text{Search effort cost}} \\
& \underbrace{+\beta(1 - \lambda_U(s(x'), \theta))U(x')}_{\text{Continuation value: no meeting}} \\
& \underbrace{+\beta\lambda_U(s(x'), \theta)\int_{\tilde{y}}(1 - A_U(x', \tilde{y}))U(x')\frac{\mu_y(\tilde{y})}{v}d\tilde{y}}_{\text{Continuation value: meeting, but no match}} \\
& \underbrace{+\beta\lambda_U(s(x'), \theta)\int_{\tilde{y}} A_U(x', \tilde{y})W\left(x', \tilde{y}, w^{\text{init}}(x', \tilde{y})\right)\frac{\mu_y(\tilde{y})}{v}d\tilde{y}}_{\text{Continuation value: new match}} \Bigg\} \\
& p(x' \mid x)\, dx'
\end{aligned}
\tag{5}
$$

Unemployed agents obtain flow utility from after-tax unemployment benefits. They also experience disutility from exerting search effort. Since the search stage takes place after productivity shocks, search effort is chosen as a function of new productivity x'. At the search stage, three scenarios can occur. First, the unemployed agent does not meet a vacant job, in which case she will be unemployed next period. Second, the agent may meet a vacant job of productivity $\tilde{y}$, but the productivities are such that no match is created, where $A_U(x', \tilde{y})$ is the probability that a match between a worker of productivity x' and a firm of productivity $\tilde{y}$ is created. Third, the worker may meet a firm with which a match is created and transition into employment, with value function $W(\cdot)$.

We can take the first order condition with respect to search effort:

$$
\begin{aligned}
c_w'\left(s\left(x'\right)\right) = \beta \Lambda_U'\left(s\left(x'\right)\right) \lambda\left(\theta\right) \times & \\
& \left\{ -U\left(x'\right) + \int_{\tilde{y}} \left(1 - A_U\left(x', \tilde{y}\right)\right) U\left(x'\right) \frac{\mu_y\left(\tilde{y}\right)}{v} d\tilde{y} \right. \\
& \left. + \int_{\tilde{y}} A_U\left(x', \tilde{y}\right) W\left(x', \tilde{y}, w^{\text{init}}\left(x', \tilde{y}\right)\right) \frac{\mu_y\left(\tilde{y}\right)}{v} d\tilde{y} \right\}.
\end{aligned} \tag{6}
$$

Search effort is costly, and with a convex cost function increasingly so the higher search effort becomes. Higher search effort, however, increases the chances of leaving unemployment and getting on the job ladder. The marginal cost and the marginal gain have to be equalized.

Value of employment. The value of employment is a function of the productivities and the wage:

$$
\begin{aligned}
W\left(x, y, w\right) = \; & \underbrace{u\left(w - T\left(w\right)\right)}_{\text{Flow utility from wage}} + \underbrace{\beta \delta\left(x\right) \int_{x'} U\left(x'\right) p\left(x' \mid x\right) dx'}_{\text{Continuation value: exogenous separation}} \\
& \underbrace{+ \beta\left(1 - \delta\left(x\right)\right) \int_{x'} \int_{y'} \left(1 - A_S\left(x', y'\right)\right) U\left(x'\right) p\left(x' \mid x\right) p\left(y' \mid y\right) dy' dx'}_{\text{Continuation value: endogenous separation}} \\
& + \left(1 - \delta\left(x\right)\right) \int_{x'} \int_{y'} A_S\left(x', y'\right) \max_{0 \leq s\left(x', y', w\right) \leq \bar{s}} \left\{ \underbrace{-c_w\left(s\left(x', y', w\right)\right)}_{\text{Search effort cost}} \right. \\
& \underbrace{+ \beta \int_{\tilde{y}} \lambda_E\left(s\left(x', y', w\right), \theta\right) A_E\left(x', y', \tilde{y}\right) W\left(x', \tilde{y}, \bar{w}_y\left(x', y'\right)\right) \frac{\mu_y\left(\tilde{y}\right)}{v} d\tilde{y}}_{\text{Continuation value: job-to-job transition}} \\
& \underbrace{+ \beta \int_{\tilde{y}} \lambda_E\left(s\left(x', y', w\right), \theta\right) A_O\left(x', y', w, \tilde{y}\right) W\left(x', y', \bar{w}_y\left(x', \tilde{y}\right)\right) \frac{\mu_y\left(\tilde{y}\right)}{v} d\tilde{y}}_{\text{Continuation value: renegotiation triggered by outside offer}} \\
& + \beta \int_{\tilde{y}} \left(1 - \lambda_E\left(s\left(x', y', w\right), \theta\right) \left[A_E\left(x', y', \tilde{y}\right) + A_O\left(x', y', w, \tilde{y}\right)\right]\right) \\
& \left. \underbrace{\times \min\left\{\max\left\{W\left(x', y', w\right), U\left(x'\right)\right\}, W\left(x', y', \bar{w}_y\left(x', y'\right)\right)\right\} \frac{\mu_y\left(\tilde{y}\right)}{v} d\tilde{y}}_{\text{Continuation value: potential renegotiation after productivity shocks}} \right\} \\
& p\left(x' \mid x\right) p\left(y' \mid y\right) dy' dx'
\end{aligned} \tag{7}
$$

Employed agents consume their after-tax wage and derive flow utility from consumption. With probability $\delta\left(x\right)$ they are exogenously separated, in which case they transition to unemployment. After productivity shocks realize, firm and worker decide whether to separate voluntarily, where $A_S\left(x, y\right)$ denotes the probability of a match between firm and worker of certain productivities being continued.[24] If there is neither an exogenous nor an endogenous separation, workers reach the search stage. They incur the utility cost of search effort. When a worker meets another firm with productivity $\tilde{y}$ and this firm can offer a higher wage than the current firm, which happens with probability $A_E(x', y', \tilde{y})$, the worker will do a job-to-job transition and receive wage $\bar{w}_y(x', y')$. Even if the potential poacher is not of sufficiently high productivity to trigger a job-to-job transition, the outside offer may still cause a renegotiation ($A_O(x', y', w, \tilde{y})$). Finally, the worker may receive no or only an irrelevant outside offer given the current wage, in which case the worker stays at the current firm. The wage may have to be renegotiated still. If at the current wage the worker prefers unemployment, the wage will be reset such that the worker will be indifferent between staying and transitioning to unemployment and the match is preserved. Similarly, if the firm prefers a vacant job relative to the filled job at the current wage, the wage will be reset to the maximum wage the firm is willing to pay. If neither side has a credible threat, the wage remains unchanged.

The first order condition for an employed searcher reads as follows:

$$\begin{aligned}
&c'_w(s(x', y', w)) = \beta \Lambda'_E(s(x', y', w))\lambda(\theta) \times \\
&\left\{ \int_{\tilde{y}} A_E(x', y', \tilde{y}) W(x', \tilde{y}, \bar{w}_y(x', y')) \frac{\mu_y(\tilde{y})}{v} d\tilde{y} \right. \\
&+ \int_{\tilde{y}} A_O(x', y', w, \tilde{y}) W(x', y', \bar{w}_y(x', \tilde{y})) \frac{\mu_y(\tilde{y})}{v} d\tilde{y} \\
&- \int_{\tilde{y}} [A_E(x', y', \tilde{y}) + A_O(x', y', w, \tilde{y})] \\
&\left. \times \min\{\max\{W(x', y', w), U(x')\}, W(x', y', \bar{w}_y(x', y'))\} \frac{\mu_y(\tilde{y})}{v} d\tilde{y} \right\}.
\end{aligned} \tag{8}$$

The increasingly higher cost of more search effort is traded off against the gains from increasing wages through more outside offers and the resulting job-to-job transitions and renegotiations.

[24] In the setup as described here, we have $A_S(x, y) = A_U(x, y)$. That is, endogenous separations are decided upon not taking into account that an employed worker has the option to search, while she transitions to unemployment without being able to participate in the search stage in the current period with an endogenous separation. We choose this modeling strategy, as employed workers have no bargaining power when meeting new firms, so if with the current firm transitioning to unemployment is optimal, the relevant outside option of the worker is unemployment in any case. We allow for the more general notation anticipating that these probabilities will differ once we explore layoff taxes as a government policy.

Value of vacant job. On the firm side, we can write the values of vacant and filled jobs. Consider first the value of a vacant job:

$$
\begin{aligned}
V(y) = \beta\left(1-\delta\left(x^{\text{med}}\right)\right)\Bigg\{ & \underbrace{(1-\lambda_f)\int_{y'} V(y')\, p(y' \mid y)\, dy'}_{\text{Continuation value: no meeting}} \\
& \underbrace{+\lambda_f p_U \Bigg\{ \int_{y'}\int_{\tilde{x}} (1-A_U(\tilde{x},y'))\frac{L_U(\tilde{x})}{L_U} V(y')\, p(y' \mid y)\, d\tilde{x} dy'}_{\text{Continuation value: meeting with unemployed, no match}} \\
& \underbrace{+\int_{y'}\int_{\tilde{x}} A_U(\tilde{x},y')\frac{L_U(\tilde{x})}{L_U} J\left(\tilde{x},y',w^{\text{init}}(\tilde{x},y')\right) p(y' \mid y)\, d\tilde{x} dy' \Bigg\}}_{\text{Continuation value: new match with unemployed}} \\
& \underbrace{+\lambda_f(1-p_U)\Bigg\{ \int_{y'}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{w}} (1-A_E(\tilde{x},\tilde{y},y'))\frac{L_E(\tilde{x},\tilde{y},\tilde{w})}{L_E} V(y')\, p(y' \mid y)\, d\tilde{w} d\tilde{y} d\tilde{x} dy'}_{\text{Continuation value: meeting with employed, no match}} \\
& \underbrace{+\int_{y'}\int_{\tilde{x}}\int_{\tilde{y}}\int_{\tilde{w}} A_E(\tilde{x},\tilde{y},y')\frac{L_E(\tilde{x},\tilde{y},\tilde{w})}{L_E} J(\tilde{x},y',\bar{w}_y(\tilde{x},\tilde{y}))\, p(y' \mid y)\, d\tilde{w} d\tilde{y} d\tilde{x} dy' \Bigg\}}_{\text{Continuation value: new match with employed}} \Bigg\}.
\end{aligned}
\tag{9}
$$

There is no flow value to having a vacant job and if the job is exogenously destroyed there is no continuation value. If the job is not destroyed and the firm does not meet a worker, the firm will still have this vacant job in the next period. The continuation value in case of meeting a worker, which happens with probability λ_f, is split into four parts. The first two terms deal with meetings with an unemployed worker. The probability of meeting an unemployed worker conditional on a meeting taking place is p_U. The likelihood of meeting an unemployed worker of a certain productivity $\tilde{x}$ depends on the efficient search units $L_U(\tilde{x})$ of workers with this productivity level. Depending on the productivity of firm and worker, a match may or may not be created. The last two cases deal with meetings with employed workers. Here, the probabilities of meeting a certain type depend on the efficient search units on the job $L_E(\tilde{x},\tilde{y},\tilde{w})$.

Value of filled job. Finally, we can write the value of a filled job:

$$
\begin{aligned}
J(x,y,w) = \; & \underbrace{f(x,y)-w}_{\text{Flow value: output net of wage}} + \beta(1-\delta(x)) \Bigg\{ \\
& \underbrace{+\int_{x'}\int_{y'} (1-A_S(x',y')) V(y') p(x' \mid x) p(y' \mid y) \, dy' dx'}_{\text{Continuation value: endogenous separation}} \\
& + \int_{x'}\int_{y'} A_S(x',y') \Bigg\{ \\
& \underbrace{\int_{\tilde{y}} \lambda_E(s(x',y',w),\theta) A_E(x',y',\tilde{y}) V(y') \frac{\mu_y(\tilde{y})}{v} d\tilde{y}}_{\text{Continuation value: separation due to job-to-job transition}} \\
& + \underbrace{\int_{\tilde{y}} \lambda_E(s(x',y',w),\theta) A_O(x',y',w,\tilde{y}) J(x',y',\bar{w}_y(x',\tilde{y})) \frac{\mu_y(\tilde{y})}{v} d\tilde{y}}_{\text{Continuation value: renegotiation triggered by outside offer}} \\
& + \int_{\tilde{y}} (1-\lambda_E(s(x',y',w),\theta) [A_E(x',y',\tilde{y}) + A_O(x',y',w,\tilde{y})]) \\
& \underbrace{\times \min\{\max\{J(x',y',w), V(y')\}, J(x',y',\underline{w}_x(x',y'))\} \frac{\mu_y(\tilde{y})}{v} d\tilde{y}}_{\text{Continuation value: potential renegotiation after productivity shocks}} \Bigg\} \\
& p(x' \mid x) p(y' \mid y) \, dy' dx' \Bigg\}.
\end{aligned}
\tag{10}
$$

The flow value of a filled job is the output of the match net of the wage paid to the worker. A filled job may become vacant in the following period either due to an endogenous separation or a job-to-job transition of the worker. An outside offer that does not lead to a job-to-job transition may still trigger a renegotiation, and wages may adjust after productivity shocks if there is a credible threat to leave the match, as detailed for the value of employment.

Wages. The value functions can be used to compute the maximum wage a firm is willing to pay, making the firm indifferent between paying the worker that wage and having a vacant job, and the minimum wage a worker is willing to accept, making the worker indifferent between working at this wage and unemployment. The lowest wage a worker of type x accepts at a firm of type y is defined as follows:

$$\underline{w}_x(x,y): \; W(x,y,\underline{w}_x(x,y)) = U(x). \tag{11}$$

The highest wage a firm of type y is willing to pay to a worker of type x is given by

$$\bar{w}_y(x,y): \; J(x,y,\bar{w}_y(x,y)) = V(y). \tag{12}$$

3.4 Distributions

In a stationary equilibrium, there are masses of vacant jobs, of unemployed, and of matches. Again, we denote at the production stage the distribution of matches/wages $\psi(x,y,w)$ and the distribution of matches $\tilde{\psi}(x,y)$. At the search stage, we have $\psi^S(x,y,w)$ and $\tilde{\psi}^S(x,y)$. The distributions of vacant jobs and unemployed are $\mu_y(y)$ and $\mu_x(x)$.

Distribution of matches. We start with the distribution of matches and wages at the production stage, where for readability we suppress the dependence of search effort on state variables.

$$\begin{aligned}
\psi(x,y,w) = & \underbrace{\lambda_U(s,\theta)\, A_U(x,y)\, \frac{\mu_y(y)}{v} \mu_x(x)\, \mathbb{1}_{w^{\text{init}}(x,y)=w}}_{\text{Inflow from unemployment}} \\
& + \underbrace{\int_{\tilde{y}} \int_{\tilde{w}} \lambda_E(s,\theta)\, A_E(x,\tilde{y},y)\, \psi^S(x,\tilde{y},\tilde{w})\, \frac{\mu_y(y)}{v} \mathbb{1}_{\bar{w}(x,\tilde{y})=w} d\tilde{w} d\tilde{y}}_{\text{Inflow through job-to-job transition}} \\
& + \underbrace{\int_{\tilde{y}} \int_{\tilde{w}} \lambda_E(s,\theta)\, A_O(x,y,\tilde{w},\tilde{y})\, \psi^S(x,y,\tilde{w})\, \frac{\mu_y(\tilde{y})}{v} \mathbb{1}_{\bar{w}(x,\tilde{y})=w} d\tilde{w} d\tilde{y}}_{\text{Renegotiation after outside offer}} + \\
& \underbrace{\int_{\tilde{y}} \int_{\tilde{w}} \left(1 - \lambda_E(s,\theta)\left[A_E(x,y,\tilde{y}) + A_O(x,y,\tilde{w},\tilde{y})\right]\right) \psi^S(x,y,\tilde{w})\, \frac{\mu_y(\tilde{y})}{v} \mathbb{1}_{w^{\text{reneg}}=w} d\tilde{w} d\tilde{y}}_{\text{No renegotiation/transition because of outside offer}}
\end{aligned} \tag{13}$$

A match between a worker of type x, a firm of type y, at wage w may be part of the distribution of matches at the production stage for the following reasons. First, an unemployed worker with productivity x from the distribution of unemployed $\mu_x(x)$ meets a vacant job with probability $\lambda_U(s,\theta)$. This vacant job is of productivity y with probability $\mu_y(y)/v$ and a match is created with probability $A_U(x,y)$. If the initial wage given these productivities is w, then the newly created match adds to $\psi(x,y,w)$. Note that neither in this case nor in any of the following cases for the distribution of matches/wages at the production stage do we have to deal with productivity transitions because these do not occur between the previous search stage and the production stage, but after the production stage before the search stage.

A worker of productivity x can also enter the distribution of matches $\psi(x,y,w)$ through a job-to-job transition. Starting from the distribution of matches at the search stage $\psi^S(x,\tilde{y},\tilde{w})$, the probability of a meeting is $\lambda_E(s,\theta)$, and the probability that this meeting is with a type y firm is $\mu_y(y)/y$. The worker becomes part of $\psi(x,y,w)$ if the maximum wage the old firm would have been willing to pay is w.

The third row deals with wage changes due to renegotiations after relevant outside offers. At the search stage the worker has to be of productivity x at a firm with productivity y, earning some wage $\tilde{w}$. An outside offer of a firm of type $\tilde{y}$ that triggers a renegotiation puts this worker into $\psi(x,y,w)$ if the maximum wage the firm with productivity $\tilde{y}$ would have been willing to pay is w.

Finally, a worker of productivity x at a firm with productivity y may not receive an offer or only receive an offer that does not trigger a job-to-job transition or renegotiation. In that case, the worker becomes part of $\psi(x,y,w)$ if the "renegotiated" wage w^{reneg} at the same firm is equal to w. This captures both the case in which there is no actual renegotiation and w was already the previous wage and the case in which there is a renegotiation to wage w because either worker or firm has a credible threat to otherwise break up the match.

The distribution of matches $\tilde{\psi}(x,y)$ is obtained by integrating over wages:

$$\tilde{\psi}(x,y)=\int_w \psi(x,y,w)\,dw. \tag{14}$$

Next, we deal with the masses of matches and wages at the search stage. This differs from the distribution at the production stage because of exogenous separations, productivity shocks, and endogenous separations following productivity shocks:

$$\psi^S(x',y',w)=\int_x\int_y (1-\delta(x))\,A_S(x',y')\,\psi(x,y,w)\,p(y'\mid y)\,p(x'\mid x)\,dydx. \tag{15}$$

Also at the search stage, we simply obtain the distribution of matches $\tilde{\psi}^S(x,y)$ as

$$\tilde{\psi}^S(x,y)=\int_w \psi^S(x,y,w)\,dw. \tag{16}$$

Distribution of unemployed. The mass of unemployed follows as the difference between the exogenous distribution of worker types and the workers in employment:

$$\mu_x(x')=\int_x\left[\phi_x(x)-\int_y\tilde{\psi}(x,y)\,dy\right]p(x'\mid x)\,dx. \tag{17}$$

This is due to the assumption that those who are separated cannot search within the same period. Therefore, they will show up in the distribution of unemployed only in the next period. However, productivity shocks apply between the production stage and the search stage.

Distribution of vacant jobs. Finally, we describe the mass of vacant jobs $\mu_y(y)$.

$$
\begin{aligned}
\mu_y(y') = & \underbrace{\int_y \left(1-\delta\left(x^{\text{med}}\right)\right) v^N(y)\,\phi_y(y)\,Fp(y' \mid y)\,dy}_{\text{Newly created jobs}} \\
& + \left(1-\delta\left(x^{\text{med}}\right)\right)\int_y \Bigg\{ \underbrace{(1-\lambda_f)}_{\text{Not filled: no meeting}} + \lambda_f \Bigg\{ \underbrace{p_U \int_x (1-A_U(x,y))\frac{L_U(x)}{L_U}dx}_{\text{Not filled: meeting with unemployed}} \\
& + \underbrace{(1-p_U)\int_{\tilde{y}}\int_x\int_w (1-A_E(x,\tilde{y},y))\frac{L_E(x,\tilde{y},w)}{L_E}dwdxd\tilde{y}}_{\text{Not filled: meeting with employed}}\Bigg\}\Bigg\} p(y' \mid y)\,\mu_y(y)\,dy \\
& + \Bigg\{ \underbrace{\int_y\int_x\int_{\tilde{y}}\int_w \left(1-\delta\left(x^{\text{med}}\right)\right)\lambda_E(s(x,y,w),\theta)\,A_E(x,y,\tilde{y}) \times \frac{\mu_y(\tilde{y})}{v}\psi^S(x,y,w)\,p(y' \mid y)\,dwd\tilde{y}dxdy}_{\text{Newly vacant due to job-to-job transition}} \Bigg\} \\
& + \Bigg\{ \underbrace{\int_{y^-}\int_{x^-}\int_y\int_x \left(1-\delta\left(x^{\text{med}}\right)\right)\left(1-\delta\left(x^-\right)\right)(1-A_S(x,y))\,\tilde{\psi}\left(x^-,y^-\right) \times p(y' \mid y)\,p\left(x \mid x^-\right)p\left(y \mid y^-\right)dxdydx^-dy^-}_{\text{Newly vacant due to endogenous separation}} \Bigg\}.
\end{aligned}
\tag{18}
$$

The distribution of vacant jobs can be computed as the sum of three components: newly created jobs, vacant jobs that were not filled in the past, and filled jobs that became vacant. Newly created jobs are denoted with $v^N(y)$. A firm with productivity y creates new vacant jobs until the marginal cost of doing so equals the value of a vacant job:

$$
\begin{aligned}
c_f'\left(v^N(y)\right) &= V(y), \\
\Rightarrow v^N(y) &= {c_f'}^{-1}(V(y)).
\end{aligned}
\tag{19}
$$

New jobs are created at the beginning of the period, so they can be exogenously destroyed. New job creation is weighted with the total mass of firms F and the exogenous distribution of firms across productivities $\phi_y(y)$. Also, between the production stage and the search and matching stage productivity shocks take place.

A vacant jobs may remain vacant for three reasons. First, it may not meet any worker. Second, it may meet an unemployed worker, but productivities are such that no match is created. Third, it may meet an employed worker, but productivities are such that there is no job-to-job transition.

A filled job may become vacant for two reasons. First, the worker may move to another firm through a job-to-job transition. Starting from the distribution of matches at the search stage, a worker may meet a vacant job and do a job-to-job transition. Then, the firm will have a vacant job. Until the search stage of the next period, however, this job can be exogenously destroyed and is subject to the firm productivity shock. Second, there can be an endogenous separation. Starting from the distribution of matches at the production stage, jobs can be exogenously destroyed. In that case they do not enter the distribution of vacant jobs. Those matches that survive exogenous destruction shocks are subject to productivity shocks. After these realize, there may be endogenous separations. However, the vacant jobs can only be refilled at the search stage of the next period, so that they are subject to exogenous destruction and productivity shocks again before entering the distribution of vacant jobs.

3.5 Equilibrium

Equilibrium definition. A stationary equilibrium consists of value functions for unemployment $U(x)$, employment $W(x,y,w)$, vacant jobs $V(y)$, and filled jobs $J(x,y,w)$, acceptance rules for match creation $A_U(x,y)$, endogenous separations $A_S(x,y)$, job-to-job transitions $A_E(x,y,\tilde{y})$, and renegotiations after outside offers $A_O(x,y,w,\tilde{y})$, search policies of unemployed workers $s(x)$, search policies of employed workers $s(x,y,w)$, job creation policies of firms $v^N(y)$, wage policies $\underline{w}_x(x,y)$, $\bar{w}_y(x,y)$, $w^{\text{init}}(x,y)$, a government policy $b, T(\cdot)$, and distributions of matches at the production stage $\psi(x,y,w)$, $\tilde{\psi}(x,y)$, at the search stage $\psi^S(x,y,w)$, $\tilde{\psi}^S(x,y)$, of unemployed $\mu_x(x)$, and of vacant jobs $\mu_y(y)$, such that

1. Value functions satisfy equations (5), (7), (9), and (10),
2. Acceptance rules are optimal given value functions,
3. Search policies are optimal, satisfying equations (6) and (8),
4. Job creation is optimal, satisfying equation (19),
5. Wages satisfy equations (4), (11), and (12),
6. Distributions are stationary, satisfying equations (13), (14), (15), (16), (17), and (18),
7. The government budget clears.

Equilibrium computation. In models with risk neutral workers it is typically possible to combine the value functions into one surplus function and solve for distributions without having to know wages.[25] However, with risk averse workers the wage is not a one-for-one transfer between workers and firms.[26] Therefore, we cannot just use the surplus function, but have to solve for the distributions, value functions, and wages simultaneously. Furthermore, with endogenous search effort it is not sufficient to only keep track of the distribution of matches, but we have to keep track of the distribution of matches and wages. We relegate the complete description of the numerical algorithm to Appendix B.1.

4 Quantification

We now bring the model to the data. We first present the calibration and then analyze how important firm- and worker-level shocks are as drivers of earnings dynamics.

4.1 Functional Forms

Utility function. For the instantaneous utility function over consumption, we assume log utility, a common choice in search models with risk averse workers (Bagger, Hejlesen, Sumiya, and Vejlin 2018; Hubmer 2018; Krusell, Mukoyama, and Şahin 2010):

$$u(c) = \log(c). \tag{20}$$

Search effort. For the cost and return of search effort we choose the functional forms proposed in Faberman, Mueller, Şahin, and Topa (2022). Consider first the cost function:

$$c_w(s) = \kappa^0 s^{1+\frac{1}{\kappa^1}}. \tag{21}$$

The return to search is given by the following functional form:

$$\Lambda_i(s) = \eta^0 + \eta_i^1 s. \tag{22}$$

Relative to Faberman, Mueller, Şahin, and Topa (2022), we simplify the functional forms slightly by keeping κ^0 and η^0 constant across employed and unemployed, as we cannot

[25] See for example Lise and Robin (2017) and Gulyas (2023).

[26] Note that even with risk neutrality, the presence of progressive taxes would prevent us from solving the model using just a surplus function.

separately identify them. However, as we show below, the functional forms are still flexible enough to match labor market flows in the Danish labor market and to also be broadly consistent with the evidence on the search behavior of employed and unemployed in Faberman, Mueller, Şahin, and Topa (2022).

Job creation. We choose the job creation cost function as in Gulyas (2023):[27]

$$c_f\left(v^{\mathrm{N}}\right) = \chi_0 \left(\frac{v^{\mathrm{N}}}{\chi_1}\right)^{\chi_1}. \tag{23}$$

Matching function. The matching function is a standard Cobb-Douglas function

$$\mathcal{M}\left(L, v\right) = \xi L^{\omega} v^{1-\omega}, \tag{24}$$

with scaling parameter ξ and matching function elasticity ω.

Productivity Processes. For both firm and worker productivity, we assume AR(1) processes in logs. Persistence parameters are denoted with ρ_y and ρ_x; the standard deviations of the innovations are σ_y and σ_x.[28]

Production function. Match output is a function of both worker and firm productivity, following Gulyas (2023):

$$f\left(x, y\right) = \nu \left(x^{\frac{1}{\gamma}} + y^{\frac{1}{\gamma}}\right)^{\gamma}. \tag{25}$$

The parameter γ captures potential complementarities in production.

Exogenous separations. For the exogenous separation probabilities, we use the functional form proposed in Faberman, Mueller, Şahin, and Topa (2022):

$$\delta\left(x\right) = \bar{\delta} - \delta_x \log\left(x\right). \tag{26}$$

[27] This or similar functional forms are widely used in the literature; see among others Merz and Yashiv (2007), Kaas and Kircher (2015), Bagger and Lentz (2019), and Bilal, Engbom, Mongey, and Violante (2022).

[28] For the numerical implementation, we discretize the productivity processes using the Tauchen (1986) procedure, with 20 grid points each for worker and firm productivity. The extremes of the productivity grid are three standard deviations above and below the median, respectively.

Tax function. The government raises revenues with the log-linear tax function popularized by Feldstein (1969), Benabou (2002), and Heathcote, Storesletten, and Violante (2017):

$$T(w) = w - \tau^L w^{1-\tau^P}, \tag{27}$$

where τ^L governs the level of taxes and τ^P the tax progressivity. If τ^P is positive, the tax system is progressive, that is marginal and average tax rates are increasing in income. A negative τ^P implies regressive taxes. With $\tau^P = 0$, taxes are flat.

4.2 Calibration

We quantify the model to be consistent with key features of the Danish economy in general and the labor market in particular. A period in the model is one month.

We calibrate the model using a standard two-step procedure, fixing a number of parameters without solving the model and calibrating the remaining parameters internally. The level parameters of the production function ν and the matching function ξ are normalizations. We also fix the curvature parameter of the matching function ω to 0.5, in line with a large literature following Petrongolo and Pissarides (2001). We now discuss which parameters are most closely associated with which moments from the data. All parameter values are summarized in Table 1.

Preferences. Given the assumption of log-utility over consumption, the discount factor β remains to be set. We choose a monthly discount factor of 0.995, corresponding to an annual discount factor slightly above 0.94, a standard value in the literature.

Inequality and risk. We want the model to be consistent with the degree of inequality and risk in the economy. The key parameters related to these moments are those of the worker and firm productivity processes. Since both these processes are assumed to be AR(1) processes in logs we need to set the persistence parameters and the standard deviations of the innovations.

Generating a sufficient amount of wage inequality in the model requires that the processes are quite persistent, with both ρ_x and ρ_y in excess of 0.99. With less persistence, firms and workers will be too similar in a lifetime value sense, which will result in a very compressed wage distribution. Table 2 reports the average wage by ventile of the monthly wage distribution. The model does very well in capturing the inequality among the bottom 90% of the distribution. In particular, the model captures well the mean-min

Table 1: Parameter Values

Parameter	**Interpretation**	**Value**
Preferences		
β	Discount factor	0.995
Labor market		
α	Bargaining power	0.500
ξ	Matching function level	0.500
ω	Matching function curvature	0.500
$\bar{\delta}$	Job destruction probability level	0.009
δ_x	Job destruction probability slope	0.010
Workers		
ρ_x	Worker productivity persistence	0.993
σ_x	Worker productivity standard deviation	0.051
Search effort		
κ^0	Search cost scale	0.150
κ^1	Search cost curvature	1.000
η^0	Search return intercept	0.010
η^1_U	Search return unemployed slope	0.375
η^1_E	Search return employed slope	0.510
Firms		
F	Mass of firms	0.110
ρ_y	Firm productivity persistence	0.994
σ_y	Firm productivity standard deviation	0.041
Production		
ν	Production function scale	0.871
γ	Production complementarity	1.200
Job creation		
χ_0	Job creation cost level	53.500
χ_1	Job creation cost curvature	1.150
Government		
b	Unemployment benefit	1.110
τ_L	Income tax level	0.571
τ_P	Income tax progressivity	0.256

Notes: Table 1 summarizes the parameter values.

Table 2: Wage Inequality

Ventile	1	2	3	4	5	6	7	8	9	10
Data	1.00	1.26	1.37	1.45	1.52	1.58	1.64	1.69	1.75	1.80
Model	1.00	1.10	1.19	1.28	1.36	1.43	1.52	1.58	1.66	1.74
Ventile	11	12	13	14	15	16	17	18	19	20
Data	1.86	1.93	2.00	2.08	2.18	2.31	2.47	2.73	3.18	5.27
Model	1.81	1.90	1.99	2.09	2.20	2.33	2.47	2.65	2.91	3.43

Notes: Table 2 shows the average wage by ventile of the monthly wage distribution in data and model, relative to the average wage of the bottom ventile.

ratio of roughly 2, a key statistic for wage dispersion in search models (Hornstein, Krusell, and Violante 2011).[29]

Two additional features of the model help in matching this statistic. First, we set the initial bargaining power parameter α to 0.5. Without this initial bargaining power it is well known that in this class of models initial wages tend to be very low, as workers accept very low starting wages in exchange for getting on the job ladder (Bagger and Lentz 2019; Postel-Vinay and Robin 2002). While this effect is somewhat muted in our model with risk averse workers relative to much of the literature, where workers are risk neutral, the degree of risk aversion is not sufficient to prevent too low starting wages. Second, we set the slope of the job losing probability δ_x to 0.01, implying an exogenous separation probability that is decreasing in worker productivity. While the model generates endogenous separations for workers who experience productivity shocks to low productivities, the model requires this effect to be strengthened through a higher incidence of exogenous separations among low productivity workers. This feature concentrates job losses among a smaller set of individuals, who regularly fall off the job ladder, and who are at the bottom of the wage distribution (Gregory, Menzio, and Wiczer 2022; Jarosch 2023; Jung and Kuhn 2019).

Next to the wage distribution in levels, the productivity process parameters are also closely related to moments of the worker earnings and the firm revenue growth distribution. Table 3 reports standard deviation and kurtosis of both the annual earnings growth distribution on the worker side and the annual revenue growth distribution on the firm side. The model captures well key features of these distributions. The dispersion in revenue growth is significantly larger than the dispersion in earnings growth. Both distributions are leptokurtic. While again all parameters determine the model values jointly,

[29] While the model performs very well in generating inequality among the bottom 90% of the wage distribution, it lacks a mechanism to generate the high incomes at the very top of the wage distribution. This could be fixed for example by adding a Pareto tail to the worker productivity distribution, but since top inequality is not the main focus of this paper, we refrain from doing this.

Table 3: Worker Earnings and Firm Revenue Growth

Earnings Growth	Std. Dev.	Kurtosis
Data	0.33	12.16
Model	0.26	16.92
Revenue Growth	Std. Dev.	Kurtosis
Data	0.56	5.20
Model	0.55	5.17

Notes: Table 3 shows moments of the annual worker earnings growth and the annual firm revenue growth distribution.

these statistics are closely related to the standard deviations of the innovations, σ_x and σ_y, respectively.

Key to produce the deviations from normality is the wage setting protocol. A large chunk of small productivity shocks is absorbed by firms because these shocks do not lead to a violation of either participation constraint, so that the wage remains the same. However, once participation constraints are hit, the resulting adjustments are large, either through an endogenous separation if the match becomes infeasible or an adjustment of the wage.

Labor market flows, search effort, and job creation. Another key set of statistics are labor market flows. The aggregate unemployment rate over the relevant time period averages 5%. The monthly employment-to-unemployment (EU) transition rate is 1.24%. The monthly employment-to-employment transition rate is 1.35%.[30]

The model generates EU transitions through both exogenous and endogenous separations. Endogenous separations can occur after negative productivity shocks if there is no wage at which both firm and worker want to proceed with the match. To generate endogenous separations, it is useful that the vacant job is preserved in case of such a separation, which gives the firm an option value of potentially filling the job with a better worker. Endogenous separations always require that worker productivity is low. They are more likely in firms of low or high rather than medium productivity. If both firm and worker are relatively unproductive, output of the match will be low and an endogenous separation may be beneficial as the worker can collect the unemployment benefit. If the worker is unproductive, but the firm has a very high productivity, the firm may not be

[30] Note that these rates are affected by our sample selection of workers with reasonably strong labor market attachment and would increase if we included marginal employment.

Table 4: Aggregate Labor Market Statistics

Moment	Unemployment rate	EU rate	EE rate
Data	5.00%	1.24%	1.35%
Model	5.20%	1.02%	1.55%

Notes: Table 4 shows key statistics of the aggregate labor market.

willing to pay a wage to the worker that the worker would accept because it would be very valuable for the firm to find a better worker.

However, endogenous separations are relatively rare, as many matches can be preserved in the hope for positive productivity shocks in the future. Hence, to match the EU transition rate, we set the parameter $\bar{\delta}$ to 0.9%. Empirically, exogenous separations are also supported in that there are separations also in strongly growing firms (Davis, Faberman, and Haltiwanger 2013), which is hard to generate endogenously.

On the job creation side, the mass of firms and the parameters of the job creation cost function are closely related to the firm size distribution. The mass of firms F does not pin down average firm size exactly because there are some firms in the simulated model economy which do not employ any workers, but it is closely related. Mean firm size in the model is 19 relative to 16 in the data, with parameter F equal to 0.11. We set the parameters of the job creation cost function to match the 10th percentile of the firm size distribution of 1 and the median of the firm size distribution of 2-3, i.e. to generate a sufficient mass of small firms.[31]

The model generates large firms with hundreds of employees, but lacks a mechanism to generate the very large firms that are present in the data. Matching this would likely require the introduction of another state variable, to model permanent next to transitory (even if very persistent) differences in firm productivity. The model also produces a very reasonable probability for a vacant job to meet a worker of 0.35, for which we do not have a direct data counterpart, but which is well within the range of plausible values (Gulyas 2023).

On the worker search effort side, the key aggregate statistics to match are the unemployment rate and the EE transition rate. We fix the cost function curvature parameter κ^1 to 1, in line with several other papers modeling the cost of search effort in this way for the Danish labor market (Bagger, Hejlesen, Sumiya, and Vejlin 2018; Bagger and Lentz 2019; Bagger, Moen, and Vejlin 2021). The scale parameter of the cost function is very closely related to the unemployment rate, as it governs how much search effort there is

[31] The median varies in the data between 2 and 3 over time; it is 3 in the model.

overall. The intercept of the return to search function is positive in order to generate positive job-to-job transition rates also for high wage earners, who have little incentive to search. What also helps in flattening the EE transition rate along the wage distribution, as required by the data, is to have positive complementarities in production ($\gamma = 1.2$), which is consistent with positive sorting in other equilibrium models of the Danish labor market (Bagger, Hejlesen, Sumiya, and Vejlin 2018; Bagger and Lentz 2019). The slope of the return to search function for the unemployed is then set to generate a share of offers unrelated to search effort of around 15%, an order of magnitude in line with Faberman, Mueller, Şahin, and Topa (2022); for the employed it is closely related to the average EE transition rate.

We also compare our model implied statistics to more evidence on search effort of the employed and the unemployed by Faberman, Mueller, Şahin, and Topa (2022). The advantage of comparing to this evidence is that it is direct evidence on search behavior rather than just observed labor market transitions. These are determined both by search behavior and the decisions of whether to create a match upon meeting. While the model accounts for both margins, it is useful to still compare to direct evidence on just the effort margin separately. The downside is that this evidence is based on a survey of workers in the U.S. and search decisions are endogenous to the environment. In line with the findings in Faberman, Mueller, Şahin, and Topa (2022), the calibrated model predicts a much higher search effort of the unemployed relative to the employed. The model is also consistent with the finding that the offer yield, the number of offers per unit of search, is higher for the employed than the unemployed. Furthermore, the model matches a lower share of offers resulting from free search for the unemployed.

Another finding of Faberman, Mueller, Şahin, and Topa (2022) is that search of the employed is strongly descreasing in wages. The paper finds this negative search wage relation by regressing the number of applications and search time on the log real wage, controlling for worker characteristics. In the model, holding constant worker and firm productivities, search is strongly decreasing in the wage. Consider as an extreme case the worker who earns the highest possible wage in the model: clearly, there is no incentive to search for a better job. However, at the same time search effort is also increasing in worker productivity, as firm competition makes outside offers more valuable for higher productivity workers. Looking at the cross-section of workers without controlling for productivity produces a relatively flat search effort profile. Obtaining a more negative relation would require EE transition rates that are strongly decreasing in wages, at odds with the data.

Government. The government runs the progressive tax system, with two parameters for the log-linear tax function. In addition, we have to set the unemployment benefit. For tax progressivity, we rely on the estimation results of Holter, Krueger, and Stepanchuk (2019), which estimates tax progressivity for a range of OECD countries. The estimation is based on the wedge in marginal tax rates at different income levels and based on labor income, appropriate for our model with just labor income. The estimated parameter is 0.26, which is the highest among the OECD countries, almost twice as high as for the U.S.[32] We set the level parameter τ_L to match an average marginal tax rate (AMTR) of 0.64 (model value 0.65), as reported by Bagger, Moen, and Vejlin (2021).[33]

We model the unemployment benefit as a fixed payment to an unemployed worker. Unemployment insurance in Denmark is voluntary, but high subsidization of the unemployment insurance funds ensures high participation. Unemployment insurance can be received for up to two years, which is why we do not model a maximum duration or risk of losing unemployment benefits. The vast majority of workers in our sample with a somewhat strong attachment to the labor force finds jobs much quicker than two years. In principle, unemployment benefits can go up to 90% of previous income. However, there is a rather tight cap on the maximum benefits, so setting a constant benefit only overstates the benefit level for some of the lowest earners. Hence, in line with the data, we set the unemployment benefit such that it is roughly 50% of the average monthly wage.

4.3 Firm Dynamics and Earnings Risk in the Model

With the quantified model at hand, we can now analyze whether and how the model reproduces the evidence presented in Section 2. In particular, through the lens of the model we can quantify the relative importance of firm- and worker-level shocks for large earnings changes.

[32]The value estimated for the U.S. is somewhat lower at 0.14 compared to the well-known 0.18 estimate of Heathcote, Storesletten, and Violante (2017). The reason is that wedge based measures only account for income taxes, whereas the estimates of Heathcote, Storesletten, and Violante (2017) account for the entire tax-and-transfer system. Wedge based measures therefore usually produce lower progressivity estimates; see also the estimates for the U.S. of Ferriere and Navarro (2022) and the discussion in Heathcote, Storesletten, and Violante (2020). A measure only based on income taxes is appropriate given our focus on individuals with some attachment to the labor market and separate modeling of unemployment benefits.

[33]This very high value for the AMTR is explained on the one hand by the high and progressive income taxes and the fact that it also accounts for consumption taxes on the other hand.

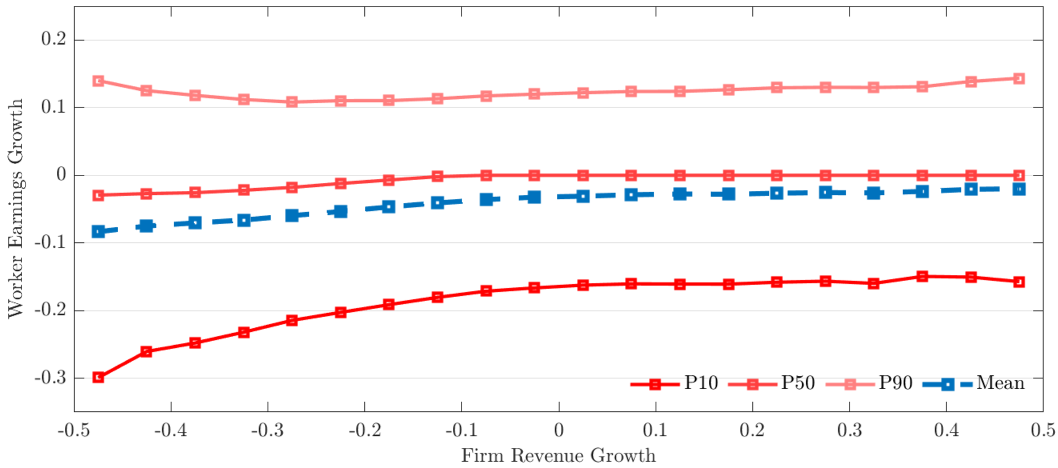

Figure 7: Worker Earnings Growth by Firm Revenue Growth – Model

Notes: This figure shows model simulated moments of the annual earnings growth distribution, conditional on the revenue growth of the firm at which workers are employed in the initial year.

Earnings dynamics by firm revenue growth. We start with worker earnings growth by firm revenue growth, not conditioning on staying in the same firm, in Figure 7, the model counterpart to Figure 2.

The model replicates quantitatively well the earnings dynamics by firm revenue growth group. Average earnings growth is increasing in firm revenue growth, and this is mostly driven by the tails of the distribution. The 10th percentile of the earnings growth distribution is increasing by roughly 15 percentage points between bottom and top of the revenue growth distribution, in line with the data.[34] The median is close to flat at zero. For the 90th percentile, the model captures the slight increase in the positive part of the firm growth distribution, but predicts a too high 90th percentile at the bottom.

What are the model features required to capture these different facts? As stated repeatedly, inaction for the median worker relies on the wage setting protocol, in which wages are only adjusted in the case of certain events: relevant outside offers or productivity shocks violating participation constraints. The former are important to generate the upper tail of the earnings growth distribution. At the bottom of the revenue growth distribution, gains for workers are mostly generated through outside offers that lead to job-to-job transitions. This is because these firms tend to be of low productivity, so that most outside offers are from better firms. At the top of the revenue growth distribution, job-to-job transitions still play a role, but since these firms tend to be of high productivity, outside offers are more often used as a threat to negotiate up wages within an existing match. For the lower tail of the earnings growth distribution exogenous separations, en-

[34] In levels, along the entire distribution the 10th percentile is slightly too low.

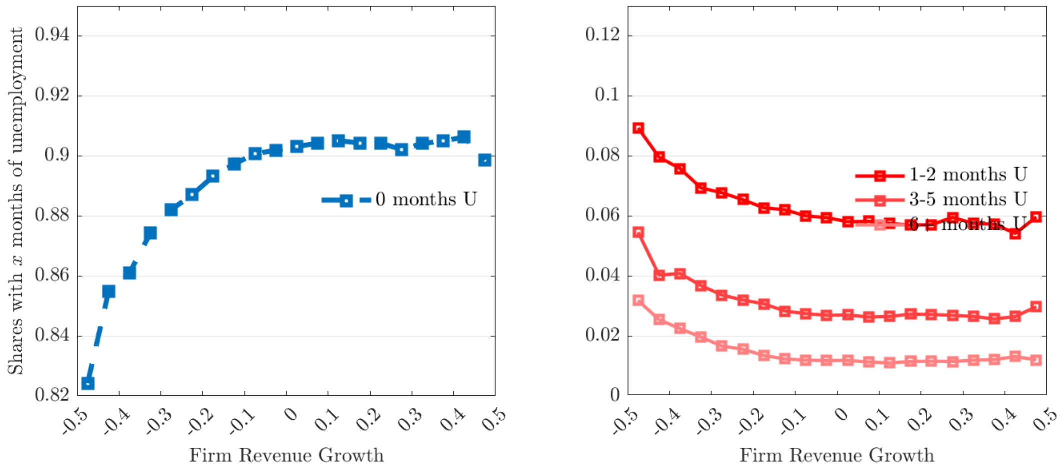

Figure 8: Unemployment Propensities by Firm Revenue Growth – Model

Notes: This figure shows the model simulated likelihood of experiencing unemployment spells of varying duration, conditional on the revenue growth of the firm at which workers are employed in the initial year.

dogenous separations, and firm demanded renegotiations leading to wage losses are the relevant margins. We now differentiate between the role of the extensive and intensive margin, and how they are driven by firm- and worker-level shocks.

Extensive margin. The probabilities of unemployment spells of different lengths are shown in Figure 8, the model counterpart to Figure 3. The model captures very well that separation rates are essentially flat across positive revenue growth firms. They become increasingly higher the more negative firm revenue growth is.

To capture this, the model relies on a relatively large share of exogenous separations, around 90% of all separations to unemployment. This is necessary to generate a high enough probability of separations in the upper half of the firm revenue growth distribution, where essentially no endogenous separations are caused by firm-level productivity shocks and few by worker level productivity shocks.

Endogenous separations are, however, important to generate the gradient in the separation probability in the bottom half of the firm revenue growth distribution. Out of all endogenous separations, 49% are triggered by a worker productivity shock, 38% by a firm level productivity shock, and 13% by a simultaneous shock to both firm and worker productivities.[35] Figure 9 shows endogenous separation probabilities split up by underlying shock along the revenue growth distribution. Towards the bottom of the revenue

[35]We do not distinguish here between positive and negative shocks because in the overwhelming majority of cases it is negative productivity shocks. In very rare instances it can be the case that a positive firm productivity shock increases the value of having a vacant job above the value of employing a relatively unproductive worker, even at a low wage.

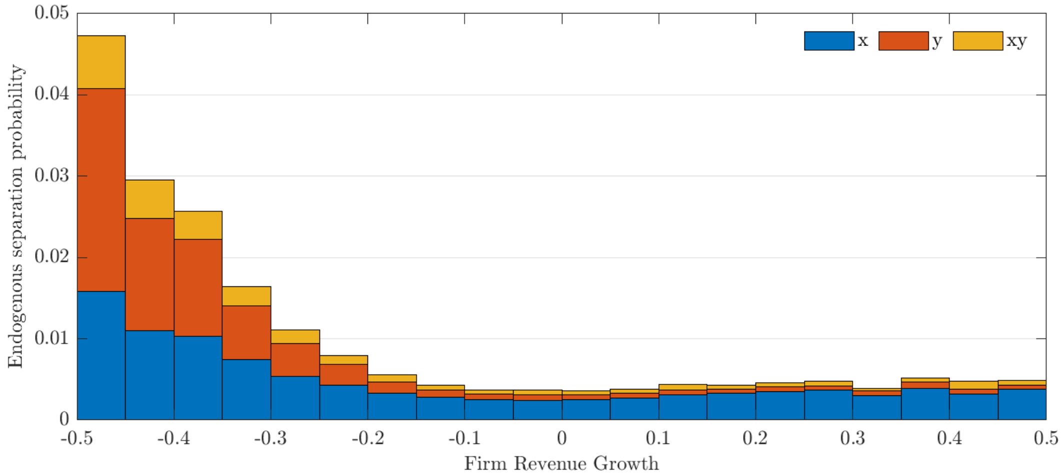

Figure 9: Reasons for Endogenous Separations

Notes: This figure splits up model generated endogenous separations into those driven by worker productivity shocks, firm productivity shocks, or both.

growth distribution, the share of separations driven by firm productivity shocks strongly increases. This is intuitive, as with falling firm productivity it becomes less likely that matches remain feasible. However, endogenous separations driven by worker productivity shocks are also increasing towards the bottom of the firm revenue growth distribution. Firms in that range tend to be of low productivity, such that it becomes also more likely for worker productivity shocks to trigger an endogenous separation. Hence, endogenous separations rise through this channel in strongly shrinking firms, even if a firm shock is not the final determinant of the separation.[36]

Intensive margin. Figure 10 in turn is the model counterpart of Figure 4, i.e. the earnings growth of stayers along the firm revenue growth distribution. As in the data, as the largest losses are driven by separations, the distribution of earnings growth along the firm revenue growth distribution is more compressed. Still, the mean of the earnings growth distribution is increasing in firm revenue growth, driven both by a lower probability of large losses at the top and a higher probability of large gains. The gradient of the 90th percentile of the worker earnings growth distribution is steeper than in the data because at the bottom of the revenue growth distribution the model tends to generate job-to-job transitions rather than gains through renegotiation after outside offers. For the 10th percentile, it is important to have both firm and worker productivity shocks. With only firm shocks large losses would be too concentrated at the bottom of the firm

[36]A portion of the gradient in the model is caused by a varying share of exogenous separations. Firms experiencing more exogenous separations are more likely to have falling revenues because they have fewer workers.

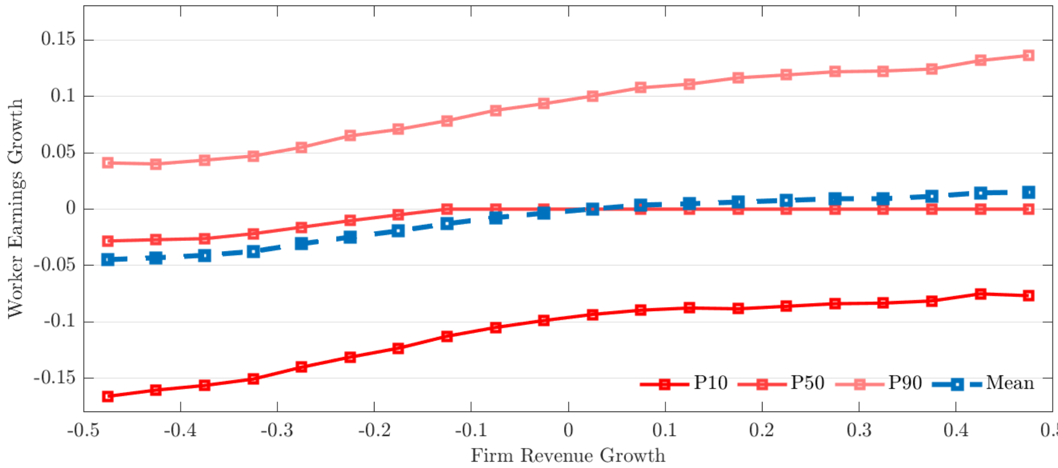

Figure 10: Worker Earnings Growth of Stayers by Firm Revenue Growth – Model

Notes: This figure shows model simulated moments of the annual earnings growth distribution for workers staying at the same firm, conditional on the revenue growth of the firm.

revenue growth distribution. Generating some wage losses for stayers also in growing firms requires productivity shocks to those workers, which may trigger a firm demanded renegotiation of the wage.

Quantitatively, out of all firm demanded renegotiations, 58% are triggered by a worker productivity shock, 31% by a firm level productivity shock, and 11% by a simultaneous shock to both firm and worker productivities. Figure 11 shows how important firm- and worker productivity shocks are along the firm revenue growth distribution in causing large earnings losses for stayers. Specifically, we consider stayers with large earnings losses of 10% or more. This is an annual outcome, and may be the consequence of several renegotiations following different types of shocks. We consider all downward wage renegotiations workers experience in base year and second year, leading to the large year-on-year earnings loss. We count how many renegotiations workers experience that are driven by worker productivity shocks (x), firm productivity shocks (y), or both simultaneously, and compute the shares of these renegotiations. To keep the figure readable, for all workers with several renegotiations, we downweight an individual renegotiation by dividing it by the total number of renegotiations that worker experiences.

In the bottom firm revenue growth groups, large earnings losses for stayers are more than twice as likely as in the top revenue growth groups. This gradient is largely driven by the differential probability of renegotiations after negative firm productivity shocks. At the bottom of the revenue growth distribution, firm and worker productivity shocks are of roughly equal importance in causing renegotiations. At the top, large earnings losses are almost exclusively driven by worker productivity shocks.

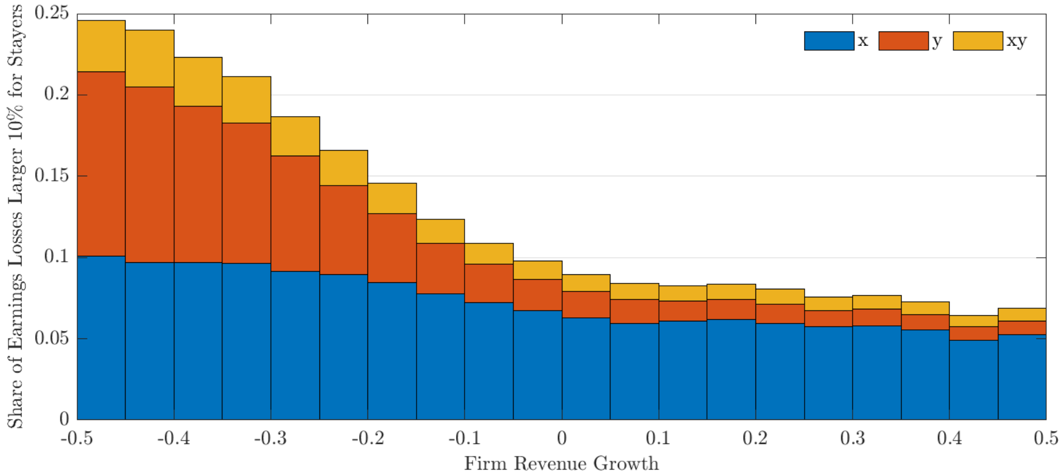

Figure 11: Reasons for Large Downward Wage Renegotiations – Model

Notes: This figure shows which share of stayers experiences earnings losses larger than 10%, by firm revenue growth group. It decomposes which share is driven by firm- vs. worker productivity shocks. We consider downward wage renegotiations in year $t-1$ and t. If a worker experiences renegotiations for different reasons, we count them with a weight inversely related to the number of renegotiations the worker experiences.

5 Policy Analysis

In the previous sections we have shown that workers are exposed to significant earnings instability. In the equilibrium framework, earnings losses are driven by exogenous separations, endogenous separations, and wage losses for stayers, where the latter two cases are driven both by firm- and worker level shocks. Also for earnings gains, firm heterogeneity is very important, as earnings gains realize through climbing the job ladder towards increasingly better firms.

With risk averse workers, there is space for redistribution and insurance by the government. Public policies can provide valuable insurance against earnings fluctuations. In the rich equilibrium framework both policies that target workers directly and policies that try to insure workers by targeting firms are conceivable. In this section, we consider unemployment benefits and layoff taxes for providing insurance against unemployment risk as well as progressive taxes as insurance against both unemployment and wage fluctuations. In particular, we evaluate what the efficiency implications of different policies are, through their impact on job creation, search effort, and the allocation of workers across firms.

In all cases, the exercise we are implementing is the following. Across a range of values for unemployment benefits, layoff taxes, and tax progressivities, we compute the

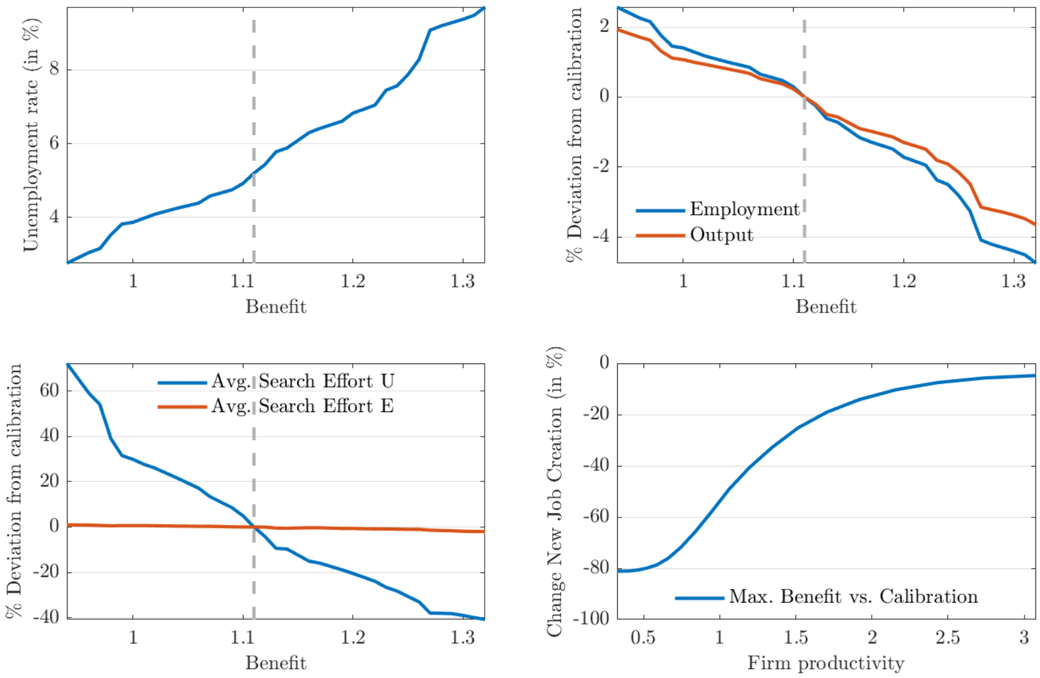

Figure 12: Policy Evaluation – Varying the Unemployment Benefit

Notes: This figure shows the unemployment rate, employment and output, and search effort across a range of unemployment benefit levels (dashed line = calibrated value). It also shows how new job creation by firm type varies in the highest benefit scenario relative to the calibration.

stationary equilibrium given a tax policy. We balance the budget by adjusting the level parameter of the income tax function.

5.1 Unemployment Insurance

The most direct way to potentially improve the welfare of the unemployed is to increase unemployment benefits. In terms of flow utility, higher benefits are clearly beneficial for the unemployed. Figure 12 illustrates how some key equilibrium objects adjust in response to higher unemployment benefits.

The upper left panel of Figure 12 shows how the unemployment rate varies with the unemployment benefit. A higher unemployment benefit leads to a much higher unemployment rate in equilibrium. This is driven by several channels. First, some matches that are feasible in the calibrated economy become infeasible in the equilibrium with higher unemployment benefits, in which unemployment is more attractive for workers. On top of this, higher benefits discourage the search effort of the unemployed, shown in the bottom left panel, where we plot the average search effort of the unemployed and the employed in deviation from the calibration. Note that this plot incorporates both a behavioral and a composition effect. For a given productivity, search effort is discouraged by higher benefits. On top of that, the pool of unemployed shifts towards lower productivity workers, who search less.

Reduced search effort and higher reservation wages would materialize in most search models as a consequence of higher unemployment benefits. However, in the rich equilibrium framework also with heterogeneous firms there are additional effects. The bottom right panel of the figure shows how new job creation by firm type is different in the equilibrium with the highest considered unemployment benefit relative to the calibration. Job creation drops across the entire firm productivity distribution, the most at relatively unproductive firms. Firms face workers with an improved outside option, so starting wages they have to pay are higher. Furthermore, the share of search coming from relatively unproductive unemployed goes up. This reduces the value of a vacant job such that job creation goes down.

In equilibrium, there is hence both a fall in total employment and output, as shown in the upper right panel. Hence, there are equilibrium repercussions for the employed across several margins. There is less job creation, slowing down reallocation of workers towards better firms. Furthermore, taxes have to rise because there are more unemployed who receive benefits and fewer employed from whom to raise taxes.

The adverse equilibrium effects are strong. Increasing benefits from the calibration is beneficial only for low productivity unemployed. Even high productivity unemployed dislike being in the economy with higher unemployment benefits because of reduced job creation and higher taxes in equilibrium.

5.2 Layoff Taxes

Given the adverse equilibrium effects of unemployment benefits, an alternative policy to consider are layoff taxes. We consider a layoff tax that firms have to pay in the case of an endogenous separation. Unlike with unemployment benefits, the aim of a layoff tax is to preserve matches that would be endogenously destroyed absent a layoff tax.

The typical argument in favor of a layoff tax is that it corrects a fiscal externality. When firms and workers decide upon whether to separate, they do not take into account that the unemployment benefit received by the worker has to be financed (Blanchard and Tirole 2008; Cahuc and Zylberberg 2008).

However, whether layoff taxes reduce or increase unemployment in equilibrium is a debated question in the literature. Theoretically, the effect is ambiguous, with layoff taxes preventing separations, but also reducing hiring as firms anticipate that they may

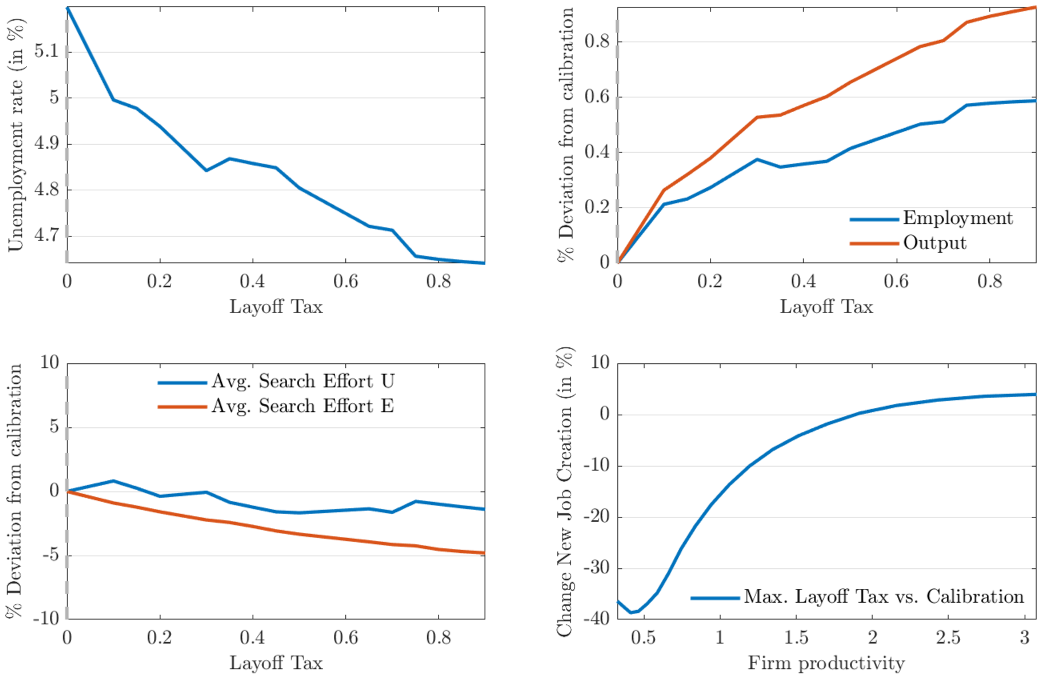

Figure 13: Policy Evaluation – Varying a Layoff Tax

Notes: This figure shows the unemployment rate, employment and output, and search effort across a range of layoff tax levels (dashed line = calibrated value). It also shows how new job creation by firm type varies in the highest layoff tax scenario relative to the calibration without layoff tax.

end up in a situation where they either keep a worker only to avoid the firing tax or have to pay it. Quantitatively, the literature has found positive and negative effects of layoff taxes on employment (Alvarez and Veracierto 1999; Bentolila and Bertola 1990; Hopenhayn and Rogerson 1993; Jung and Kuester 2015; Ljungqvist 2002; Mortensen and Pissarides 1999; Saint-Paul 1995).

Figure 13 shows how the equilibrium changes with positive layoff taxes. In our framework, the unemployment rate falls slightly and aggregate employment goes up. Key to this result is how job creation adjusts across heterogeneous firms. Recall from the previous section that endogenous separations are much more likely in firms with negative productivity shocks, or generally low productivities. Hence, new job creation by those firms is most strongly negatively affected. In equilibrium, high productivity firms face less competition from less productive firms, such that their job creation goes in fact up. Search effort of workers changes only modestly, so that the equilibrium allocation of workers to firms improves and output rises even more than employment.

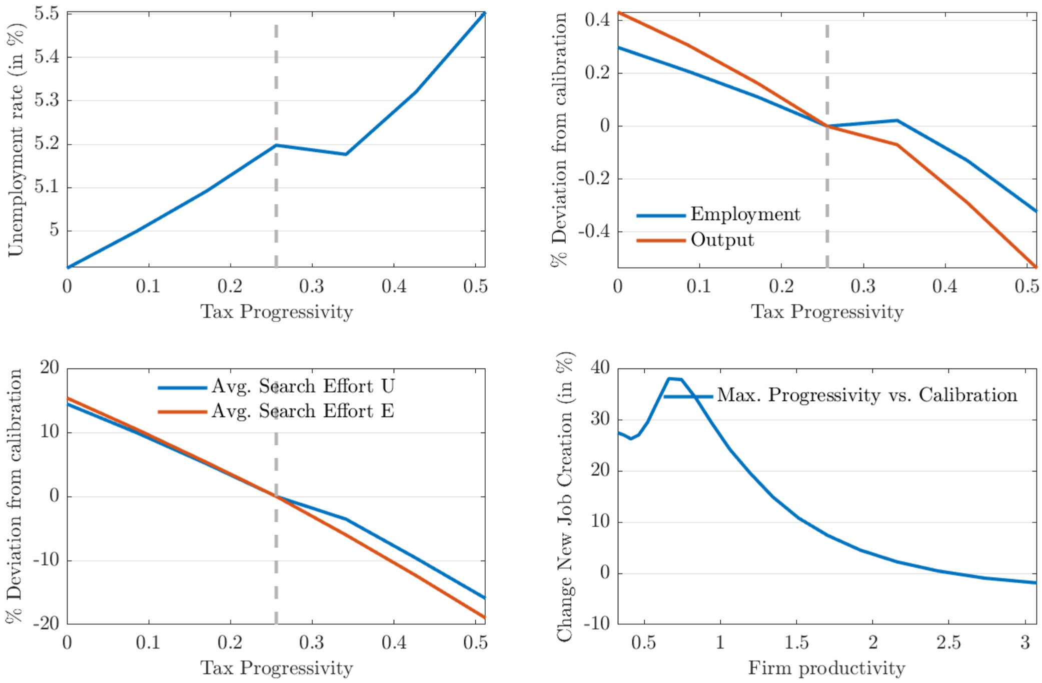

Figure 14: Policy Evaluation – Varying Tax Progressivity

Notes: This figure shows the unemployment rate, employment and output, and search effort across a range of tax progressivities (dashed line = calibrated value). It also shows how new job creation by firm type varies in the highest progressivity scenario relative to the calibration.

5.3 Progressive Taxation

The two policies considered so far are mostly targeted towards reducing very large earnings losses through unemployment. A policy that reduces after-tax inequality and earnings changes across the entire distribution are more progressive taxes.

This policy is widely studied in incomplete markets models, where the income process is exogenous. The typical trade-off is between reducing inequality and providing insurance on the one hand and distortions to labor supply, skill investment, and capital accumulation on the other hand.[37] In our framework, the benefits in reducing consumption inequality and fluctuations are similar, but the set of distortions is quite different.

[37]See Heathcote, Storesletten, and Violante (2017) and Badel, Huggett, and Luo (2020) for skill investment choices; and Kindermann and Krueger (2022), Boar and Midrigan (2022), and Ferriere, Grübener, Navarro, and Vardishvili (2023) for savings decisions. In more related frameworks, Kreiner, Munch, and Whitta-Jacobsen (2015) quantifies efficiency losses from misallocation due to labor taxation with a partial equilibrium search model; Bagger, Hejlesen, Sumiya, and Vejlin (2018) studies the effects of Danish tax reforms on equilibrium labor allocations, but in a model with fixed firm and worker types and households' income fluctuations insured through complete markets; Bagger, Moen, and Vejlin (2021) studies optimal affine redistributive taxation, but in a framework with risk-neutral workers, where the equilibrium allocation absent distortive taxes is constrained efficient; and Kaas, Lalé, and Siassi (2023) studies progressive taxation in an incomplete markets model with a job ladder, but where workers and firms are homogeneous ex ante, and productivity heterogeneity is constant at the match level.

Equilibria across a range of tax progressivities are shown in Figure 14. More progressive taxes reduce search effort of households, and in contrast to higher unemployment benefits they do so for both unemployed and employed workers. With more progressive taxes, tax payments are lower at the bottom of the wage distribution and higher at the top, reducing the benefit from moving up the job ladder. Hence, employment is falling, but output is falling even more strongly, as the reallocation of employed towards better firms is slowed down. This effect is amplified by less job creation by the most productive firms.

In terms of welfare, low productivity unemployed and low productivity, low wage employed are generally better off with more progressive taxes, whereas high productvity and high wage individuals are worse off.

6 Conclusion

In this paper, we study the role of firms for the earnings dynamics of workers, focusing on the joint distribution of firm revenue growth and worker earnings growth. Using Danish matched employer-employee data we establish a strong connection between the revenue growth of firms and the probability of large earnings changes of workers. In particular, workers in shrinking firms are more likely to transition to unemployment and to experience large earnings losses even when staying. Workers in growing firms are more likely to experience large earnings gains. However, while there is a strong connection between firm revenues and large worker earnings changes, these can occur along the entire firm distribution and median earnings growth is remarkably stable.

We rationalize these findings in an equilibrium search model, where risk averse workers, choosing their search effort, contract with risk neutral firms, choosing job creation, under two-sided limited commitment. Exogenous and endogenous separations generate earnings losses through unemployment spells across the entire firm distribution, but more concentrated in shrinking firms. Productivity shocks to firms and workers generate wage losses for stayers, also most often in shrinking firms, but potentially also in strongly growing firms. Outside offers and job-to-job transitions generate large earnings gains. Firm productivity shocks account for roughly a third of large earnings losses through endogenous separations and renegotiations for stayers.

We use the equilibrium model to study the effects of policy interventions targeted to reduce workers' earnings fluctuations. The interaction of heterogeneous workers adjusting their search effort and heterogeneous firms adjusting their job creation is key to understand the aggregate and distributional consequences of policies in equilibrium.

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A Data Appendix

A.1 Overview of Data Sources

For the empirical investigation we combine data from several Danish registry data sets provided by Statistics Denmark.

BFL. The main data source on earnings and employment in this paper is the BFL (*Beskæftigelse for Lønmodtagere*). We use information on employment spells within each month from 2008 until 2016. For each of these spells we observe earnings, hours, and the start and end date of the spell. Also, for each individual there is an identifier based on the anonymized social security number, which can be used to link this individual to other data sets. Similarly, for each firm there is an identifier that also can be used to match the data to other data sets. It also contains information on occupation, industry, and location.

BEF. The BEF (*Befolkningen*) data set contains information on demographics, which can be merged to with the BFL data. A control variable we use from this data source is an individual's sex.

IDAP. Another individual level data set is IDAP (*Integreret Database for Arbejdsmarkedsforskning - Persondata*) from the integrated database for labor market research. From this data set we take variables on individuals' age and labor market experience.

UDDA. The UDDA (*Uddannelser*) database contains information on individuals' education.

FIKS. The main source for firm revenue information is the FIKS (*Firmaernes køb og salg*) database. This data set includes information on firms' purchases and sales from value added tax data. In addition to the purchase and sales numbers it contains information on the frequency at which a firm settles its VAT accounts. For the largest firms this is done at monthly frequency; for smaller firms it is done at quarterly or half-yearly frequency.

FIRM. The FIRM (*Generel firmastatistik*) database contains general information on firms. From this database we take information on firm-level employment and information on the source of firm-level accounting data in the FIRE database (see below).

FIRE. FIRE (*Regnskabsstatistikken*) is the database for firm-level accounting information, a complementary source of information on firm revenues. Non-imputed revenue information is available only for a much smaller set of firms than what is available in the FIKS data, so we use FIKS as our main source for revenues.

A.2 Summary Statistics

Table A.1 shows statistics of the distribution of annual earnings growth, both for raw earnings and residualized earnings after taking out education, gender, location, industry, and occupation.

A.3 Additional Figures

Table A.1: The Annual Earnings Growth Distribution

	Mean	SD	Skewness	Kurtosis	P10	P50	P90
Raw earnings	0.02	0.34	0.08	13.70	-0.23	0.01	0.25
Raw earnings [-2,2]	0.02	0.33	0.12	12.16	-0.23	0.01	0.25
Res. earnings [-2,2]	0.02	0.31	0.15	12.26	-0.23	0.01	0.25

Notes: Table A.1 shows statistics of the annual earnings growth distribution using raw earnings, raw earnings for earnings changes restricted between -2 and 2, and for residualized earnings with changes also restricted between -2 and 2.

In this section we reproduce some figures from the main text using alternative cuts of the data. Figure A.1 reproduces Figure 2 using an alternative measure of firm revenues from accounting data (FIRE database) rather than value added tax data (FIKS database). All key patterns are as in Figure 2, with the largest earnings drops in shrinking firms even slightly bigger.

Figures A.2 and A.3 split the sample into base years 2008-2010 vs. 2013-2015, i.e. recession and expansion samples. While large earnings losses are generally more common during the recession years, the key relationship between firm revenue growth and the left tail of the worker earnings growth distribution is clearly present in both samples.

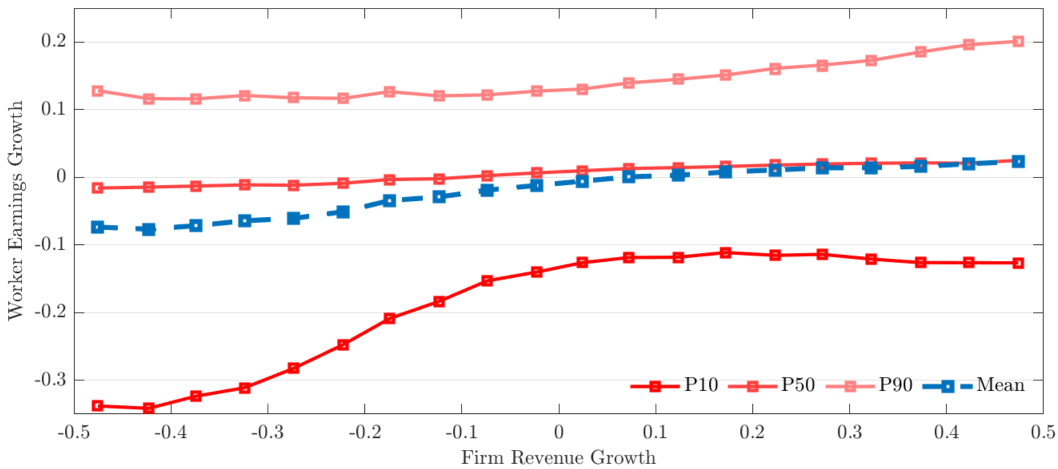

Figure A.1: Worker Earnings Growth by Firm Revenue Growth – Alternative Revenue Measure

Notes: This figure shows moments of the annual earnings growth distribution, conditional on the revenue growth of the firm at which workers are employed in the initial year. The data for revenue is taken from accounting information (FIRE) rather than value added tax data (FIKS), as in the main text, covering a smaller sample of firms.

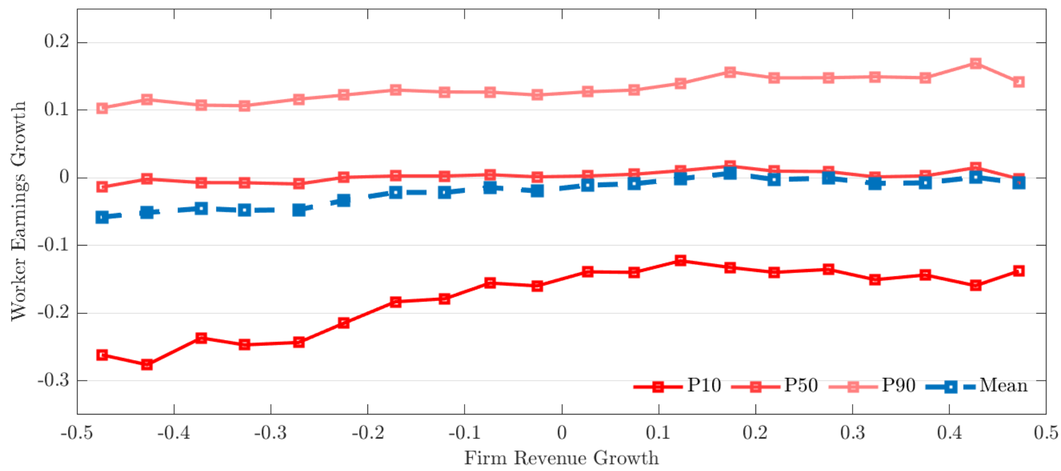

Figure A.2: Worker Earnings Growth by Firm Revenue Growth – Recession

Notes: This figure shows moments of the annual earnings growth distribution, conditional on the revenue growth of the firm at which workers are employed in the initial year. The sample is restricted to base years 2008 to 2010.

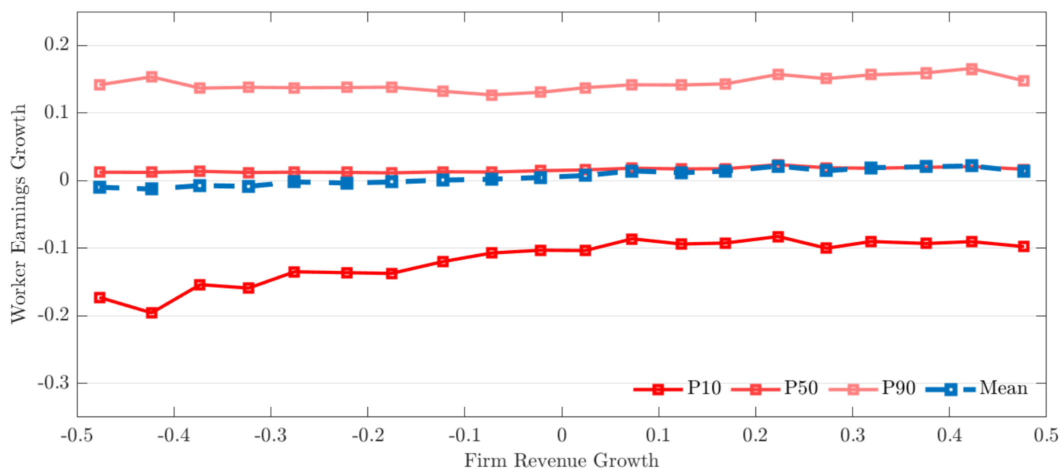

Figure A.3: Worker Earnings Growth by Firm Revenue Growth – Expansion

Notes: This figure shows moments of the annual earnings growth distribution, conditional on the revenue growth of the firm at which workers are employed in the initial year. The sample is restricted to base years 2013 to 2015.

B Model Appendix

B.1 Numerical Algorithm

We use the following algorithm to compute the stationary equilibrium of the model, given a government policy.

1. Guess value functions and distributions of matches, unemployed, and vacant jobs.
2. Compute implied aggregate labor market variables.
3. Compute wage policies given value functions.
4. Compute acceptance rules.
5. Compute search effort policies given value functions and distributions.
6. Compute new value functions and deviations from guesses.
7. Compute new distributions and deviations from guesses.
8. If errors between guessed and computed value functions and distributions are small enough, stop. Otherwise, update value functions and distributions and restart from step 2.