A Theory of Worker Betas:

Aggregate Demand Incidence across the Labor Income Distribution

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Abstract

This paper models the incidence of aggregate demand shocks on relative labor income across households. Incidence patterns are central to policy debates and to aggregate monetary transmission. Building on a New Keynesian framework with multiple industries and segmented factor markets, I find that labor income responses follow a Phillips curve logic. Workers and industries facing more nominal rigidity or more elastic factor supply exhibit larger employment responses and smaller price responses to aggregate demand. The model rationalizes several empirical patterns. Low-income occupations face stronger employment cyclicality through more elastic labor supply, while high-income occupations face stronger earnings cyclicality through more flexible compensation. When properly accounting for capital-skill complementarities, the model generates a U-shaped incidence across the wage distribution. The development of new technologies that substitute mid-skill occupations would reverse the incidence profile, dampening aggregate monetary transmission.

1 Introduction

Policy makers and researchers are increasingly attentive to the differential effects of aggregate demand fluctuations across income and demographic groups. Empirical studies (Romer and Romer, 1998; Guvenen et al., 2017; Coibion et al., 2017) documented that low-income households have more cyclical employment, while high-income households

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have more cyclical labor compensation, determining a U-shaped profile of labor income cyclicality. To prevent a rise in unemployment for low-income workers, some policymakers advocate for a more aggressive monetary response to aggregate demand shortfalls.

Even frontier models, however, cannot rationalize a heterogeneous incidence of aggregate demand and monetary policy on labor income. The HANK literature (Kaplan et al., 2018; Auclert, 2019) modeled the incidence of monetary policy on portfolio valuations and consumption-saving choices. Some HANK models allow for relative labor incomes to be correlated with aggregate output, but they take the relationship as given exogenously.

As a key theoretical contribution, this paper establishes when and why aggregate demand endogenously affects relative labor incomes. I first isolate a class of aggregate demand drivers, including monetary policy and discount factor shocks, whose cross-sectional incidence only depends on the aggregate output gap that they open with respect to the economy's production potential. I then turn to studying the economic mechanisms causing differential labor income responses. Finally, I calibrate the model to match disaggregated production and labor market features of the US economy. The calibrated model generates a U-shaped incidence of aggregate demand shocks, as found in empirical studies.

By modeling the endogenous incidence of aggregate demand on relative labor incomes, this paper fills an important gap in the literature on monetary transmission. Previous work treats labor income incidences as exogenous, and relies on empirical estimates. This paper points out which economic mechanisms generate heterogeneous incidences, and how the incidence profile responds to changing economic conditions. While my baseline calibration matches the U-shaped incidence pattern observed in the data, I show that a higher degree of substitutability between middle-income occupations and non-labor factors – arising for example from the automation of white collar tasks – would instead lead to an increasing incidence profile along the income distribution. This is important for aggregate monetary transmission. Through the lens of HANK models, monetary policy has larger real effects when households with high marginal propensities to consume (MPC) also have more cyclical labor income (Patterson, 2023). Automation of middle-income occupations would shift the incidence toward higher-income, low-MPC households, thereby weakening the real effects of monetary policy. My framework also allows me to point out which fiscal tools can complement monetary policy in managing aggregate demand, to ensure a heterogeneous incidence across households.

Section 2 illustrates the economic mechanisms generating heterogeneous incidences of aggregate demand on labor income. It focuses on a simplified setting, with two industries and two factors of production. The fundamental premise is that households specialize in different occupations and consume different baskets of goods. For example, low-income households are likely employed in heavy manual tasks or personal services, while high-income households are likely employed in management or professional services. Workers in different occupations participate in segmented labor markets. Labor markets are heterogeneous in their employment volatility and compensation flexibility, as determined by occupation-specific labor supply elasticities and wage adjustment probabilities. In the data, low-income occupations tend to have more elastic labor supply and more rigid wages (Volpe, 2025).

Heterogeneity in these occupation-level characteristics creates a differential sensitivity of industry-level prices to changes in employment, captured by different supply slopes. Industries whose workers adjust labor supply more elastically and/or renegotiate wages less frequently have flatter supply curves. My model allows us to connect heterogeneous supply slopes with occupation-specific exposures to aggregate demand fluctuations in general equilibrium. Consistent with the conventional Phillips curve logic, occupations with flatter slopes have smaller wage responses and larger employment responses. The magnitude of relative wage and employment responses depends on how easily consumers can shift expenditure toward flat-supply industries, with the employment margin being stronger when goods are more substitutable. Through this logic, the model rationalizes more cyclical employment for low-income occupations and more cyclical compensation for high-income occupations.

The simple model of section 2 captures the central role of heterogeneous supply slopes, but it misses several realistic elements that are important in a quantitative analysis. First, there are non-labor factors of production alongside labor (such as land, equipment, structures). Non-labor factors are less elastically supplied than labor in the short run, and have different degrees of complementarity with different labor occupations. Second, occupations are hired by industries which differ in their price adjustment frequency, their reliance on intermediate inputs, and their complementarity with other industries. Production chains of different lengths result in differential network-adjusted nominal rigidities, while relative labor demand depends on complementarities at each stage of the production process (between labor and capital, intermediate inputs, final goods). All these features create complex relationships between primitive parameters (nominal rigidities, supply elasticities, input requirements, substitution elasticities) and the relevant supply slopes for each labor market.

Sections 3 and 4 set up a general framework incorporating all these elements, and show that more complex economies are described by the same equilibrium equations as the simple model, expressed using an appropriate vector-matrix notation. Therefore the fundamental role of supply slopes directly generalizes from the simple setting of section 2 to the full model. Key to the generalization is encoding the parameters describing individual agents (primary factors, firms, final users) into appropriate elasticity matrices, which capture own effects and spillovers at all stages of production and consumption. I provide illustrative examples of how mis-specifying these elasticity matrices leads to incorrect measures of supply and demand slopes.

The general framework allows us to isolate which features determine a heterogeneous incidence of aggregate demand fluctuations. Heterogeneous nominal rigidities and/or supply elasticities are essential. In a knife-edge case where all primary factors of production have the same supply elasticity and face the same network-adjusted nominal rigidity, incidences are identical. In this knife-edge case all other parameters are irrelevant, including industry size and centrality, home bias in consumption, and demand elasticities. Once supply elasticities are heterogeneous, however, demand elasticities also become important. In particular, when non-labor factors (such as land, equipment, structures) are less elastically supplied than labor, occupations that are directly substitutable with non-labor factors face more cyclical labor demand. This will be important in determining differential labor demand responses for low-and middle-skill occupations in the quantitative model.

Section 5 restricts the focus to monetary policy. Monetary policy is one specific driver of aggregate demand fluctuations, therefore its cross-sectional incidence follows the patterns outlined above. This raises two questions. First, how does factor market segmentation affect the transmission of monetary policy into aggregate variables? I show that factor market segmentation flattens the aggregate Phillips curve through a composition effect, thereby amplifying the real effects of monetary policy. Second, can appropriate fiscal policies help manage aggregate demand while maintaining identical incidences across households? I show that such fiscal policies exists when all primary factors have the same supply elasticity.

Section 6 calibrates the model to the US economy. I identify households with differentiated labor occupations, and introduce non-labor factors such as land, equipment, structures, and intellectual property. The model can exactly match employment shares across industry-occupation pairs, input-output linkages, and the use of non-labor factors across industries. I parameterize more flexible wages at the top of the income distribution, to generate higher compensation cyclicality. Likewise I parameterize more elastic labor supply at the bottom, to generate higher employment cyclicality. Using measures of occupation-level automatability from Autor and Dorn (2013), I set highly automatable occupations to be substitutable with non-labor factors, while all other tasks are complementary. With this parameterization, the model generates a U-shaped incidence pattern consistent with empirical measures.

In a counterfactual calibration I show that, if mid-skill labor also became substitutable with non-labor assets – for example due to automation of white collar tasks – incidence would shift up for middle-income occupations. The incidence profile in turn matters for aggregate dynamics. Following Patterson (2023), the transmission of monetary policy and other aggregate demand shocks into aggregate output is amplified when incidence is concentrated on high-MPC, low-income households. Thus, higher substitutability between middle-income occupations and non-labor assets would dampen aggregate responses.

1.1 Related Literature

This paper extends frontier input-output models (Acemoglu et al., 2012; Baqaee and Rubbo, 2023; La'O and Tahbaz-Salehi, 2022; Rubbo, 2023) by introducing labor market segmentation and heterogeneous supply, demand, and pricing elasticities. My framework is closest to Baqaee and Farhi (2019). While they study aggregation of industry-specific shocks, I focus on cross-sectional responses to aggregate demand shocks. I also contribute an explicit characterization of disaggregated price and employment responses.

My theoretical framework is akin to currency union models (Benigno, 2004; Huang and Liu, 2007; Gali and Monacelli, 2008) and regional models (Nakamura and Steinsson, 2014; Beraja et al., 2019; Hazell et al., 2020), which also feature multiple segmented factor markets and multiple final users. I extend these models by allowing for heterogeneous elasticities and for a fully general input-output structure. In my application I identify labor markets with differentiated occupations, but the same framework could be used to describe country- or region-specific labor markets. The currency union literature emphasized that a single monetary authority cannot stabilize country-specific supply shocks. I argue that aggregate demand shocks and monetary policy themselves have a heterogeneous incidence across labor markets, thereby destabilizing local employment and consumption and creating an additional source of variation – beyond idiosyncratic local shocks.

A vast empirical literature (Storesletten et al., 2004; Guvenen et al., 2014, 2017) has studied the cyclical behavior of inequality in the US.[1] Romer and Romer (1998); Jaimovich and Siu (2012); Coibion et al. (2017); Broer et al. (2022); Amberg et al. (2022); Coglianese et al. (2023); Lee et al. (2024) specifically analyzed income responses to monetary shocks, finding that monetary expansions disproportionately benefit low income and minority workers. My calibrated model matches these empirical results.

Much attention has been devoted to the heterogeneous incidence of monetary policy on borrowing constraints and portfolio valuations.[2] Several authors moved from empirical evidence (Guvenen and Smith, 2014; Heathcote et al., 2014; Doepke and Schneider, 2006; Del Canto et al., 2023) to model how cyclical inequality caused by borrowing constraints feeds back into the aggregate Euler equation (Werning, 2015), and into the aggregate response to shocks and policies, such as credit tightenings (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017), forward guidance (McKay et al., 2016), and monetary policy (Kaplan et al., 2018; Auclert, 2019).[3] This sparked an interest in the optimal monetary policy with borrowing constraints and idiosyncratic income shocks (Bhandari et al., 2021; Acharya et al., 2023; Davila and Schaab, 2023; Nuno and Thomas, 2022; Yang, 2022; Le Grand et al., 2022; McKay and Wolf, 2023). Relatedly, LaO and Morrison (2024) study optimal monetary policy in an economy where households have systematically different, but exogenous, income sensitivity to aggregate output.

My approach is complementary to this literature, in that it endogenizes the cross-sectional incidence of monetary policy on labor income. A related paper is Schaab and Tan (2023), who calibrate a fully-specified HANK-IO model. They take labor income incidences as given, and argue that there can be an interaction between borrowing constraints and labor income incidences only if labor markets are segmented.

My study of neutral fiscal policies is reminiscent of Farhi et al. (2014). I focus on different shocks (aggregate demand as opposed to local productivity), and consider a more general setting with arbitrary input-output linkages and heterogeneous demand, supply, and pricing elasticities.

[1] Storesletten et al. (2004) documented systematic differences in cyclical income risk, while Guvenen et al. (2014, 2017) further documented that the households' occupation is an important determinant of the regressive incidence of recessions, supporting my modeling approach.

[2] Guvenen and Smith (2014); Heathcote et al. (2014) document that households can insure a significant share of cyclical income risk, while Doepke and Schneider (2006) show that inflation benefits borrowers through debt devaluation. Relatedly, Glover et al. (2020) study redistribution across age groups through asset prices during the Great Recession, and Del Canto et al. (2023) argue that asset revaluations are main channel through which monetary shocks affects relative welfare across income quintiles. Montecino and Epstein (2015); Casiraghi et al. (2018) study the effect of quantitative easing on inequality through labor and asset income. Pourpourides (2011); Gornemann et al. (2012); Motta and Tirelli (2013); Ko (2015); Areosa and Areosa (2016) show that realistic aggregate dynamics and countercyclical inequality can be generated by an interaction between limited asset market participation and features such as habits in consumption, labor market frictions, capital-skill complementarity, and wealth effects in labor supply.

[3] Auclert and Rognlie (2018); Bilbiie (2020); Alves et al. (2020); Patterson (2023) further emphasize how aggregate outcomes depend on the correlation between marginal propensities to consume and the incidence of shocks and policies on incomes. Relatedly, Andersen et al. (2022) underscore that aggregate responses are determined by the marginal propensity to consume domestic goods, which can be different across groups. Orchard (2022); Olivi et al. (2023); Lan et al. (2024); Beraldi and Malgieri (2024) make the case that heterogeneity in consumption bundles is an important determinant of incidence.

2 A Simple Model of Cross-Sectional Incidence

This section introduces the paper's core economic mechanism using a simplified two-household, two-factor model. To generate heterogeneous employment and labor income responses, the model must feature two segmented labor markets. For aggregate demand shocks to generate asymmetric responses, the two labor markets must face different labor supply elasticities and/or different exposures to nominal rigidities in employment and consumption. For concreteness, we can think of the two labor markets as low-skill and high-skill workers. Low-skill workers have more elastic labor supply, capturing more cyclical employment, while high-skill workers have more flexible wages, capturing more cyclical labor compensation (Guvenen et al., 2017).

Production: Two industries L(ow-skill) and H(igh-skill) produce different intermediate goods using distinct labor occupations, according to $Y_{it} = L_{it}$, $i = H, L$. A single final good is produced by combining the two intermediate goods, with constant elasticity of substitution θ and equal steady-state expenditure shares: $Y = \frac{1}{2}\left(Y_H^{\frac{\theta-1}{\theta}} + Y_L^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}$.

Price Setting: Wages face Calvo-style nominal rigidities, with adjustment probabilities $\delta_H > \delta_L$, while intermediate input prices are flexible. To model wage rigidity, I introduce fictitious labor unions which hire workers and resell their labor services to good producers. Accordingly, I denote by W_i the (flexible) wage paid by the union to workers i, and by P_i the (sticky) wage charged by the labor union to intermediate good producers. Labor unions are owned by their respective workers. Optimal price setting implies real marginal cost Phillips curves for the labor unions $p_{it} = \hat{\delta}_i mc_t + \left(1 - \hat{\delta}_i\right)(p_{t-1} + \rho\mathbb{E}\pi_{t+1})$ with slopes $\hat{\delta}_i \equiv \frac{\delta_i(1-\rho(1-\delta_i))}{1-\rho\delta_i(1-\delta_i)}$. I denote by $\bar{\delta}$ the average slope across occupations. Intermediate good prices are flexible, therefore they coincide with the labor union prices.

Preferences: Two distinct households supply the two labor occupations, consume the final good, and trade a risk-free asset with no borrowing constraints. The economy begins in a steady state with equal income shares. Households have GHH preference[4] $\mathcal{U}_i \equiv \sum_{t=0}^{\infty} e^{-\rho t}\left(C_{it} - \frac{L_{it}^{1+\varphi_i}}{1+\varphi_i}\right)^{1-\gamma}$ over consumption C and labor L, where φ_i is the inverse Frisch elasticity of labor supply for occupation i, with $\varphi_L < \varphi_H$. Both households consume the same final good, but purchase it from household-specific retailers with price adjustment probabilities $\delta_H^R \neq \delta_L^R$.

Monetary policy: The central bank sets nominal interest rates, according to a conventional Taylor rule.

2.1 Log-Linearized Equilibrium

Proposition 2.1 characterizes the log-linearized equilibrium of this economy.

Proposition 2.1. Log-linearized equilibrium. *Equilibrium is described by the five equations below. Aggregate employment ($\bar{\ell}$), real GDP (y), and consumption ($\bar{c}$) are given by $\bar{\ell} \equiv \frac{\ell_1+\ell_2}{2} = y = \bar{c}$. The aggregate GDP deflator $\bar{p}_t$*

[4]The GHH assumption is relaxed in the general model.

is the weighted average of household retail prices $\bar{p}_{it}$: $\bar{p}_t = \frac{1}{2}(\bar{p}_{Lt} + \bar{p}_{Ht})$.

$$w_{it} = \varphi_i \ell_{it} + \bar{p}_{it} \tag{1a}$$

$$p_{it} = \hat{\delta}_i w_t + \left(1 - \hat{\delta}_i\right)(p_{it-1} + \rho \mathbb{E}\pi_{it+1}) \tag{1b}$$

$$\ell_{it} = y - \frac{1}{2}\theta (p_{it} - p_{jt}) \tag{1c}$$

$$c_{it+1} = c_{it} + \frac{1}{\gamma}(i_t - \log\rho - \bar{\pi}_{it+1}) + \epsilon^D_{it} \tag{1d}$$

$$i_t - \log\rho = \phi_y y_t + \phi_\pi \bar{\pi}_t \tag{1e}$$

The first three equations in Proposition 2.1 describe the supply side of the economy, meaning how labor is transformed into real GDP. The last two equations describe aggregate demand. I start by analyzing the drivers of aggregate demand, then turn to discussing the heterogeneous propagation of aggregate demand changes through supply-side adjustments.

Aggregate Demand Determination The Euler equations (1d) imply that consumption growth is equalized across households,[5] yielding a standard aggregate Euler equation

$$\bar{y}_{t+1} = \bar{y}_t + \frac{1}{\gamma}(i_t - \bar{\pi}_{t+1}) + \bar{\epsilon}^D_t. \tag{2}$$

Equation (2) illustrates how monetary policy affects the dynamics of aggregate real GDP $\bar{y}$, by setting the nominal interest rate. Aggregate real GDP also depends on the demand shifter $\bar{\epsilon}^D_t$, originating from reduced-form shocks to the households' Euler equations (for example, discount factor shocks or fiscal shocks). Below I show that, given aggregate output $\bar{y}$, disaggregated price and employment responses do not depend on the path of nominal interest rates i_t and aggregate demand shifters $\bar{\epsilon}^D_t$. This motivates using aggregate output as a proxy for aggregate demand.

Heterogeneous Supply Slopes Throughout the next two paragraphs I take aggregate output $\bar{y}$ as given, and study how aggregate demand shocks – meaning changes in $\bar{y}$ – propagate into relative labor demand for the two occupations. This is determined by general equilibrium adjustments working through heterogeneous supply slopes and substitution elasticities.

I start by describing the connection between supply slopes, relative demand adjustments, and equilibrium outcomes. I will later derive supply slopes as a function of primitive parameters. Combining consumption-leisure

[5] Consumption growth is equalized up to changes in relative retail prices. See appendix A below for detailed derivations.

tradeoffs (1a) and real marginal cost Phillips curves (1b) yields the supply curves

$$p_{it} = \kappa_i^\ell \ell_{it} + \frac{1}{2}\kappa_i^p \left(p_{Lt} + p_{Ht}\right) + \Upsilon_{it}, \tag{3}$$

where the elasticities κ^ℓ and κ^p are functions of underlying price adjustment probabilities and supply elasticities, derived below. Υ_{it} depends on lagged prices and inflation expectations. Intuitively, low-skill workers – with more elastic labor supply and stickier wages – have flatter slopes $\kappa_L^\ell < \kappa_H^\ell$. Workers who consume sticky-priced goods also have flatter slopes κ_p. Using equation (3) we can connect relative prices with relative employment and aggregate output ($\ell_i - \ell_j$ and $\bar{y}$), as follows:

$$p_{Ht} - p_{Lt} = \bar{\kappa}^\ell \left[\left(1 + \frac{1}{4}\frac{\kappa_H^\ell - \kappa_L^\ell}{\bar{\kappa}^\ell}\frac{\kappa_H^p - \kappa_L^p}{1 - \bar{\kappa}^p}\right)\left(\ell_{Ht} - \ell_{Lt}\right) + \left(\frac{\kappa_H^\ell - \kappa_L^\ell}{\bar{\kappa}^\ell} + \frac{\kappa_H^p - \kappa_L^p}{1 - \bar{\kappa}^p}\right)\bar{y}_t \right] + \Upsilon_t, \tag{4}$$

where $\bar{\kappa}^\ell$ and $\bar{\kappa}^p$ denote averages across households. In equation (4), the price of low-skill labor – which is more elastically supplied and has a more rigid wage – responds less to changes in employment. This in turn leads to a heterogeneous incidence of aggregate demand on employment and labor income, as explained next.

Expenditure switching, relative employment, and labor income As relative prices change, the relative demand for labor also changes. CES preferences over final goods imply that relative demand for intermediate inputs is decreasing in relative prices, with elasticity θ. Demand for inputs translates into demand for high- and low-skill labor:

$$\ell_H - \ell_L = -\theta\left(p_H - p_L\right). \tag{5}$$

Combining relative supply (4) and relative demand (5) allows us to solve for the incidence of aggregate real GDP on relative employment across the two occupations:

$$\ell_H - \ell_L = -\frac{\theta\bar{\kappa}^\ell\left(\frac{\kappa_H^\ell - \kappa_L^\ell}{\bar{\kappa}^\ell} + \frac{\kappa_H^p - \kappa_L^p}{1 - \bar{\kappa}^p}\right)}{1 + \theta\bar{\kappa}^\ell\left(1 + \frac{1}{4}\frac{\kappa_H^\ell - \kappa_L^\ell}{\bar{\kappa}^\ell}\frac{\kappa_H^p - \kappa_L^p}{1 - \bar{\kappa}^p}\right)}\bar{y}. \tag{6}$$

Equation (6) tells us that, whenever low-skill labor has a flatter supply curve ($\kappa_L^\ell < \kappa_H^\ell$), low-skill employment is more sensitive to aggregate demand.

Changes in relative prices play a key role in shaping the propagation of aggregate demand shocks. If relative prices remained fixed, increasing aggregate demand would raise labor demand proportionally for both occupations (equation 5). All incomes would increase proportionally, and all households would scale up demand for all goods by the same amount, because of homothetic preferences. Relative prices, however, respond to changes in aggregate demand, as governed by factor-specific supply slopes (equation 4). Wages and prices increase by less for goods and factors with flatter supply slopes. Relative price changes then set off relative demand adjustments, whereby consumers shift expenditure toward cheaper goods. As a result, an aggregate demand expansion leads to higher employment and lower prices for flat-supply goods and factors (equation 6).

The implied change in relative labor income is:

$$p_H - p_L + \ell_H - \ell_L = \frac{\theta - 1}{\theta} \left(\ell_H - \ell_L \right). \tag{7}$$

Equation (7) shows that employment and wages move in opposite directions, due to expenditure switching. Thus, employment and wage effects tend to offset each other. Labor income is dominated by the employment response if and only if the two labor occupations are substitutable ($\theta > 1$).

The quantitative model in section 6 builds on these observations, to reproduce a U-shaped correlation between individual labor income and aggregate GDP along the wage distribution. Replicating a U-shaped pattern will require introducing at least three occupation groups – unlike this simple example – with appropriate patterns of supply elasticities, wage adjustment probabilities, and degrees of complementarity.

When are Factor Supply Slopes Heterogeneous? Supply slopes can differ among labor occupations for three reasons: different Frish elasticities ($\varphi_L < \varphi_H$), different wage rigidities ($\delta_L < \delta_H$), or different consumption price rigidities. Below, I illustrate each channel in turn.

Heterogeneous wage rigidities. When retail prices are flexible, and workers have identical supply elasticities $\varphi_1 = \varphi_2$, equation (3) specializes to

$$w_i = \hat{\delta}_i \varphi \ell_i + \hat{\delta}_i \bar{p} + \Upsilon_{it},$$

implying $\kappa_i^\ell = \hat{\delta}_i \varphi$ and $\kappa_i^p = \hat{\delta}_i$. Equation (6) then tells us that workers with more rigid wages experience larger employment responses:

$$\ell_H - \ell_L \propto -\frac{\hat{\delta}_H - \hat{\delta}_L}{\bar{\delta}\left(1 - \bar{\delta}\right)} \bar{y}.$$

In our example, low-skill workers have more rigid wages ($\delta_L < \delta_H$) and hence more cyclical employment.

Heterogeneous labor supply elasticities. If retail prices are flexible and workers have identical adjustment probabilities $\delta_H = \delta_L$, equation (3) specializes to

$$w_i = \hat{\delta}_i \varphi \ell_i + \hat{\delta}_i \bar{p} + \Upsilon_{it},$$

yielding $\kappa_i^\ell = \hat{\delta} \varphi_i$ and $\kappa_i^p = \hat{\delta}$. Equation (6) then implies that workers with more elastic labor supply see larger employment responses:

$$\ell_H - \ell_L \propto -\frac{\varphi_H - \varphi_L}{\bar{\varphi}} \bar{y}.$$

In our example, low-skill workers have more elastic labor supply ($\varphi_L < \varphi_H$) and hence more cyclical employment.

Heterogeneous nominal rigidities in consumption. Suppose both households face the same wage adjustment probabilities ($\delta_H = \delta_L = 1$) and labor supply elasticities ($\varphi_H = \varphi_L$), while retailers for the two households face different price adjustment probabilities $\delta_H^R \neq \delta_L^R$. The supply curve (3) specializes to

$$p_i = \varphi \ell_i + \hat{\delta}_i^R \bar{p},$$

yielding $\kappa_i^\ell = \varphi$ and $\kappa_i^p = \hat{\delta}_i^R$. Equation (6) then implies a larger employment incidence of aggregate demand on households who face more flexible consumption prices:

$$\ell_H - \ell_L = -\frac{\theta\varphi}{1+\theta\varphi} \frac{\hat{\delta}_H^R - \hat{\delta}_L^R}{1-\bar{\delta}^R} \bar{y}$$

All specifications reveal the same fundamental principle: households with more elastic labor supply (lower φ_i), stickier wages (lower $\hat{\delta}_i$), or stickier consumption prices (lower $\hat{\delta}_i^R$) have flatter supply curves. The price of the goods they produce responds less to changes in employment. Therefore demand for these goods responds more to changes in aggregate demand, through expenditure switching.

2.2 Limitations of the Two-Factor Model

Different from our simple model, in realistic economies there is no perfect overlap between factor market boundaries and industries. Different industries use the same production factors, although in different proportions, and direct factor contents are different from total contents due to intermediate inputs. As a result the supply slopes κ are complex functions of nominal rigidities, supply elasticities, and input-output linkages. Likewise, the relevant substitution elasticities Θ are combinations of demand elasticities at different nodes of the production network.

A more general theoretical framework is key to mapping disaggregated primitive parameters into the relevant supply and demand slopes. Moreover, allowing for more than two occupations is necessary to reproduce non-monotonic labor income responses along the income distribution, as observed in empirical studies (Guvenen et al., 2017; Patterson, 2023).

Nonetheless the core economic mechanism – formalized by equation (6) – is fully general. As explained in section 4, the incidence of shocks on relative employment can still be described analytically by a cross-sectional multiplier $(I - \Theta\kappa)^{-1}$. This multiplier captures the fixed-point interaction between supply slopes and expenditure switching, generalizing the denominator $1 + \theta\bar{\kappa}^\ell \left(1 + \frac{1}{4} \frac{\kappa_i^\ell - \kappa_j^\ell}{\bar{\kappa}^\ell} \frac{\kappa_i^p - \kappa_j^p}{1-\bar{\kappa}^p}\right)$ in (6). The economic intuition remains the same.

3 General Environment

This section extends the simple model from section 2 to a quantitatively realistic economy with multiple heterogeneous industries and primary factors. The full model allows for enough flexibility to match real-world value added shares of labor occupations and non-labor factors across industries, input-output linkages, and household-specific consumption baskets and ownership shares.

This generalization is essential to paint a full picture of aggregate demand incidence across households, through their multifaceted interactions with the production side of the economy. Even though it features many sources of heterogeneity, the generalized model preserves analytical tractability, thanks to a vector representation of the equilibrium equations. This allows a clear parallel with conventional aggregated models.

My analytical results highlight the same propagation mechanism described in section 2. With segmented factor markets aggregate demand changes have differential incidences on relative factor employment, through the interaction between factor supply slopes and expenditure switching. Compared with the stylized setting of section 2, a quantitatively-realistic framework is necessary to correctly measure the relevant supply slopes and substitution elasticities for each labor occupation given primitive parameters.

Section 3.1 describes the primitives and defines the model equilibrium, while section 3.2 introduces the vector-matrix notation which will enable a transparent representation of the log-linearized model.

3.1 Model Primitives

Production. There is a set $\mathcal{N}$ of good-producing industries in the economy, indexed by i, j. Within each industry there is a continuum of firms producing differentiated varieties.

All firms z in industry i have the same constant returns to scale production function, taking as inputs intermediate goods and primary factors (labor occupations and non-labor factors, such as land, equipment, structures, intellectual property, etc.):

$$Y_{izt} = G_i(\{L_{ifzt}\}_{f \in \mathcal{F}}, \{X_{ijzt}\}_{j=1}^{N})$$

where L_{ifzt} is the quantity of primary factor f hired by firm z in industry i at time t, and X_{ijzt} is the quantity of intermediate input j used by the firm. These production functions generalize the simple model by allowing each industry to use multiple primary factors, granting enough flexibility to match real-world value added shares.

The elasticities of substitution between goods j and k in the production of good i are

$$\theta^i_{jk} \equiv -\frac{d \log \frac{X_{ij}}{X_{ik}}}{d \log \frac{P_j}{P_k}}.$$

These elasticities generalize the parameter θ from Section 2, allowing for rich substitution patterns across the production network.

Customers buy CES bundles of varieties, with elasticity of substitution ϵ_i, yielding industry output and prices:

$$Y_i = \left(\int Y_{if}^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}, \quad P_i = \left(\int P_{if}^{1-\epsilon_i} df \right)^{\frac{1}{1-\epsilon_i}}$$

All producers minimize costs given input prices. With constant returns to scale, firms within industry i use inputs in the same proportions and have equal marginal costs

$$MC_i = \min_{\{X_{ij}\},\{L_{if}\}} \sum_{f\in\mathcal{F}} W_f L_{if} + \sum_{j\in\mathcal{N}} P_j X_{ij} \;\; \text{s.t. } G_i\left(\{L_{if}\}_{f\in\mathcal{F}}, \{X_{ij}\}_{j\in\mathcal{N}}\right) = 1 \tag{8}$$

Producers face Calvo price rigidities. In each period t, only a fraction δ_i of firms in industry i can update their price. Adjusting firms maximize expected discounted profits:

$$P^*_{ixt} = \frac{\epsilon_{it}}{\epsilon_{it}-1} \frac{\mathbb{E}\sum_s \left[SDF_{t+s}\left(1-\delta_i\right)^s Y_{ixt+s}(P^*_{ixt})\left(1-\tau_{it+s}\right)MC_{it+s}\right]}{\mathbb{E}\sum_s \left[SDF_{t+s}\left(1-\delta_i\right)^s Y_{ixt+s}(P^*_{ixt})\right]} \tag{9}$$

where SDF_{t+s} is the stochastic discount factor, τ_{it+s} represents subsidies, and demand functions are $Y_{ixt+s}(P_{xt}) = Y_{it+s}\left(\frac{P^*_{ixt}}{P_{it+s}}\right)^{-\epsilon_{it}}$. Non-adjusting firms keep prices unchanged and absorb cost changes in their markups.

I follow a standard practice in the literature, and assume that the government provides industry-specific subsidies to eliminate markup distortions arising from monopolistic competition. This is stated in Assumption 3.1.

Assumption 3.1. Producers charge unit post-subsidy markups in the initial equilibrium.

Primary factors and final users. The economy has a set $\mathcal{F} = \mathcal{L}\cup\mathcal{K}$ of primary factors, consisting of differentiated labor occupations $\mathcal{L}$ and non-labor factors $\mathcal{K}$ (land, equipment, structures, intellectual property). This generalizes the two labor types from section 2 to multiple heterogeneous factors. Factor markets are segmented, meaning that workers cannot change occupation, and non-labor assets cannot be repurposed.

There is also a set $\mathcal{Z} = \mathcal{H}\cup\mathcal{U}$ of final users, consisting of households $\mathcal{H}$ and asset utilization producers $\mathcal{U}$. Households represent different income or demographic groups. They own shares in industries and non-labor factors, and supply labor to different occupations.[6] Households of type $h \in \mathcal{H}$ maximize the present discounted value of utility flows, with preferences:

$$\sum_{t=0}^{\infty} e^{-\rho t}\left[\frac{C_{ht}^{1-\gamma}}{1-\gamma} - \sum_{f\in\mathcal{L}} \frac{L_{hft}^{1+\varphi_f}}{1+\varphi_f}\right] \tag{10}$$

The parameter γ governs intertemporal substitution and wealth effects in labor supply. The parameters φ_f are inverse Frisch elasticities determining labor supply slopes for each occupation $f \in \mathcal{L}$, generalizing the occupation-specific parameters φ_h from section 2.

The consumption aggregator $C_{ht} \equiv \mathcal{C}_{ht}\left(c_{h1t}, ..., c_{hNt}\right)$ is homothetic over the set $\mathcal{N}$ of goods produced in the economy. The elasticity of substitution between goods i and j in consumption generalizes the parameter θ from section 2:

$$\theta^C_{ij} \equiv -\frac{d\log\frac{c_i}{c_j}}{d\log\frac{P_i}{P_j}}.$$

[6]The model does not impose a one-to-one correspondence between household groups and labor occupations. For example, lower quintile households may predominantly work in service occupations but also work in manufacturing, while some higher quintile households may also work in service occupations.

Households are subject to per-period budget constraints

$$P_{ht}^C C_{ht} \leq \sum_{f \in \mathcal{F}} \hat{\Xi}_{hf} W_{ft} L_{ft} + \sum_{i \in \mathcal{N}} \hat{\Xi}_{hi} \Pi_{it} + T_{ht}, \tag{11}$$

where P_{ht}^C is the household-specific price index, W_{ft} is the nominal return to factor f, $\hat{\Xi}_{hf}$ and $\hat{\Xi}_{hi}$ are ownership shares in non-labor factors and industries, Π_{it} are industry profits, and T_{ht} is net borrowing.

Households freely trade a risk-free asset paying nominal interest rate i_{t+1}. The asset is in zero net supply with zero initial borrowing. Optimal consumption-saving follows standard Euler equations:

$$\frac{C_{h,t+1}}{C_{h,t}} = \left[\frac{1 + i_{t+1}}{e^{-\rho}} \frac{P_{h,t}^C}{P_{h,t+1}^C} \right]^{\frac{1}{\gamma}}, \ \forall h \in \mathcal{H} \tag{12}$$

Households can also trade shares in firms and semi-fixed assets, whose rates of return follow analogous Euler equations. Optimal trade of the risk-free asset, firm shares, and shares in non-labor factors determine the households' net borrowing $\{T_{ht}\}_{h \in \mathcal{H}}$. The intertemporal budget constraint requires

$$\sum_{t=0}^{\infty} \mathbb{E} \frac{T_{ht}}{\prod_{s=0}^{t} (1 + i_s)} = 0 \ \forall h \in \mathcal{H}. \tag{13}$$

The households' optimal labor supply choices generalize equation (1a) from section 2:

$$C_{ht}^{\gamma} L_{hft}^{\varphi_f} = \frac{W_{ft}}{P_{ht}^C}, \ \forall h \in \mathcal{H}, f \in \mathcal{L}. \tag{14}$$

Labor is hired through occupation-specific unions facing Calvo price rigidities, with potentially different wage adjustment probabilities across occupations. This generalizes the wage adjustment probabilities δ_H and δ_L from Section 2. Wage rigidities create a wedge between desired wages W_{ft} in (14) and the price paid to labor unions. I assume that households own unions proportional to their occupation headcount, guaranteeing that all workers in a given occupation end up earning the same sticky wage.

Besides labor occupations, industries use non-labor factors such as land, equipment, structures, intellectual property, etc. I model the supply of non-labor factors through a stylized utilization framework, which parsimoniously accounts for non-labor shares in value added in a tractable way. The modeling approach is described in detail in appendix A. Accounting for non-labor factors is important. Non-labor shares in value added vary widely across industries, while labor occupations differ in their industry exposures, resulting in a differential incidence of aggregate demand across occupations.

Policy and Aggregation The central bank sets nominal interest rates i_t following a Taylor rule. The government provides industry-level cost subsidies financed through lump-sum profit taxes. I adopt standard definitions for aggregate real GDP, the aggregate GDP deflator, and income shares, reported in appendix A.

3.2 Equilibrium

The equilibrium definition adapts Baqaee and Farhi (2020) to account for endogenous markups with pricing frictions. This definition nests the standard flexible-price equilibrium when $\delta_i = 1$ in all industries.

Definition 3.1. Given price adjustment probabilities δ_i and policies, general equilibrium consists of firm markups $\mathcal{M}_{ixt}$, sectoral prices P_{it}, factor wages W_{ft}, factor supplies L_{ft}, sectoral outputs Y_{it}, intermediate quantities X_{ijt}, and final demands C_{izt} such that: (i) fraction δ_i of firms in sector i charge profit-maximizing prices (9); (ii) adjusting firms' markups equal price-to-marginal cost ratios while non-adjusting firms maintain constant prices; (iii) agents maximize utility subject to budget constraints; (iv) producers minimize costs and charge relevant markups; (v) all markets clear.

Vector Variables and Elasticity Matrices For analytical tractability, I work with a log-linearized version of the model. Table 1 introduces the model variables. In order to obtain interpretable formulas, I collect variables pertaining to the same group of agents (primary factors, industries, or final users) into vectors, whose components correspond to individual agents. This will allow me to directly extend the equilibrium equations from section 2, while providing the necessary flexibility to match real world value added shares and consumption patterns.

All throughout, steady-state variables are denoted with a star. I use lower case letters to denote log-deviations from the initial steady state. Vectors are boldfaced.

Variable	**Definition**
Aggregate output	$\bar{y}_t \equiv \log \frac{Y_t}{Y^*}$
Employment of primary factors	$\boldsymbol{\ell}_t = (\ell_{1t}, ..., \ell_{Ft})^T$ where $\ell_{ft} \equiv \log \frac{L_{f,t}}{L_f^*}$
Final users consumption	$\boldsymbol{c}_t = (c_{1t}, ..., c_{Zt})^T$ where $c_{zt} \equiv \log \frac{C_{zt}}{C_z^*}$
Industry-level prices	$\boldsymbol{p}_t = (p_{1t}, ..., p_{Nt})^T$ where $p_{it} \equiv \log \frac{P_{i,t}}{P_i^*}$
Industry-level inflation	$\boldsymbol{\pi}_t = (\pi_{1t}, ..., \pi_{Nt})^T$ where $\pi_{it} \equiv p_{it} - p_{i,t-1}$
Nominal interest rate	i_{t+1}

Table 1: Model variables

Income flows and expenditure shares. Shock propagation in the log-linearized model is fully described by steady-state expenditure shares and elasticities, introduced in tables 2 and 3. Section 2 relied on simplifying assumptions about the income flows and expenditure shares in the economy (each industry uses a single primary factor, households fully own the industry where they work). To describe a more general setting, table 2 introduces flexible notation for the industries' expenditure shares on primary factors and intermediate inputs, and for the final users' ownership shares in industries and primary factors.

Importantly, I encode direct expenditure shares in the matrix Ω, whose ij-th element corresponds to the expenditure share of agent i on j. I define Ω to include factor marketplaces and final users as well, and extend the set $\mathcal{N}$ accordingly. I instead encode total expenditures (direct and through intermediate inputs) of i on j in the matrix $\Psi_{ij} \equiv (I - \Omega)^{-1}_{ij}$. As a convention, I denote by $\Omega_{\mathcal{Z}:}$ and $\Psi_{\mathcal{Z}:}$ the rows corresponding to final users, and by $\Omega_{:\mathcal{F}}$ and $\Psi_{:\mathcal{F}}$ the columns corresponding to labor unions and factor marketplaces. In particular, the block $\Psi_{\mathcal{Z}\mathcal{F}}$ represents the total content of primary factors in final uses. I adopt the same notation to indicate the rows and columns corresponding to final users and primary factors in all matrices.

Parameter	**Notation**	**Definition**
Expenditure shares	$\Omega \in \mathbb{R}^{\lvert\mathcal{N}\rvert\times\lvert\mathcal{N}\rvert}$	$\Omega_{ij} = \frac{P_j^* X_{ij}^*}{P_i^* Y_i^*}$
Leontief inverse	Ψ	$\Psi \equiv (I - \Omega)^{-1}$
Final users' income shares	$\mathbf{s} \in \mathbb{R}^{\lvert\mathcal{Z}\rvert}$	$s_h \equiv \frac{P_h^{C*} C_h^*}{GDP^*}$
Primary factors' income shares	$\bar{\mathbf{\Psi}}_{\mathcal{F}} \in \mathbb{R}^{\lvert\mathcal{F}\rvert}$	$\bar{\Psi}_f \equiv \frac{W_f^* L_f^*}{GDP^*}$
Ownership shares	$\Xi \in \mathbb{R}^{\lvert\mathcal{Z}\rvert\times\lvert\mathcal{N}\rvert}$	See Appendix A

Table 2: Income Flows and Expenditure Shares

Elasticity Matrices Table 3 introduces a compact notation for demand, supply, and pricing elasticities, which generalize the scalar elasticities from section 2. These matrices will describe supply and demand relations between the vector variables in Table 1, capturing both own-effects and cross-effects among different agents. The table presents matrix notation for the full model, alongside the corresponding scalar notation from the two-sector model. Full definitions are reported in Appendix A.

Parameter	**Notation**	**Simple model**
Wealth effects in factor supply	$\Gamma \in \mathbb{R}^{\lvert\mathcal{F}\rvert\times\lvert\mathcal{Z}\rvert}$	γ
Factor supply elasticities	$\Phi \in \mathbb{R}^{\lvert\mathcal{F}\rvert\times\lvert\mathcal{F}\rvert}$	φ_i
Factor price deflators	$\beta^T \boldsymbol{p}$, $\beta \in \mathbb{R}^{\lvert\mathcal{N}\rvert\times\lvert\mathcal{F}\rvert}$	$\bar{p}$
Nominal rate deflator	$\pi^{IS} \in \mathbb{R}$	$\bar{\pi}$
Price pass-through	$\mathcal{P} \in \mathbb{R}^{\lvert\mathcal{N}\rvert\times\lvert\mathcal{N}\rvert}$	$\hat{\delta}_i$
Demand elasticity	$\Theta \in \mathbb{R}^{\lvert\mathcal{F}\rvert\times\lvert\mathcal{N}\rvert}$	θ

Table 3: Elasticity Matrices

4 Incidence of Aggregate Demand on Relative Employment

This section presents the main theoretical results, characterizing the heterogeneous incidence of aggregate demand on employment and income across households.

Section 4.1 begins by laying out the log-linearized equilibrium system, using the vector-matrix notation introduced in section 3. Section 4.2 then analyzes the key economic mechanisms. I start by describing the determination of aggregate demand, which is similar to conventional aggregated models. The economy admits an aggregate Euler equation, because households can freely trade a risk-free asset. Aggregate output dynamics therefore depend on nominal interest rates and exogenous shifters (such as discount factor shocks or fiscal policy). I then turn to constructing generalized factor supply curves and substitution elasticities, which extend the simpler supply and demand relations from section 2 to an arbitrary multi-sector, multi-factor economy.

Finally, section 4.3 connects changes in aggregate output with relative employment and income. Like in the simple model of section 2, increasing aggregate output has a uniform *direct* effect on factor demand, meaning that employment would increase proportionally for all factors if prices remained fixed. Changes in aggregate output however have heterogeneous incidences on relative prices, resulting in heterogeneous *propagation* effects. Industries with steeper supply curves see larger price increases. Customers respond by shifting expenditures away from these industries, according to the relevant substitution elasticities, thereby reducing demand for the primary factors that they employ. The adjustment process between price changes and expenditure switching has a fixed point, which I represent through a cross-sectional multiplier.

4.1 Log-Linearized Model in Vector Notation

Lemma 4.1 characterizes the log-linearized equilibrium of the economy. Using the vector-matrix notation introduced in section 3 allows me to preserve a direct correspondence with aggregated models, while accommodating factor market segmentation and heterogeneous elasticities.

Lemma 4.1. *The log-linearized equilibrium is defined by the following equations:*

$$\boldsymbol{w}_t = \Phi \boldsymbol{\ell}_t + \beta^T \boldsymbol{p}_t + \Gamma \boldsymbol{\xi} \tag{15a}$$

$$\boldsymbol{p}_t = \mathcal{P}_{:\mathcal{F}} \boldsymbol{w}_t + \left(I - \mathcal{P}\Psi^{-1}\right)\left(\boldsymbol{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{t+1}\right) \tag{15b}$$

$$\boldsymbol{\ell}_t = \mathbf{1}\bar{y}_t - \Theta \boldsymbol{p}_t + \mathcal{D}_\xi \boldsymbol{\xi} \tag{15c}$$

$$\boldsymbol{c}_{\mathcal{H},t+1} = \boldsymbol{c}_{\mathcal{H},t} + \frac{1}{\gamma}\left[(\mathbf{1} i_t - \log \rho) - \Omega_{\mathcal{H}:} \boldsymbol{\pi}_{t+1}\right] \tag{15d}$$

$$i_t - \log \rho = \phi_y \bar{y}_t + \boldsymbol{\phi}_\pi \boldsymbol{\pi}_t + \epsilon_t \tag{15e}$$

The vector $\boldsymbol{\xi}$ denotes relative present discounted values of final user incomes, satisfying $\boldsymbol{\xi}_{\mathcal{U}} = \mathbf{0}$ for utilization producers and $\sum_{h \in \mathcal{H}} \boldsymbol{\xi}_h = 0$ for households.

Proof. The system (15a) - (15e) is obtained by log-linearizing the equilibrium equations (8), (9), (11), (12), (13), (14) and the optimal utilization of non-labor factors, and by solving for consumption using the Euler equations and intertemporal budget constraints. □

All elasticities in lemma 4.1 are defined in table 3. The first two equations describe factor supply and pricing, extending the standard consumption-leisure tradeoff and real marginal cost Phillips curve to accommodate multiple heterogeneous primary factors. The third equation generalizes relative factor demand (1c) from section 2. The fourth line contains the households' Euler equations, while the last equation is a standard Taylor rule.

Changes in relative present values of income $\boldsymbol{\xi}$ are derived in section 4.2. These changes arise because – even though households can smooth income fluctuations ex-post by trading the risk-free asset – they cannot insure ex-ante against unanticipated shocks. Therefore, even though the Euler equations imply that income *growth* is equalized, relative *permanent* incomes $\boldsymbol{\xi}$ still fluctuate.[7] In response to one-off, unanticipated shocks, $\boldsymbol{\xi}$ is constant across time periods t.

In equation (15a), the matrix Φ is a generalized inverse supply elasticity that combines Frisch elasticities, wealth effects in labor supply, and asset utilization costs. Similarly, the generalized factor price deflators $\beta^T \boldsymbol{p}$ combine consumer prices and wealth effects in labor supply from profit rebates. Labor supply also depends on permanent income $\boldsymbol{\xi}$ through wealth effects, captured by the elasticity Γ.

In (15b), the matrix $\mathcal{P} = (\mathcal{P}_{ij})$ describes the pass-through of a cost shock or desired price change in industry j into the price of industry i, through production chains with sticky prices. Desired price changes are driven by factor costs ($\boldsymbol{w}$), relative price distortions inherited from previous periods ($\boldsymbol{p}_{t-1}$), and expected future inflation ($\mathbb{E}\boldsymbol{\pi}_{t+1}$).

Equation (15c) is central to understanding how aggregate demand propagates differently across primary factors. The matrix Θ is a generalized substitution elasticity, that relates relative factor demand with relative good and factor prices. This generalized substitution elasticity accounts for expenditure switching at each node of the production network, and for changes in the relative cost of living of households who consume different bundles of goods. The matrix $\mathcal{D}_\xi$ instead relates factor demand with the households' relative permanent income. These relative demand coefficients are discussed in detail in section 4.2.

4.2 Aggregate Demand, Supply Slopes, and Expenditure Switching

The equilibrium equations (15a) - (15e) can be combined to isolate two model blocks: supply and aggregate demand. I start by describing the aggregate demand block, and then turn to the supply side. The aggregate demand block pins down the dynamics of aggregate real GDP $\bar{y}$ given nominal interest rates, price dynamics, and exogenous demand shifters ϵ_t. The supply block describes how primary factors are transformed into real GDP. This is key to understanding how relative employment responds to aggregate demand.

[7] When $\gamma \neq 1$, income growth is equalized up to changes in relative consumption prices. This is accounted for in the definition of factor price deflators β and substitution elasticities Θ.

Consumption-saving and aggregate demand Households freely trade a risk-free asset, following standard Euler equations (15d). Therefore the economy admits an aggregate Euler equation

$$\bar{y}_{t+1} = \bar{y}_t + \frac{1}{\gamma}\left(\mathbf{1} i_t - \pi^R_{t+1}\right). \tag{16}$$

Equation (16) is obtained by defining π^{IS} as in Table 3, and combining the Euler equations, the log-linearized budget constraint (11) and the relative demand equation (15c).

Even though ex-post consumption growth is equalized across households[8], they cannot insure ex-ante against unexpected monetary shocks. Households must therefore tolerate changes in their relative present values of income $\boldsymbol{\xi}$. These can be computed directly from per-period and intertemporal budget constraints (11) and (13):

$$\boldsymbol{\xi} = (1-\rho)\sum_{t=0}^{\infty}\rho^t\left[\left(\Xi_{\mathcal{HF}} - \mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{HF}}\right)\boldsymbol{\ell}_t + \frac{1}{\gamma}\left[\gamma\left(\Xi_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\right)\Psi^{-1} + (1-\gamma)\left(\Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H},:}\right)\right]\boldsymbol{p}_t\right]. \tag{17}$$

In contrast with HANK models, I purposefully keep consumption-saving choices as simple as possible to better highlight redistributive mechanisms operating through the supply side.

Accordingly, the transmission of monetary policy described in equation (16) is the same as in textbook New Keynesian models. Given aggregate output dynamics, as implied by equation (16), the supply-side of the economy (equations (15a) - (15c)) does not depend on interest rates i_t or exogenous demand shocks $\bar{\epsilon}^D$. This justifies using aggregate output $\bar{y}$ as a measure of aggregate demand. In the next two paragraphs I take $\bar{y}$ as given, and study how it propagates into the relative demand for different primary factors.

Expenditure switching, income reallocation, and relative factor demand. This subsection and the next one lay the foundations to study the heterogeneous propagation of aggregate demand into factor employment. They derive two essential elements: generalized substitution elasticities, describing how factor demand responds to relative prices, and generalized supply slopes, describing how industry-level prices respond to changes in employment. Equilibrium employment responses then emerge as the fixed-point of supply and demand adjustments, as explained in section 4.3.

Lemma 4.2 derives generalized substitution elasticities and relative factor demand responses to income reallocation.

[8] Consumption growth is exactly equalized only if $\gamma = 1$. Changes in relative consumption prices are accounted for in the definitions of β and Θ.

Lemma 4.2. *The elasticities* Θ *and* $\mathcal{D}_\xi$ *in* (15c) *are given by:*

$$\begin{aligned}
\Theta \equiv & \mathcal{D}_\ell \left[\mathcal{X}(\Psi^T_{:\mathcal{F}}, I) + Cov_{\boldsymbol{s}} \left(\Psi^T_{\mathcal{Z}\mathcal{F}}, \begin{pmatrix} \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:} \\ \Omega_{\mathcal{U},:} \end{pmatrix} - \Xi\Psi^{-1} + \Gamma^{-1} \begin{pmatrix} \Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:} \\ \mathbb{O} \end{pmatrix} \right) \right] \\
\mathcal{D}_\xi \equiv & \mathcal{D}_\ell Cov_{\boldsymbol{s}} \left(\Psi^T_{\mathcal{Z}\mathcal{F}}, I \right) \\
\mathcal{D}_\ell \equiv & \frac{1}{\boldsymbol{\Psi}_{\mathcal{F}}} \left(I - Cov_{\boldsymbol{s}} \left(\Psi^T_{\mathcal{Z}\mathcal{F}}, \frac{\Xi_{:\mathcal{F}}}{\boldsymbol{\Psi}_{\mathcal{F}}} \right) \right)^{-1} \\
\left[\mathcal{X}(X,Y)\right]_{fi} \equiv & \sum_{j\in\mathcal{N}} \bar{\boldsymbol{\Psi}}_j \sum_{n,q\in\mathcal{N}} \Omega_{jn}\Omega_{jq}\theta^j_{nq} \left(X_{fn} - X_{fq}\right)\left(Y_{nj} - Y_{qj}\right)
\end{aligned}$$

The elasticities Θ and $\mathcal{D}_\xi$ capture two key economic forces: expenditure switching and income reallocation. Expenditure switching is described by the operator $\mathcal{X}$,[9] which is a generalized substitution elasticity. This operator maps changes in relative good prices into changes in relative factor employment, accounting for substitution at all nodes of the production network. Income reallocation is described by the covariance $Cov_{\boldsymbol{s}}\left(\Psi^T_{\mathcal{Z}\mathcal{F}}, \cdot\right)$. This covariance relates factor contents in consumption or utilization with real income changes across households and utilization producers. In turn, real income depends on relative present values of income (the term $\mathcal{D}_\xi\boldsymbol{\xi}$), current relative prices (a component of $\Theta\boldsymbol{p}$), and current factor employment (through the multiplier $\mathcal{D}_\ell$).

The term $\mathcal{D}_\xi\boldsymbol{\xi}$ implies higher demand for primary factors used more intensively by households whose relative permanent income increased. In the expression for Θ, employment increases for factors consumed more intensively by agents whose income increased more than consumption prices. The multiplier $\mathcal{D}_\ell$ implies that changes in relative demand are amplified if the owners of factors whose income increased also consume disproportionately more of those factors, and dampened otherwise.

Examples B.1 and B.2 in appendix B illustrate how the generalized Θ extends the substitution elasticity θ from section 2, by aggregating across stages of production and accounting for heterogeneous consumption bundles. The demand relationship described by Θ and $\mathcal{D}_\xi$ will interact with the supply curves derived in the next paragraph to determine equilibrium employment responses.

Generalized Supply Curves Lemma 4.3 derives industry-by-factor supply curves, which extend the simpler supply relationships from Section 2. Different from the simple model, prices in each industry depend on the employment of all primary factors, either directly, through input-output linkages, or through the feedback between factor prices, good prices, and factor price deflators.

Lemma 4.3. *Generalized supply curves relate industry prices with factor employment, and are given by*

$$\boldsymbol{\pi}_t = \kappa \left[\boldsymbol{\ell}_t + \Phi^{-1}\Gamma\boldsymbol{\xi}\right] - \mathcal{V}\boldsymbol{p}_{t-1} + \rho\left(I - \mathcal{V}\right)\mathbb{E}\boldsymbol{\pi}_{t+1}, \tag{18}$$

[9] $\mathcal{X}$ is akin to the substitution operators introduced by Baqaee and Farhi (2020).

where

$$\kappa \equiv \mathcal{P}_{:\mathcal{F}} \left(I - \beta^T \mathcal{P}_{:\mathcal{F}}\right)^{-1} \Phi$$
$$\mathcal{V} \equiv \left(I - \mathcal{P}_{:\mathcal{F}} \beta^T\right)^{-1} \left(\mathcal{P}\Psi^{-1} - \mathcal{P}_{:\mathcal{F}} \beta^T\right).$$

Proof. Equation (18) follows directly from the factor supply and pricing equations. See Appendix A for derivations. □

The matrix $\kappa = (\kappa_{if})$ describes how a change in the employment of each factor f affects the price of each industry i. The supply slopes encoded in κ are central to understanding why aggregate demand changes propagate asymmetrically into relative prices.

Following lemma 4.3, κ has three components. First, the inverse factor supply elasticity $\Phi_{:f}$ describes how factor prices respond to changes in the employment of each factor f. Second, the pass-through $\mathcal{P}_{:f}$ of factor prices into good prices captures nominal rigidities along the production chains in which each factor f is employed. Third, the price-wage multiplier $\left(I - \beta^T \mathcal{P}_{:\mathcal{F}}\right)^{-1}$ captures nominal rigidities in consumption and factor utilization, described by the product of factor price pass-through $\mathcal{P}_L$ and factor price deflators β. This multiplier represents a feedback loop whereby increasing employment requires raising real factor prices, which puts upward pressure on good prices, requiring even higher nominal factor prices, and so on.

Through the supply slopes κ, a uniform factor demand shift ($\boldsymbol{\ell} = \mathbf{1}\bar{y}$) translates into differential price changes, whenever $\kappa\mathbf{1} \not\propto \mathbf{1}$. Corollaries 4.1 and 4.2 characterize which industries and factors have steeper supply curves.

Corollary 4.1. *The incidence of a proportional increase in employment on industry-level prices is given by* $\kappa\mathbf{1}$*. Denoting average factor price pass-through* $\bar{\mathcal{P}}_{\mathcal{F}} \equiv \bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \mathcal{P}_{:\mathcal{F}} \mathbf{1}$ *and supply elasticities* $\bar{\Phi} \equiv \bar{\boldsymbol{\Psi}}_{\mathcal{F}} \Phi \mathbf{1}$*, the price incidence on industry* i *is larger than the incidence on the GDP deflator* $\bar{p} \equiv \bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \boldsymbol{p}$ *if (i) industry* i*, or its suppliers, suppliers' suppliers, etc., have more flexible prices (*$\mathcal{P}_{i\mathcal{F}}\mathbf{1} > \bar{\mathcal{P}}_{\mathcal{F}}$*); (ii) industry* i *uses inelastically supplied primary factors more intensively (*$\boldsymbol{\Psi}_{i\mathcal{F}}\Phi\mathbf{1} > \bar{\Phi}$*); or (iii) industry* i *uses primary factors whose wage deflators are more responsive to changes in aggregate employment (*$\boldsymbol{\Psi}_{i\mathcal{F}}\beta^T \mathcal{P}_{:\mathcal{F}}\mathbf{1} > \bar{\mathcal{P}}_{\mathcal{F}}$*).*

Proof. See Appendix A. □

Corollary 4.2. *The incidence of factor employment on the GDP deflator is given by* $\bar{\boldsymbol{\kappa}} \equiv \bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \kappa$*. Primary factors* f *that are inelastically supplied (*$\bar{\boldsymbol{\Psi}}_{\mathcal{F}} \Phi_f > \bar{\Phi}$*), have high price pass-through into the GDP deflator (*$\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \mathcal{P}_{:f} > \bar{\mathcal{P}}_{\mathcal{F}}$*), or have a high incidence on factor price deflators (*$\bar{\boldsymbol{\Psi}}_{\mathcal{F}} \beta^T \mathcal{P}_{:f} > \bar{\mathcal{P}}_{\mathcal{F}}$*) affect the aggregate GDP deflator more than proportionally to their income shares.*

Proof. See Appendix A. □

Examples B.3 and B.4 in appendix B illustrate how the supply slopes from Section 2 generalize to accommodate input-output linkages and more than two primary factors.

Equilibrium dynamics Building on the relative demand equation (15c) and supply curves (18) derived above, lemma 4.4 characterizes the joint dynamics of industry-level prices and aggregate output.

Lemma 4.4. *Define π^{IS} as in table 3. The equilibrium dynamics of industry-level prices and aggregate output are governed by the following system of difference equations:*

$$\begin{cases} \boldsymbol{\pi}_t & = (I + \kappa\Theta)^{-1} \left\{ \kappa \left[\mathbf{1}\bar{y}_t + \left(\Phi^{-1}\Gamma + \mathcal{D}_\xi \right) \boldsymbol{\xi} \right] - (\mathcal{V} + \kappa\Theta) \, \boldsymbol{p}_{t-1} + \rho \, (I - \mathcal{V}) \, \mathbb{E} \boldsymbol{\pi}_{t+1} \right\} \\ \bar{y}_{t+1} & = \bar{y}_t + \frac{1}{\gamma} \left(\mathbf{1} i_t - \pi^{IS}_{t+1} \right) + \epsilon^{IS}_t \\ i_t & = \boldsymbol{\phi}_\pi \boldsymbol{\pi}_t + \phi_y \bar{y}_t + \epsilon^i_t \end{cases} \tag{19}$$

The system (19) and the relative demand equation (15c) together are sufficient to characterize the equilibrium path of prices and employment for all industries and primary factors. In order to shed light on the asymmetric propagation of aggregate demand changes, the next subsection explicitly solves for the relative employment of primary factors at each point in time, taking lagged relative prices and expected future inflation as given.

4.3 The Cross-Sectional Multiplier

Proposition 4.1 provides the paper's main theoretical result. By combining generalized supply slopes and substitution elasticities, it characterizes how changes in aggregate output translate into heterogeneous employment and income responses.

Proposition 4.1. *Relative employment $\boldsymbol{\ell}$ across primary factors and relative nominal incomes $\boldsymbol{\varsigma}$ across households are given by*

$$\begin{aligned} \boldsymbol{\ell} - \mathbf{1}\bar{y} &= (I + \Theta\kappa)^{-1} \left[-\Theta\kappa\mathbf{1}\bar{y} + \left(\mathcal{D}_\xi - \Theta\kappa\Phi^{-1}\Gamma \right) \boldsymbol{\xi} - \Theta \, (I - \mathcal{V}) \left(\boldsymbol{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{+1} \right) \right] \\ \boldsymbol{\varsigma} - \mathbf{1}\bar{y} &= \left[\Xi\Psi^{-1} - \Xi_{:\mathcal{F}}\Theta \right] \left[\kappa \boldsymbol{\ell} + (I - \mathcal{V}) \left(\boldsymbol{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{+1} \right) \right] + \Xi_{:\mathcal{F}} \mathcal{D}_\xi \boldsymbol{\xi}. \end{aligned} \tag{20}$$

Proof. The first line is obtained by combining the supply equations (18) with the relative demand equation (15c). The second line follows from the Euler equations (15d) and budget constraints (11). □

Proposition 4.1 formalizes the central mechanism of the paper. Cross-sectional employment responses have three components, determined by aggregate output $\bar{y}$, relative permanent incomes $\boldsymbol{\xi}$, and price dynamics. The key insight emerges from the first term, $-\Theta\kappa\mathbf{1}\bar{y}$. Aggregate demand, captured by aggregate output $\bar{y}$, has a uniform *direct* effect across all primary factors, given by the vector $\mathbf{1}$. If prices remained unchanged, employment would increase proportionally.[10]

Proportional changes in factor demand, however, have unequal incidences on factor prices, leading to heterogeneous *propagation* effects. Differential price responses are due to heterogeneous supply slopes across industries and factors ($\kappa\mathbf{1} \not\propto \mathbf{1}$). Industries i that are more flex-priced or capital-intensive are characterized by steeper supply slopes $\kappa_{i:}\mathbf{1}$, and experience larger price increases. This triggers expenditure switching, captured by the substitution elasticities Θ. Consumers and producers substitute away from goods and factors whose relative prices have increased, reducing demand for these factors.

The interaction between supply and demand adjustments creates further relative price changes, which cause additional demand adjustments, leading to further price adjustments, and so on. The cross-sectional multiplier $(I + \Theta\kappa)^{-1}$ describes the fixed point of this adjustment process, capturing how a uniform direct effect of aggregate demand changes propagates heterogeneously across primary factors.

The remaining terms in equation (20) capture additional effects through permanent income changes and price dynamics. The term $\mathcal{D}_\xi \boldsymbol{\xi}$ tells us that factors consumed more intensively by households whose permanent income increases see a larger demand increase. At the same time, these households reduce their labor supply ($-\Theta\kappa\Phi^{-1}\Gamma\boldsymbol{\xi}$).

Nominal income responses depend on a combination of employment and price responses. Given that relative prices and employment move in opposite directions, the two tend to offset each other. This is captured by the difference $\Xi\Psi^{-1} - \Xi_{:\mathcal{F}}\Theta$, which generalizes the term $1 - \theta$ from section 2. The substitution elasticities encapsulated in Θ determine whether price or employment effects dominate. If Θ is small, meaning that primary factors are complementary in a network-adjusted sense, employment and income move in opposite directions.

Heterogeneous supply slopes are necessary. Corollary 4.3 summarizes the knife-edge conditions under which monetary policy has a uniform incidence on employment and real income across households.

Corollary 4.3. *Changing aggregate output $\bar{y}$ has no effect on relative factor employment and relative real incomes if and only if either (a) factors have identical supply slopes ($\kappa\mathbf{1} \propto \mathbf{1}$), or (b) industries use factors with the same intensity ($\Psi_{:\mathcal{F}} = \mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T$) and ownership shares are equal to total contents in consumption ($\Xi \propto \Psi_{\mathcal{Z}:}$). Condition (a) is satisfied if and only if supply elasticities are such that $\Phi\mathbf{1} \propto \mathbf{1}$ and nominal rigidities are such that $\mathcal{P}_{:\mathcal{F}}\mathbf{1} \propto \mathbf{1}$. Condition (b) is equivalent to having a single primary factor.*

This corollary demonstrates that heterogeneous incidences on labor demand and income are the norm, not the exception. Cross-sectional neutrality requires either that all primary factors face identical supply elasticities and nominal rigidities, or that factor markets are effectively unsegmented. From a quantitative point of view, heterogeneous nominal rigidities and labor shares across industries are prominent features of the data. Thus, aggregate demand shocks will typically have differential effects on labor demand and factor income across households.

[10] Homothetic preferences and production functions are crucial to ensure a uniform direct effect.

The cross-sectional multiplier $(I + \Theta\kappa)^{-1}$ is central to understanding these distributional consequences.

When substitution elasticities become important. A fundamental reason why supply elasticities are heterogeneous is the presence of non-labor factors, that are less elastically supplied than labor. Once supply elasticities differ, substitution elasticities also become important. Consider two labor occupations employed in different industries, where both industries combine labor with an inelastically supplied, industry-specific non-labor factor. If labor is more substitutable with the non-labor factor in one industry than in the other, that industry can adjust more easily along the employment margin during contractions and expansions. The other industry, where labor is complementary with its fixed factor, maintains more stable employment. Thus, substitutability with non-labor factors implies more cyclical employment. Example 4.1 illustrates the mechanism in a simple setting. This mechanism plays a central role in the quantitative analysis of Section 6, where high incidence at the bottom of the income distribution arises precisely from greater substitutability between low-income occupations and non-labor factors.

Example 4.1. Capital-Labor Complementarity and Employment Cyclicality.

Consider an economy with two industries: manufacturing (M) and consulting (C). Each industry combines a labor occupation with a semi-fixed factor. Manufacturing uses blue-collar workers (L_M) and machinery (K_M), while consulting uses analysts (L_C) and office capital (K_C). Labor markets are segmented across industries, so blue-collar workers cannot supply labor to consulting and vice versa.

Both labor occupations have inverse supply elasticity φ_L, while both semi-fixed factors have inverse supply elasticity $\varphi_K > \varphi_L$. A representative household has GHH preferences, owns all industries and primary factors, and consumes a final good produced by combining the two industry outputs with equal expenditure shares. Each industry has equal steady-state expenditure shares on its two inputs.

Production and consumption are CES, with elasticity of substitution σ between final goods and θ_i between inputs in each industry $i \in \{M, C\}$. Industries differ in the substitutability between labor and capital: in manufacturing, labor and machinery are more substitutable than labor and office capital in consulting ($\theta_M > \theta_C$).

Let's study how the relative employment of blue-collar workers versus analysts responds to a change in aggregate demand $\bar{y}$.

In each industry i, the supply of labor and semi-fixed factors is governed by

$$w_i = \varphi_L \ell_i + \bar{p}$$
$$r_i = \varphi_K k_i + \bar{p}.$$

This implies a positive correlation between relative employment and relative labor income:

$$(w_M + \ell_M) - (w_C + \ell_C) = (1 + \varphi_L)(\ell_M - \ell_C).$$

Moreover, production and consumption functions imply

$$y_i = \frac{1}{2}\left(\ell_i + k_i\right)$$
$$\ell_i - k_i = -\theta_i\left(w_i - r_i\right)$$
$$y_M - y_C = -\sigma\left(p_M - p_C\right).$$

Combining factor supply and producer optimization yields the result that employment is more cyclical in the manufacturing industry, where labor and capital are more substitutable:

$$\ell_M = \ell_C \propto \left(\theta_M - \theta_C\right)\left(\varphi_K - \varphi_L\right)\bar{y}$$

5 Aggregate Non-Neutrality and Distributionally-Neutral Fiscal Policies

Section 4 discussed the cross-sectional incidence of generic aggregate demand changes. Its results hold regardless of what drives aggregate demand. In particular, the economic mechanisms described in section 4 apply to the cross-sectional propagation of monetary policy across households and labor occupations. Sections 5.1 and 5.2 then address two natural follow-up questions. First, do the cross-sectional effects of monetary policy on labor income matter for aggregate monetary transmission? Second, can fiscal policy eliminate the redistributive incidence of aggregate demand shocks?

5.1 Aggregate Monetary Non-Neutrality

Let us begin by considering how the cross-sectional incidence of monetary policy affects aggregate inflation dynamics. In multi-industry models there are many possible ways to define an aggregate inflation index, depending on the reference price index. For any aggregate price index $\bar{P}$ (such as consumer prices, or the GDP deflator), consider the Phillips curve slope $\bar{\kappa} \equiv \frac{d\log\bar{P}}{d\log Y}$. We can express $\bar{\kappa}$ as the sum of an average slope, plus the covariance between factor-specific employment responses to $\bar{y}$, and factor-specific slopes:

$$\bar{\boldsymbol{\kappa}}\frac{\partial\boldsymbol{\ell}}{\partial\bar{y}} = \left[\bar{\boldsymbol{\kappa}}\mathbf{1} + Cov_{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}}\left(\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\Psi}}}, \frac{\partial\boldsymbol{\ell}}{\partial\bar{y}}\right)\right]. \tag{21}$$

If employment changed proportionately for all primary factors ($\frac{d\log\boldsymbol{L}}{d\log Y} = \mathbf{1}$), prices $\bar{P}$ would increase by $\bar{\boldsymbol{\kappa}}\mathbf{1}$. If instead the employment of factors with a flatter slope increased relatively more ($Cov_{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}}\left(\frac{\bar{\boldsymbol{\kappa}}}{\bar{\boldsymbol{\Psi}}}, \frac{\partial\boldsymbol{\ell}}{\partial\bar{y}}\right) < 0$), prices would increase by less than $\bar{\boldsymbol{\kappa}}\mathbf{1}$. In other words, a negative covariance in (21) would imply a flatter aggregate Phillips curve.

The covariance in (21) depends on the full distribution of supply slopes and substitution elasticities, therefore it cannot be signed in general. Solving for relative employment responses using Proposition 4.1 yields

$$Cov_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left(\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}}, \frac{\partial \boldsymbol{\ell}}{\partial \bar{y}}\right) = -Cov_{\bar{\mathbf{\Psi}}_{\mathcal{F}}}\left(\frac{\bar{\boldsymbol{\kappa}}}{\bar{\mathbf{\Psi}}}, (I + \Theta\kappa)^{-1}\Theta\kappa\mathbf{1}\right). \tag{22}$$

The covariance in (22) indicates that factors f with a stepper slope ($\bar{\kappa}_f > \bar{\boldsymbol{\kappa}}\mathbf{1}\bar{\Psi}_f$) are likely to be employed by industries whose price responds more to changes in aggregate demand. Consumers and producers substitute away from these industries ($\Theta_{f:}\kappa\mathbf{1} > 0$), pushing toward a negative covariance. Through this channel, (22) is negative in the illustrative economy of section 2 (see example B.5 in appendix B) and in the full calibration of section 6.

Following the same logic as in conventional aggregated models, a flatter Phillips curve implies a stronger response of aggregate output to monetary policy. In our disaggregated economy, however, the dynamics of aggregate output depend on price responses across all industries. Therefore no single price index is a sufficient statistic. Quantitatively, heterogeneity increases the response of aggregate output to monetary policy shocks in the calibrated model.

5.2 Neutral Fiscal Policies

This section considers whether there exist fiscal tools to manage aggregate demand while insuring equal incidences on all households. The question can be approached from two different angles. First, proposition 4.1 implies that monetary policy has a heterogeneous incidence on employment and income across households, except in knife-edge cases. Can appropriate fiscal policies change aggregate demand while preserving a uniform incidence on all prices and quantities? Second, in case aggregate output and prices are constrained to follow an exogenously determined path (for example because of a zero lower bound), can industry-specific cost subsidies ensure uniform cross-sectional incidences?

Proposition 5.1 considers an economy where all primary factors have the same supply elasticity. In this economy, fiscal policy can always implement identical incidences. Any increase (decline) in real GDP can be implemented via a uniform subsidy (tax) on factor prices, while maintaining good prices, relative employment, and relative incomes unchanged. Likewise, for any given path of aggregate output and aggregate inflation, there is a set of cost subsidies which implements proportional changes in all prices and quantities.

Proposition 5.1. *Denote by* $\hat{\Phi} \equiv \bar{\mathbf{\Psi}}_{\mathcal{F}}^T\Phi\mathbf{1}$. *If* $\Phi\mathbf{1} = \hat{\Phi}\mathbf{1}$, *a cost subsidy* $\tau_c = \hat{\Phi}\bar{y}$ *to all factor prices, financed through lump-sum taxes proportional to households' factor ownership shares, implements constant good prices, and identical log-changes in the aggregate real GDP, the income of all households, and the employment of all primary factors.*

Given a path of aggregate real GDP $\{\bar{y}_t\}$ *and aggregate prices* $\{\bar{p}_t\}$, *industry-specific cost subsidies* $\boldsymbol{\tau_c} = \left(\mathcal{P}^{-1}\mathbf{1} - \Omega_{\mathcal{F}}\mathbf{1}\right)\bar{p} - \Omega_{\mathcal{F}}\mathbf{1}\hat{\Phi}\bar{y}$ *ensure that all prices and quantities change proportionally.*

Proof. See Appendix A. □

Proposition 5.2 turns to the case where factors have different supply elasticities ($\Phi\mathbf{1} \neq \hat{\Phi}\mathbf{1}$). It first derives the optimal fiscal policy according to a utilitarian planner, constrained to implement a given path of aggregate real GDP. It then computes two alternative sets of cost subsidies. Given exogenous paths of aggregate output and aggregate prices, the first set of subsidies ensures that all prices and quantities change proportionally. The second instead ensures that employment changes proportional to factor-specific supply elasticities ($\Phi\boldsymbol{\ell} \propto \mathbf{1}$).

Proposition 5.2. *Suppose that $\Phi\mathbf{1} \neq \hat{\Phi}\mathbf{1}$, and consider the problem of a utilitarian planner subject to implementing a given path of real GDP $\{\bar{y}_t\}_{t=0}^{\infty}$. Like in Proposition 5.1, it is optimal for the planner to introduce an aggregate factor price subsidy proportional to $\bar{y}$, financed through lump-sum taxes proportional to the households' factor ownership shares. In addition, the planner must ensure that producer prices remain constant by using appropriate industry-specific cost taxes and subsidies, which offset changes in relative factor prices due to heterogeneous supply elasticities. To guarantee that purchaser prices remain undistorted given factor prices, relative cost subsidies must be offset by purchase taxes. Finally, the planner implements lump-sum transfers such that permanent incomes change proportionally across households.*

Given a path of aggregate real GDP $\{\bar{y}_t\}$ and aggregate prices $\{\bar{p}_t\}$, industry-specific cost subsidies $\boldsymbol{\tau_c} = \left(\mathcal{P}^{-1}\mathbf{1} - \Omega_{\mathcal{F}}\mathbf{1}\right)\bar{p} - \Omega_{\mathcal{F}}\Phi\mathbf{1}\bar{y}$ ensure that all prices and quantities change proportionally. Appendix A reports factor cost subsidies which instead ensure $\Phi\boldsymbol{\ell} \propto \mathbf{1}$.

Proof. See Appendix A for a formal proof, and explicit expressions for all taxes and subsidies. □

6 Quantitative Results

This section calibrates the model to the US economy and presents quantitative results. Section 6.1 describes the data sources and calibrated parameters. Section 6.2 examines how factor market segmentation affects aggregate monetary transmission. Section 6.3 analyzes the cross-sectional incidence of aggregate demand shocks on labor income.

6.1 Data

I calibrate the model to match expenditure flows reported in US national accounts, and external measures of the relevant supply, demand, and pricing elasticities. I identify production factors with 30 differentiated labor occupations and 31 semi-fixed assets (equipment, structures, and intellectual property). Households are identified with labor occupations. The main data sources are summarized in Table 4, with additional detail in Appendix C.

Data	Source
Industries' expenditure on intermediate inputs	BEA input-output tables (2019, 71 industries)
Industries' expenditure on labor occupations	BEA IO tables, Occupational Employment and Wage Statistics (2019, 30 occupations)
Industries' expenditure on non-labor factors	NIPA tables, following Vom Lehn and Winberry (2021) (2019, 31 assets)
Expenditure shares in asset utilization	Capital Flow Tables (2019)
Expenditure shares in consumption	BEA IO tables, Consumer Expenditure Survey (2019)
Ownership shares in industries and non-labor factors	World Inequality Database (WID)

Table 4: Data sources for expenditure flow calibration.

Calibrated Parameters. Table 5 reports the calibrated elasticities.

Parameter	Calibrated Value
Intertemporal elasticity of substitution	$\gamma = 1$
Inverse Frisch elasticity	Increasing in job tenure, from Current Population Survey (CPS) $\varphi = 1$ for bottom tercile, $\varphi = 2$ for middle, $\varphi = 3$ for top
Wage adjustment frequency	Increasing in self-employment share (from CPS) and increasing in bonus share of compensation (from PSID) Bottom self-employment tercile: $0.1 - 0.25$ (quarterly) Mid self-employment tercile: 0.35 Top self-employment or top bonus tercile: 0.4
Inverse supply elasticity of semi-fixed assets	Heavy equipment: 10 Light equipment: 2 Structures: 20 Intellectual property: 5
Substitution elasticities for non-labor inputs	From Atalay (2017) Consumption goods: 0.9 Intermediates vs. non-labor factors: 0.5 Across intermediate inputs: 0.1 Across non-labor factors: 0.1
Substitution elasticities involving labor	Increasing in Autor and Dorn (2013) automatability measure Top automatable tercile vs. non-labor factors: 2 Mid automatable tercile vs. non-labor factors: 0.9 Bottom automatable tercile vs. non-labor factors: 0.1 High-skill vs. other occupations: 0.1 Mid- vs. low-skill occupations: 0.9
Price adjustment frequencies	Pasten et al. (2019)

Table 5: Calibrated model elasticities.

The relative supply elasticity of non-labor factors versus labor is calibrated to match the differential price responses to identified monetary shocks across industries with high versus low labor shares in value added (see Appendix C for detail). Substitution elasticities in consumption and production follow Atalay (2017).

Labor supply elasticities, wage adjustment probabilities, and substitution elasticities across labor occupations are difficult to measure directly. I set the average parameters equal to conventional estimates, and proxy for dispersion across occupations using measurable features. My goal is to demonstrate that under a plausible parameterization, informed by external measures of labor market characteristics, the model can replicate the incidence patterns documented by Guvenen et al. (2017) and Patterson (2023). This allows me to highlight the essential parameters, and to discuss how incidence patterns would change as technology and labor market institutions evolve over time.

I set a lower value of the inverse Frisch elasticity φ for labor occupations with shorter average job tenure, to replicate a greater employment cyclicality. I calibrate the average inverse Frish to be around 2, as conventional in the New Keynesian literature. Likewise I set a higher wage flexibility for occupations with larger self-employment shares or larger bonus shares of compensation, to generate a greater compensation cyclicality. Following Beraja et al. (2019), I set the baseline quarterly wage adjustment probability to 0.25. Low-tenure occupations are concentrated at the bottom of the wage distribution, while high bonus shares are concentrated at the top. These patterns are consistent with evidence from Guvenen et al. (2017) and Patterson (2023), as well as Grigsby et al. (2019) on the flexibility of bonus-adjusted compensation.

Substitution elasticities across labor types are set to match external measures of automatability across occupations (Autor and Dorn, 2013). I impose that highly automatable occupations are more substitutable with non-labor assets and intermediate inputs. This will result in higher employment cyclicality for occupations that are more substitutable with non-labor assets, consistent with the evidence in Jaimovich and Siu (2012).

6.2 Factor Market Segmentation and Aggregate Monetary Transmission

Figures 1 and 2 connect factor market segmentation with aggregate monetary transmission. The solid lines represent impulse-responses of aggregate prices and quantities to an unexpected monetary shock. The shaded areas represent dispersion across industries, workers, and households. Figure 1 compares the baseline model with segmented labor markets (in blue) against a counterfactual economy with a single unsegmented labor market (in orange). Figure 2 instead compares the baseline economy (blue) with a model that omits non-labor factors (orange).

As a basic premise, figure 1 makes the case that accounting for labor market segmentation is essential to obtain heterogeneous employment and labor income responses to aggregate demand. While the baseline model with segmented labor markets generates sizable dispersion in employment responses, the single-labor model cannot generate variation in employment and labor income responses. Aggregate transmission, instead, is not much affected by labor market segmentation.

Figure 2 instead shows that accounting for non-labor factors affects aggregate dynamics, while also amplifying price dispersion and the range of employment responses across occupations. Industries with higher labor shares in value added exhibit smaller price responses, because labor is more elastically supplied than non-labor factors. When averaging across industries, this generates a smaller but more persistent aggregate price response, and a larger but less persistent output response.

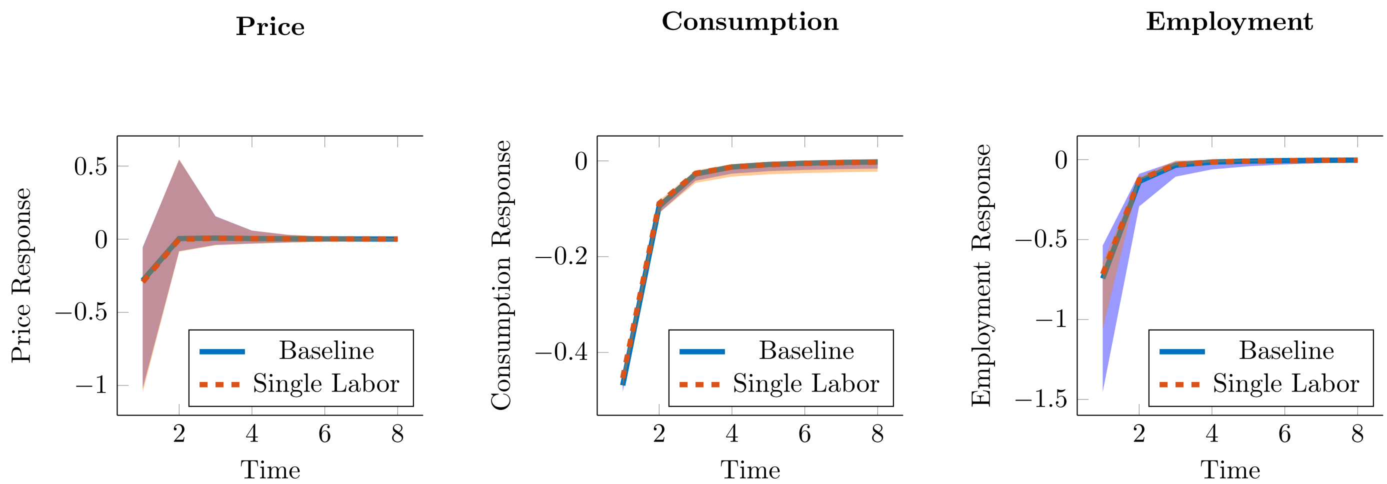

Figure 1: Baseline vs Single Labor: Impulse Responses

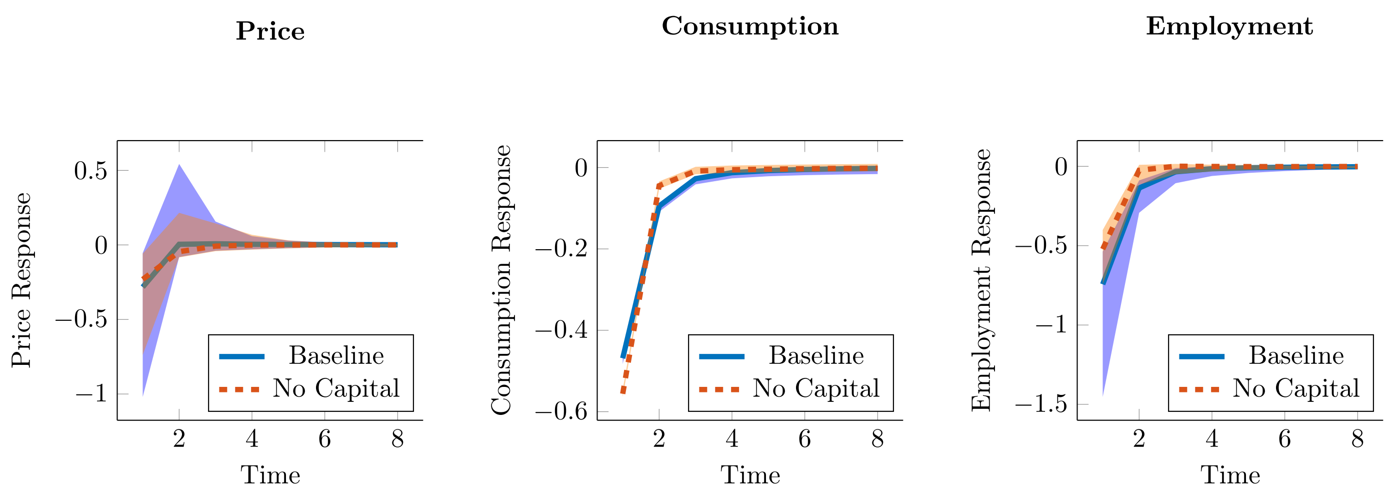

Figure 2: Baseline vs No Capital: Impulse Responses

6.3 Cross-Sectional Incidence of Aggregate Demand Changes

This section examines the incidence of aggregate demand shocks on labor income and total income across the distribution. For labor income, I present results for two different incidence measures. The first consists in the covariance between real labor income and aggregate output, when fluctuations are entirely driven by aggregate demand shocks. The second consists in cumulative impulse responses of labor income to aggregate demand shocks. Both measures have similar patterns across income groups and occupations. The covariance-based incidence is the one adopted by empirical studies (Guvenen et al., 2017; Patterson, 2023; Alves et al., 2020). Cumulative impulse-responses are useful because they can be additively decomposed between employment and wage components. The two measures are related but not identical. The covariance-based incidence captures the correlation between individual and aggregate income given a distribution of aggregate demand shocks. Cumulative impulse responses summarize the dynamic effect of a single shock.

Incidence on Labor Income. Figure 3 reports covariance-based incidence elasticities, aggregated by income percentile groups (P1-15, P15-30, P30-55, P55-85, P85-100). The covariance-based incidence β_f for a labor occupation f is the regression coefficient of the occupation's real labor income response on aggregate output:

$$\beta_f = \frac{\text{Cov}\left(\Delta \log Y_f^{\text{real}}, \Delta \log \bar{Y}\right)}{\text{Var}\left(\Delta \log \bar{Y}\right)}. \tag{23}$$

The figure compares two parameterizations of capital-labor substitutability. In the left panel (teal bars), only highly automatable occupations are substitutable with non-labor assets. All other occupations are complementary. In the middle panel (purple bars), mid-skill occupations also become substitutable with non-labor assets. This second parametrization represents the diffusion of automation technologies that substitute white collar tasks.

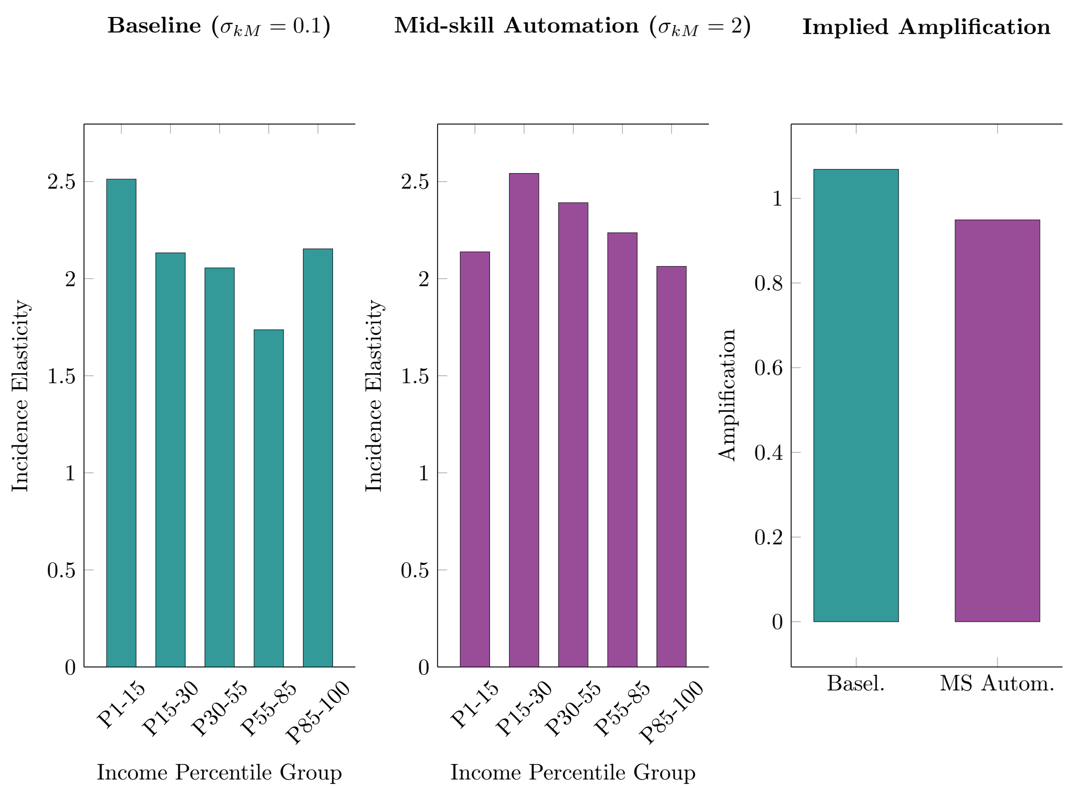

Figure 3: Incidence elasticities by income percentile group and implied amplification. The left panel shows incidence under the baseline calibration where mid-skill labor is complementary with capital ($\sigma_{kM} = 0.1$). The middle panel shows incidence when mid-skill labor is substitutable with capital ($\sigma_{kM} = 2.0$). The right panel reports implied Keynesian amplification factors under each parameterization, computed following Alves et al. (2020).

When only highly automatable occupations are substitutable with capital, the incidence profile exhibits a U-shape. High-income occupations have above-average sensitivity to aggregate demand, because they are complementary with other occupations and non-labor factors. Complementarity implies that their relative labor income response is determined by their relative wage cyclicality (as illustrated in section 2), which is higher than average due to larger bonus and self-employment shares. Low-income occupations also have above-average sensitivity to aggregate demand, because they are substitutable with capital. Substitutability implies that their relative labor income is determined by relative employment cyclicality, which is higher than average due to more elastic labor supply. If mid-skill labor also became substitutable with non-labor assets, incidence would shift up for middle-income occupations. The U-shaped pattern would reverse, with incidence becoming monotonically increasing along the income distribution.

In HANK models, incidence patterns are important for the aggregate response to monetary policy. Aggregate output depends on the average marginal propensity to consume ($\overline{\text{MPC}}$) and on the covariance between labor income responses and marginal propensities to consume (Patterson, 2023; Alves et al., 2020). This is measured by an amplification factor:

$$\text{Amplification} = \frac{\overline{\text{MPC}} + Cov(\text{MPC}, \text{incidence})}{\overline{\text{MPC}}} \tag{24}$$

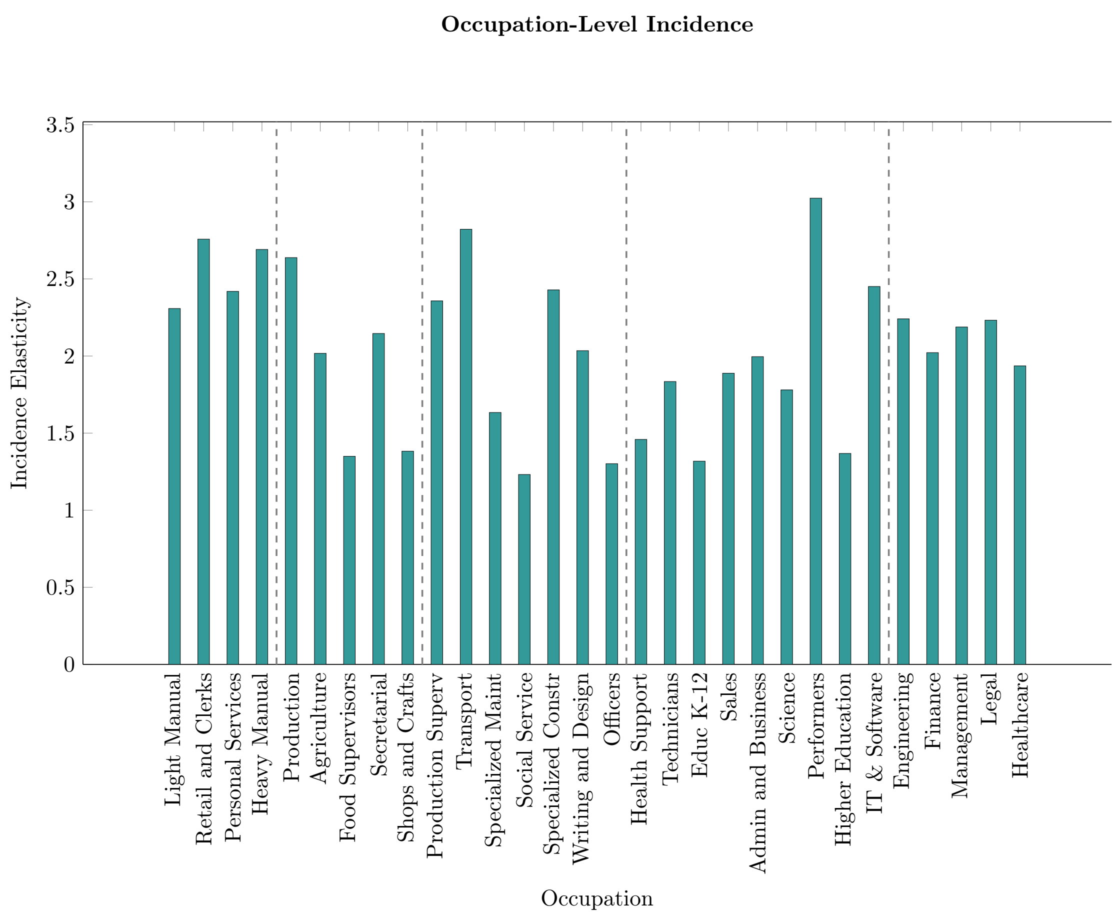

Figure 4: Occupation-level incidence elasticities under the baseline calibration (mid-skill occupations are complementary with capital). Occupations are sorted by increasing average income. Vertical dashed lines indicate income percentile group boundaries (P15, P30, P55, P85).

The right panel of figure 3 reports implied amplification factors. Aggregate demand shocks are amplified when incidence is concentrated on high-MPC, low-income households (teal bar). Amplification instead is attenuated when incidence shifts toward middle-income, lower-MPC households (purple bar).

Figure 4 disaggregates incidence to the occupation level, revealing substantial heterogeneity within income groups. Incidence elasticities vary considerably, even among occupations in the same percentile group. These within-group differences underscore that the specific occupation matters, and income rank alone is not a sufficient statistic for cyclical exposure.

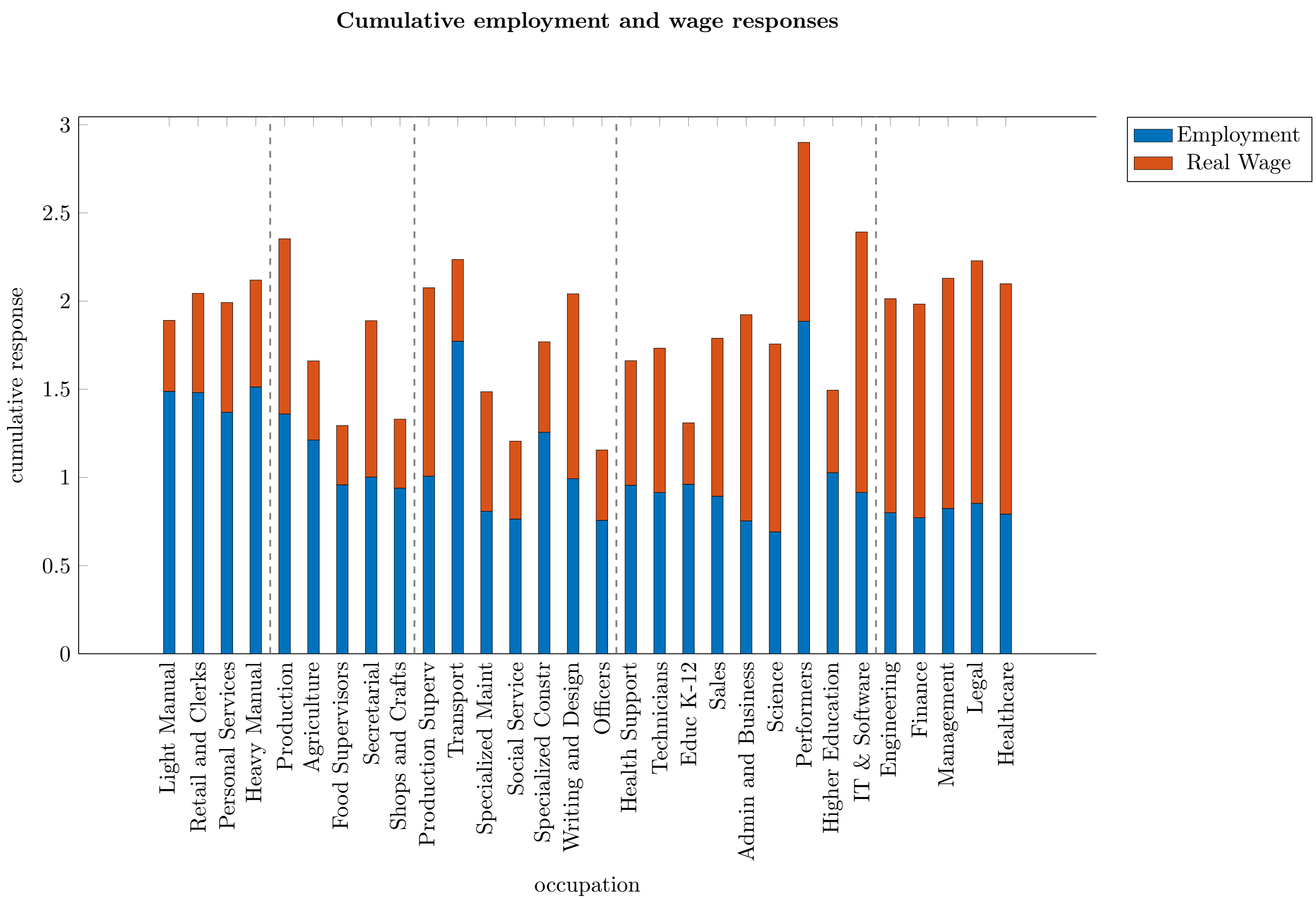

Figure 5: Cumulative labor income responses by occupation, decomposed into employment (blue) and real wage (orange) components. Occupations are sorted by increasing average income. Vertical dashed lines indicate income percentile group boundaries (P15, P30, P55, P85)

Figure 5 presents cumulative impulse responses, defined as the present discounted value of income changes following an aggregate demand shock. Cumulative responses provide a different scalar summary of the full impulse responses of labor income, complementary to the covariance-based incidence measure. In particular, they allow us to decompose total income changes into employment and wage components (for labor income), and into labor versus non-labor income (for total income).

The figure decomposes cumulative labor income responses into employment and wage components for each occupation. The U-shaped pattern in total labor income responses mirrors the incidence results, and the decomposition reveals the mechanisms at play. At the bottom of the distribution, employment responses dominate: low-income occupations experience large increases in hours worked, with relatively modest wage gains. At the top of the distribution, wage responses dominate: high-income occupations see smaller employment changes but substantial increases in compensation, reflecting the flexibility of bonuses and performance pay.

Cumulative nominal income changes

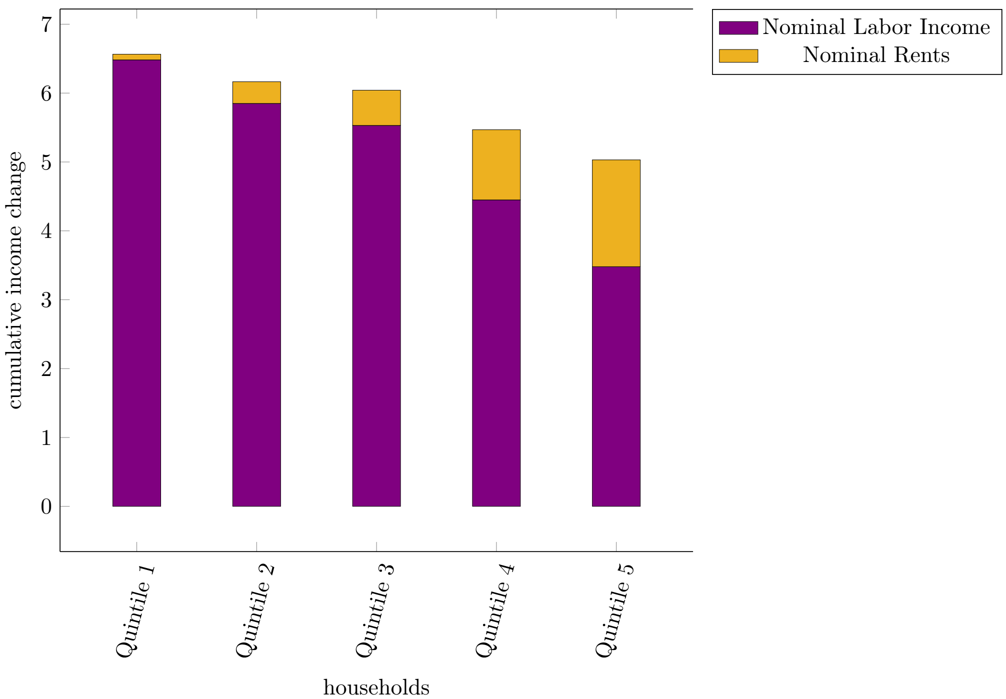

Figure 6: Cumulative income responses by income quintile, decomposed into labor income (purple) and income from asset ownership and profits (gold).

Incidence on Total Income. Figure 6 decomposes total income responses into labor income and non-labor income (rents from firm ownership and non-labor factors). Non-labor income becomes increasingly important at higher income levels, reflecting the concentration of asset ownership among high-income households. The response of non-labor income is smaller than the response of labor income. This reflects the fact that good prices increase by less than wages following an aggregate demand expansion, thereby reducing profits. As a result the total income response for high-income households is attenuated, contributing to a progressive overall incidence of aggregate demand shocks.

Full Impulse Response Functions. Figure 7 reports full impulse response functions of wages and employment by occupation. Darker colors correspond to lower income occupations. Wages exhibit a larger and anti-persistent response at the top of the income distribution, reflecting the higher frequency of wage adjustment for high-income occupations. At the bottom of the distribution, wage responses are smaller and more persistent. Employment responses are uniformly larger at the bottom of the distribution, consistent with the more elastic labor supply calibrated for low-income occupations.

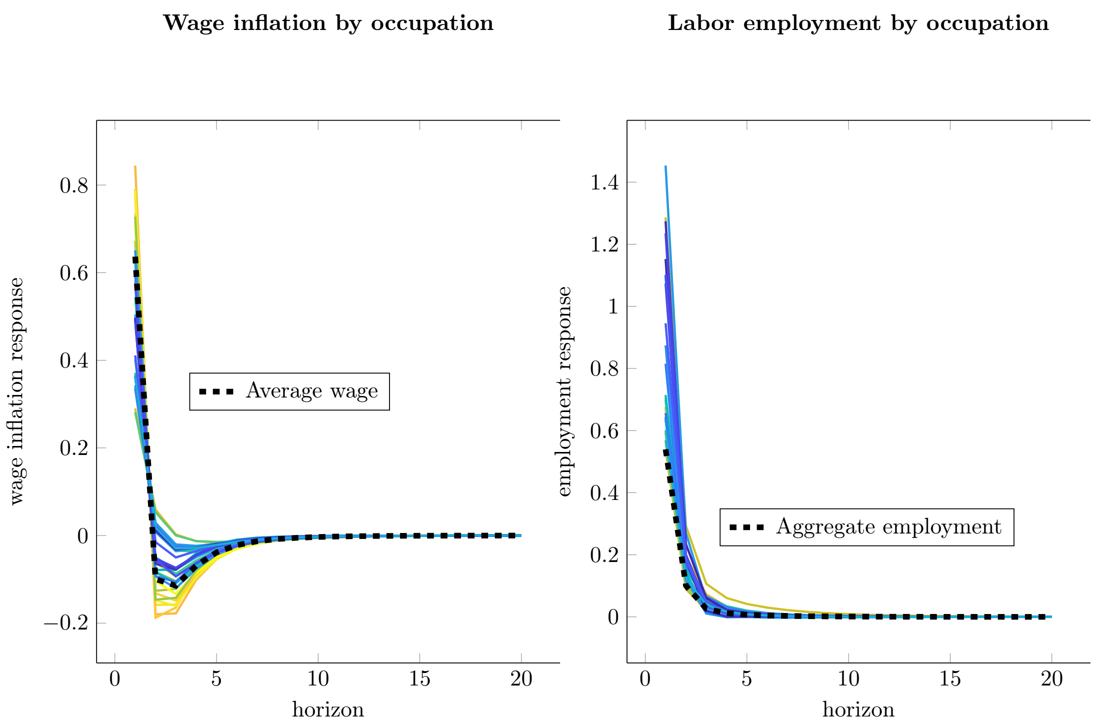

Figure 7: Impulse-response of wages and employment by occupation. Colors indicate relative income (dark blue = low income, yellow = high income).

7 Conclusion

This paper develops a framework for understanding the heterogeneous incidence of aggregate demand shocks on labor income across the wage distribution. This is important for redistributive concerns, as well as for the aggregate transmission of monetary policy.

The key theoretical insight is that cross-sectional incidences follow a Phillips curve logic. Workers and industries facing more nominal rigidity or more elastic factor supply exhibit larger employment responses and smaller price responses to changes in aggregate demand. When matching labor market characteristics, input-output linkages, and non-labor shares in value added, the model replicates a U-shaped incidence pattern documented in empirical studies.

These results highlight that aggregate and distributional consequences of monetary policy are not fixed features of the economy. They depend on production technologies, labor market characteristics, and the evolving complementarity between labor occupations and non-labor factors. In particular, factor market segmentation flattens the aggregate Phillips curve and amplifies the real effects of monetary policy. In addition, counterfactual analysis suggests that technological changes enabling substitution between mid-skill labor and capital would shift incidence toward middle-income workers. This in turn would dampen the response of aggregate output to monetary policy.

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Appendix to Monetary Non-Neutrality in the Cross-Section

A Derivations and Additional Theory Results

Non-Labor Factors

I model non-labor factors using a stylized utilization framework that delivers constant-elasticity supply curves, similar to the consumption-leisure tradeoff for labor occupations. Each asset $f \in \mathcal{K}$ combines a fixed endowment $\bar{K}_f$ – which never depreciates – with utilization U_f – which fully depreciates within one period:

$$L_{ft} = U_{ft} \bar{K}_f.$$

The production function for utilization is

$$U_{ft} \equiv \left[(1 + \varphi_f) \, C_{ft} \right]^{\frac{1}{1+\varphi_f}} ,$$

where the homothetic utilization bundle $C_{ft} = \mathcal{C}_f(c_{f1,t}, ..., c_{fN,t})$ uses goods $i \in \mathcal{N}$ with substitution elasticities

$$\theta_{ij}^f \equiv - \frac{d \log \frac{c_{fi}}{c_{fj}}}{d \log \frac{P_i}{P_j}}.$$

Asset retailers are owned by households in proportion to their asset ownership shares. They combine asset endowments with utilization, and sell asset services at rental rate W_f. Profit maximization yields supply curves with constant elasticity $\frac{1}{\varphi_f}$:

$$U_f^{\varphi_f} = \frac{W_f}{P_f^C} \bar{K}_f, \ \forall f \in \mathcal{K} \tag{25}$$

Inverse supply elasticities φ_f can vary across assets and between labor and non-labor factors. Payments $W_f L_f$ are split between utilization expenditures (share $\frac{1}{1+\varphi_f}$) and rebates to the households (share $\frac{\varphi_f}{1+\varphi_f}$).

Aggregation and Income Shares

Denote steady-state variables with stars.

Nominal GDP equals total final expenditures:

$$GDP = \sum_{z \in \mathcal{Z}} P_z^C C_z$$

Real GDP changes are share-weighted consumption and utilization changes:

$$d\log Y_t = \sum_{z\in\mathcal{Z}} s_z d\log C_{zt}$$

GDP deflator changes are share-weighted price index changes:

$$d\log P_t^Y = \sum_{z\in\mathcal{Z}} s_z d\log P_{zt}^C$$

Steady-state income shares are:

$$s_z \equiv \frac{P_z^{*C} C_z^*}{GDP^*},\ z\in\mathcal{Z} \quad \text{and} \quad \bar{\Psi}_f \equiv \frac{W_f^* L_f^*}{GDP^*},\ f\in\mathcal{F}$$

Remark A.1. Around an efficient equilibrium, real GDP changes equal income-weighted primary factor quantity changes:

$$d\log Y_t = \sum_{f\in\mathcal{F}} \bar{\Psi}_f d\log L_{ft}$$

I now derive steady-state income shares of primary factors and final users as a function of ownership and expenditure shares. Throughout the paper, I maintain the following assumption.

Assumption A.1. There is a connected path of direct or indirect income flows between any two agents in the economy:

$$rank\left(I - \hat{\Xi}_{:L}\Psi_{CL}^T\right) = \mid \mathcal{L} \mid + \mid \mathcal{K} \mid -1.$$

Lemma A.1. *Denoting by* $\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T \equiv \mathbf{s}^T\Psi_{CL}$*, the steady-state income shares of final users* $\mathbf{s}$ *and Domar weights* $\bar{\boldsymbol{\Psi}}$ *solve the following system:*

$$\begin{cases} \bar{\boldsymbol{\Psi}} = \Psi_{C,:}^T \mathbf{s} \\ \mathbf{s} = \hat{\Xi}_{:L}\bar{\boldsymbol{\Psi}}_{\mathcal{F}} + \frac{\mathbf{T}}{GDP} \end{cases} \tag{26}$$

Under Assumption A.1, the system (26) has a unique solution $\boldsymbol{s} \geq \mathbf{0}$ *and* $\bar{\boldsymbol{\Psi}}_{\mathcal{F}} \geq \mathbf{0}$ *such that* $\sum_{h\in\mathcal{H}} s_h = 1$ *and* $\sum_{f\in\mathcal{L}\cup\mathcal{K}} \bar{\Psi}_f = 1$.

Proof. The system (26) is an accounting identity, which follows immediately from the definition of income and expenditure shares. Existence and uniqueness of the solution follows from the Perron-Froboenius theorem applied to the matrix $I - \hat{\Xi}_{:L}\Psi_{CL}^T$, noting that $\hat{\Xi}_{:L}\Psi_{CL}^T \geq 0$ and $\hat{\Xi}_{:L}\Psi_{CL}^T\mathbf{1} = \mathbf{1}$. □

The second equation in (26) states that, in a steady-state with zero profits, the income of final users consists of factor income rebates plus transfers. In turn, the first equation states that each producers' (primary factor's) income depends on the total content of this producer (primary factor) in final demand, weighting final users according to their income shares. The system (26) is not invertible.[11] Nonetheless Assumption A.1 guarantees that all relative prices and income shares are well defined, and hence the system has a unique solution such that $\sum_{h\in\mathcal{H}} s_h = 1$.

Notation

Full notation for income flows and expenditure shares is reported in table 6 below.

Parameter	Notation	Definition
Expenditure shares	$\Omega \in \mathbb{R}^{\|\mathcal{N}\|\times\|\mathcal{N}\|}$	$\Omega_{ij} = \frac{P_j^* X_{ij}^*}{P_i^* Y_i^*}$
Leontief inverse	Ψ	$\Psi \equiv (I-\Omega)^{-1}$
Final user income shares	$\mathbf{s} \in \mathbb{R}^{\|\mathcal{Z}\|}$	$s_h \equiv \frac{P_h^{C*} C_h^*}{GDP^*}$
Primary factor income shares	$\bar{\boldsymbol{\Psi}}_{\mathcal{F}} \in \mathbb{R}^{\|\mathcal{F}\|}$	$\bar{\Psi}_f \equiv \frac{W_f^* L_f^*}{GDP^*}$
Domar weights	$\bar{\boldsymbol{\Psi}} \in \mathbb{R}^{\|\mathcal{N}\|}$	$\bar{\Psi}_i \equiv \frac{P_i^* Y_i^*}{GDP^*}$
Ownership shares	$\Xi \in \mathbb{R}^{\|\mathcal{Z}\|\times\|\mathcal{N}\|}$	$\Xi \equiv \mathcal{S}^{-1}\left(\hat{\Xi} + \boldsymbol{T^*}\mathbf{1}^T\right) diag\left(\bar{\boldsymbol{\Psi}}\right)$ $T_z^* \equiv \left(\frac{s_G^h}{s_G^{\mathcal{C}}} - \frac{s_h}{s_{\mathcal{H}}}\right) s_G^{\mathcal{C}} - s_G^{\mathcal{G}} \quad z \in \mathcal{H}$ $T_z^* = 0 \quad z \in \mathcal{U}$
Industries' weights in GDP deflator	$\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \in \mathbb{R}^{\|\mathcal{N}\|}$	$\bar{\boldsymbol{\Omega}}_{\mathcal{Z}} \equiv \boldsymbol{s}^T \Omega_{\mathcal{Z},:}$

Table 6: Income flows and shares

Table 7 reports the full definitions of model elasticities.

Parameter	Notation	Definition
Wealth effects in factor supply	$\Gamma \in \mathbb{R}^{\|\mathcal{F}\|\times\|\mathcal{Z}\|}$	$\Gamma = \hat{\Xi}_{:\mathcal{F}}^T diag\left(\boldsymbol{\gamma}\right)$ $\boldsymbol{\gamma} \equiv \begin{pmatrix} \gamma\mathbf{1}_{\mathcal{H}} & \mathbf{0}_{\mathcal{U}} & 0 \end{pmatrix}$
Heterogeneous factor supply elasticities	$\Phi \in \mathbb{R}^{\|\mathcal{F}\|\times\|\mathcal{F}\|}$	$\Phi \equiv diag\left(\boldsymbol{\varphi}\right) +$ $\Gamma \begin{pmatrix} \mathbf{1}_{\mathcal{H}}\bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}} & \Xi_{\mathcal{U}\mathcal{F}} & \mathbf{0} \end{pmatrix}^T$
Factor price deflators	$\beta \in \mathbb{R}^{\|\mathcal{N}\|\times\|\mathcal{F}\|}$	$\beta^T \equiv (I-\Gamma)\begin{pmatrix} \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}} & \Omega_{\mathcal{U}} \end{pmatrix}^T$ $+\Gamma\begin{pmatrix} \mathbf{1}_{\mathcal{H}}\bar{\boldsymbol{\Xi}}_{\mathcal{H},:} & \Xi_{\mathcal{U},:} & \mathbf{0} \end{pmatrix}^T \Psi^{-1}$
Industry shares in nominal rate deflator	$\pi^R \equiv \hat{\boldsymbol{\beta}}^T \boldsymbol{\pi}$	$\hat{\boldsymbol{\beta}}^T \equiv (1-\gamma)\,\bar{\boldsymbol{\Omega}}_{\mathcal{H}:}$ $+\gamma\left[\bar{\boldsymbol{\Xi}}_{\mathcal{H}:}\Psi^{-1} - \bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\Theta\right]$
Substitution elasticities	θ_{jk}^i	$\theta_{jk}^i \equiv -\frac{d\log\frac{X_{ij}}{X_{ik}}}{d\log\frac{P_j}{P_k}}$
Real marginal cost Phillips curve slopes	$\Delta \equiv diag\left(\hat{\delta}_1, ..., \hat{\delta}_{\bar{N}}\right)$	$\hat{\delta}_i \equiv \frac{\delta_i(1-\rho(1-\delta_i))}{1-\rho\delta_i(1-\delta_i)}$
Price pass-through	$\mathcal{P}$	$\mathcal{P} \equiv \Delta(I-\Omega\Delta)^{-1}$
Demand elasticity	$\Theta \in \mathbb{R}^{\|\mathcal{F}\|\times\|\mathcal{N}\|}$	See lemma 4.2

Table 7: Elasticities

[11] The system implies

$$\left(I - \mathcal{W}\Psi_{CL}^T\right)\mathbf{s} = \mathbf{T}$$

The matrix $I - \mathcal{W}\Psi_{CL}^T$ is not invertible, because $\mathbf{1}^T\left(I - \mathcal{W}\Psi_{CL}^T\right) = \mathbf{1}^T - \mathbf{1}^T\Psi_{CL}^T)$, and $\mathbf{1}^T - \mathbf{1}^T\Psi_{CL}^T = \mathbf{1}^T - \mathbf{1}^T = \mathbf{0}^T$.

Log-Linearized Equilibrium (Lemma 4.1 and Lemma 4.4)

Consumption-saving problem. Log-linearizing the households' Euler equations gives

$$\boldsymbol{c}_{t+1} = \boldsymbol{c}_t + \Gamma^{-1}\left(\mathbf{1} i_{t+1} - \Omega_{H:}\boldsymbol{\pi}_{t+1}\right) \tag{27}$$

while the intertemporal budget constraint imposes

$$\sum_{t=0}^{\infty} \rho^t \boldsymbol{c}_t = \sum_{t=0}^{\infty} \rho^t \left[\mathbf{1}\bar{y}_t - \left[\Omega_{C:} + \Xi_{:L}\mathcal{D}_p - \Xi\Psi^{-1}\right]\boldsymbol{p}_t\right]. \tag{28}$$

Together, (27) and (28) imply

$$\boldsymbol{p}_{\mathcal{H}t} + \boldsymbol{c}_{\mathcal{H}t} = \mathbf{1}\left(\bar{c}_{\mathcal{H}t} + \bar{p}_{\mathcal{H}t}\right) - \left(\Gamma^{-1} - I\right)\left(\Omega_{\mathcal{H}:} - \mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}}\right)\boldsymbol{p}_t + \boldsymbol{\xi}, \tag{29}$$

where the subscript $\mathcal{H}$ denotes the components of a vector or matrix corresponding to households ($h \in \mathcal{H}$) and the value of $\boldsymbol{\xi}$ is given in the statement of Lemma 4.1.

Finally, combining the households' Euler equations with per-period budget constraints and with the relative demand equation from Lemma 4.1, and aggregating across households, yields the aggregate Euler equation

$$\bar{y}_{t+1} = \bar{y}_t + \frac{1}{\gamma}\left(\mathbf{1} i_{t+1} - \bar{\boldsymbol{\beta}}^T \boldsymbol{\pi}_{t+1}\right)$$

from Lemma 4.4.

Factor supply. The log-linearized consumption-leisure tradeoff is given by

$$\Gamma \boldsymbol{c} + diag\left(\boldsymbol{\varphi}\right)\boldsymbol{\ell} = \boldsymbol{w} - \Omega_{C:}\boldsymbol{p}.$$

Combining with equation (29) above implies

$$\Gamma \boldsymbol{c}_{\mathcal{H}} = \Gamma \mathbf{1}\bar{\boldsymbol{\Xi}}_{\mathcal{H}\mathcal{F}}\boldsymbol{\ell} - \left[\Omega_{\mathcal{H}:} - \left(I - \Gamma\right)\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}} - \Gamma\mathbf{1}\bar{\Xi}_{\mathcal{H}:}\right]\boldsymbol{p}. \tag{30}$$

Plugging into the log-linearized consumption-leisure tradeoff yields the factor supply equation.

Pricing. Replicating the derivation in Gali (2015) industry-by-industry, we can express industry-level prices as a function of the vector of industry-level marginal costs $\boldsymbol{mc}_t$, lagged prices $\boldsymbol{p}_{t-1}$, and expected inflation $\mathbb{E}\boldsymbol{\pi}_{t+1}$:

$$\boldsymbol{p}_t = \Delta \boldsymbol{mc}_t + \left(I - \Delta\right)\left(\boldsymbol{p}_{t-1} + \rho \mathbb{E}\boldsymbol{\pi}_{t+1}\right).$$

Further expressing marginal costs as a function of factor and input prices,

$$\boldsymbol{mc}_t = \Omega_{:,\mathcal{F}}\mathbf{w}_t + \Omega\boldsymbol{p}_t,$$

and rearranging, yields the pricing equation

$$\boldsymbol{p}_t = \mathcal{P}_{:L}\boldsymbol{w}_t + \left(I - \mathcal{P}\Psi^{-1}\right)\rho\mathbb{E}\boldsymbol{\pi}_{t+1}.$$

The matrix $I - \Omega\Delta$ (used in the definition of $\mathcal{P}$) is always invertible. The result follows from the Perron-Froboenius theorem, observing that $\Omega\Delta \geq \mathbb{O}$ and $\Omega\Delta\mathbf{1} \leq \mathbf{1}$.

Factor demand. To derive the relative demand equation, start by differentiating the accounting relation

$$\bar{\boldsymbol{\Psi}}_{\mathcal{F}} = \Psi^T_{\mathcal{Z}\mathcal{F}}\mathbf{s}$$

to obtain

$$d\bar{\boldsymbol{\Psi}}_{\mathcal{F}} = d\left(\Psi^T_{\mathcal{Z}\mathcal{F}}\right)\mathbf{s} + \Psi^T_{\mathcal{Z}\mathcal{F}}d\boldsymbol{s}.$$

Next denote by $\boldsymbol{\mu}$ the vector of log-changes in industry-level markups, and use the definition of $\Psi \equiv (I - \Omega)^{-1}$ to solve

$$d\left(\Psi^T_{\mathcal{Z}\mathcal{F}}\right)\mathbf{s} = \Psi^T_{:\mathcal{F}}d\Omega^T\mid_{Sub}\bar{\boldsymbol{\Psi}} - \Psi^T_{:\mathcal{F}}diag\left(\bar{\boldsymbol{\Psi}}\right)\boldsymbol{\mu} + s_{\mathcal{H}}Cov_{\boldsymbol{s}_{\mathcal{H}}}\left(\Psi^T_{:\mathcal{F}}, \boldsymbol{\xi}\right),$$

where

$$\Psi^T_{:\mathcal{F}}d\Omega^T\mid_{Sub}\bar{\boldsymbol{\Psi}} \equiv \sum_{i,j,k}\bar{\Psi}_i\omega_{ij}\omega_{ik}\left(1 - \theta^i_{jk}\right)\left(\Psi_{:\mathcal{F}}\right)_{j,:}\left(p_j - p_k\right) =$$

$$= \sum_{i,j,k}\bar{\Psi}_i\omega_{ij}\omega_{ik}\left(\Psi_{:\mathcal{F}}\right)_{j,:}\left(p_j - p_k\right) - \mathcal{X}\left(\Psi_{:\mathcal{F}}, \mathbf{p}\right) =$$

$$= \left[\Psi^T_{:\mathcal{F}}\left(I - \bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\mathbf{1}^T\right)diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1} - Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}, \Omega_{\mathcal{Z}:}\right) - \mathcal{X}\left(\Psi_{:\mathcal{F}}, I\right)\right]\mathbf{p}$$

and we defined

$$\left[\mathcal{X}\left(X, Y\right)\right]_{fi} \equiv \sum_j\bar{\boldsymbol{\Psi}}_j\sum_{nq}\Omega_{jn}\Omega_{jq}\theta^j_{nq}\left(X_{fn} - X_{fq}\right)\left(Y_{nj} - Y_{qj}\right).$$

Solving for markups as a function of prices and wages,

$$\boldsymbol{\mu} = \Psi^{-1}\boldsymbol{p} - \Omega_{:\mathcal{F}}\boldsymbol{w},$$

yields

$$d\left(\Psi_{\mathcal{Z}\mathcal{F}}^{T}\right)\mathbf{s}=-\left[\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}+Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}},\Omega_{\mathcal{Z}:}\right)+\mathcal{X}\left(\Psi_{:\mathcal{F}},I\right)\right]\mathbf{p}-diag(\bar{\boldsymbol{\Psi}}_{\mathcal{F}})\boldsymbol{w}+s_{\mathcal{H}}Cov_{\boldsymbol{s}_{\mathcal{H}}}\left(\Psi_{:\mathcal{F}}^{T},\boldsymbol{\xi}\right)$$

and hence

$$\begin{aligned}
d\bar{\boldsymbol{\Psi}}_{\mathcal{F}}&=diag(\bar{\boldsymbol{\Psi}}_{\mathcal{F}})\left(\boldsymbol{w}+\boldsymbol{\ell}-\mathbf{1}y\right)-\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}\boldsymbol{p}\\
&=-\left[\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}+Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}},\Omega_{\mathcal{Z}:}\right)+\mathcal{X}\left(\Psi_{:\mathcal{F}},I\right)\right]\mathbf{p}-diag(\bar{\boldsymbol{\Psi}}_{\mathcal{F}})\boldsymbol{w}+\Psi_{\mathcal{Z}\mathcal{F}}^{T}d\boldsymbol{s}+s_{\mathcal{H}}Cov_{\boldsymbol{s}_{\mathcal{H}}}\left(\Psi_{:\mathcal{F}}^{T},\boldsymbol{\xi}\right)
\end{aligned}$$

so that

$$diag(\bar{\boldsymbol{\Psi}}_{\mathcal{F}})\left(\boldsymbol{\ell}-\mathbf{1}y\right)=-\left[Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}},\Omega_{\mathcal{Z}:}\right)+\mathcal{X}\left(\Psi_{:\mathcal{F}},I\right)\right]\mathbf{p}+\Psi_{\mathcal{Z}\mathcal{F}}^{T}d\boldsymbol{s}+s_{\mathcal{H}}Cov_{\boldsymbol{s}_{\mathcal{H}}}\left(\Psi_{:\mathcal{F}}^{T},\boldsymbol{\xi}\right)$$

Moreover we have

$$\Psi_{\mathcal{Z}\mathcal{F}}^{T}d\boldsymbol{s}=Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^{T},d\log\boldsymbol{s}\right)$$

and

$$d\log\boldsymbol{s}=\Xi\left(\boldsymbol{\ell}-\mathbf{1}y\right)+\left[\Xi\Psi^{-1}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{Z}}-\left(\Gamma^{-1}-I\right)\begin{pmatrix}\Omega_{\mathcal{H}:}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:}\\ \mathbb{O}\end{pmatrix}\right]\boldsymbol{p}$$

which yields

$$\begin{aligned}
\boldsymbol{\ell}-\mathbf{1}y&=-\Theta\boldsymbol{p}+\boldsymbol{\mathcal{D}}_{\xi}\boldsymbol{\xi} &(31)\\
\Theta&\equiv\mathcal{D}_{\ell}\left[\mathcal{X}(\Psi_{:\mathcal{F}}^{T},I)+Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^{T},\begin{pmatrix}\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:}\\ \Omega_{\mathcal{U},:}\end{pmatrix}-\Xi\Psi^{-1}+\Gamma^{-1}\begin{pmatrix}\Omega_{\mathcal{H}:}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}:}\\ \mathbb{O}\end{pmatrix}\right)\right] &(32)\\
\mathcal{D}_{\xi}&\equiv\mathcal{D}_{\ell}Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^{T},I\right) &(33)\\
\mathcal{D}_{\ell}&\equiv diag(\bar{\boldsymbol{\Psi}}_{\mathcal{F}})^{-1}\left(I-Cov_{\boldsymbol{s}}\left(\Psi_{\mathcal{Z}\mathcal{F}}^{T},\Xi_{:\mathcal{F}}diag(\bar{\boldsymbol{\Psi}}_{\mathcal{F}})^{-1}\right)\right). &(34)
\end{aligned}$$

Proof of corollaries 4.1 and 4.2. Let's begin by providing a more detailed statement of corollary 4.1.

Corollary A.1. *Denote by $\bar{\boldsymbol{\kappa}}\equiv\kappa\mathbf{1}$ the incidence of a proportional increase in employment on industry-level prices, and by $\bar{\Phi}\equiv\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^{T}\Phi\mathbf{1}$, $\bar{\boldsymbol{\beta}}\equiv\beta\bar{\boldsymbol{\Psi}}_{\mathcal{F}}$, $\bar{\boldsymbol{\mathcal{P}}}\equiv\boldsymbol{s}^{T}\mathcal{P}_{C:}$, $\bar{\mathcal{P}}\equiv\boldsymbol{s}^{T}\mathcal{P}_{CL}\mathbf{1}$. The incidence κ_i on industry i is larger than the incidence on the GDP deflator ($\bar{\boldsymbol{\Omega}}_{C}\bar{\boldsymbol{\kappa}}$) if industry i (or its suppliers, its suppliers' suppliers, etc.) have more flexible prices ($\mathcal{P}_{iL}\mathbf{1}>\bar{\mathcal{P}}_{\mathcal{F}}$), or if industry i uses inelastically supplied primary factors more intensively ($\boldsymbol{\Psi}_{iL}\Phi\mathbf{1}>\bar{\Phi}$).*

Proof. Using the definition of the slope κ, we can compute

$$\begin{aligned}
\bar{\boldsymbol{\kappa}}-\mathbf{1}\left(\bar{\boldsymbol{\Omega}}_{C}\bar{\boldsymbol{\kappa}}\right)&=\left(\mathcal{P}_{:L}\mathbf{1}-\bar{\mathcal{P}}_{CL}\mathbf{1}\right)\left(\bar{\Phi}+\bar{\beta}^{T}\kappa\mathbf{1}\right)+\bar{\mathcal{P}}_{CL}\left(\Psi_{:L}\Phi\mathbf{1}-\bar{\Phi}\mathbf{1}\right)\\
&\quad+\left(\mathcal{P}_{:L}-\mathbf{1}\bar{\boldsymbol{\mathcal{P}}}_{CL}\right)\left(\beta^{T}\kappa\mathbf{1}-\left(\bar{\boldsymbol{\beta}}^{T}\kappa\mathbf{1}\right)\mathbf{1}\right)+\bar{\mathcal{P}}_{CL}\left(I-\mathbf{1}\bar{\boldsymbol{\Omega}}_{C}\right)\left(\frac{\mathcal{P}_{:L}}{\bar{\mathcal{P}}_{CL}}-\Psi_{:L}\right)\left(\Phi\mathbf{1}-\bar{\Phi}\mathbf{1}\right)
\end{aligned}$$

The terms in the first line are explained in the statement of the proposition. The second line is a covariance term, capturing the notion that increasing employment has an even larger price incidence on industries which face both more flexible prices and more inelastic factor supply. Likewise, increasing employment has a larger price incidence on industries i which are exposed to factors f ($\mathcal{P}_{if} > \bar{\mathcal{P}}_{Cf}$) whose wage deflators are more sensitive to aggregate employment ($\boldsymbol{\beta}_{f:}^T \kappa \mathbf{1} > \bar{\boldsymbol{\beta}}^T \kappa \mathbf{1}$). $\square$

Let's now turn to a more detailed statement of corollary 4.2.

Corollary A.2. *Denote the incidence of primary factors on the GDP deflator by* $\boldsymbol{\kappa}^Y \equiv \bar{\boldsymbol{\Omega}}_C \kappa$. *Factor incidences on the GDP deflator are proportional to factor income shares* ($\boldsymbol{\kappa}^Y \propto \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T$) *whenever (i) the factor price pass-through into the GDP deflator is proportional to factor income shares* ($\bar{\boldsymbol{\mathcal{P}}}_{\mathcal{F}} = \bar{\mathcal{P}} \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T$)*; (ii) factor supply elasticities are such that* $\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T \Phi = \bar{\Phi} \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T$*; and (iii) the aggregate factor price deflator is equal to the GDP deflator* ($\bar{\boldsymbol{\beta}}^T = \bar{\boldsymbol{\Omega}}_{\mathcal{Z}:}$)*. Otherwise, primary factors f that are inelastically supplied* ($\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T \Phi_{:f} > \bar{\Phi} \bar{\Psi}_f$)*, have high price pass-through* ($\bar{\mathcal{P}}_{\mathcal{Z}f} > \bar{\mathcal{P}} \bar{\Psi}_f$)*, or have a high incidence on factor price deflators* ($\left(\bar{\boldsymbol{\beta}}^T \kappa\right)_f > \left(\bar{\boldsymbol{\beta}}^T \kappa \mathbf{1}\right) \bar{\Psi}_f$)*, affect the aggregate GDP deflator more than proportionally to their income share.*

Proof. Using the definition of κ, we can compute

$$
\begin{aligned}
\boldsymbol{\kappa}^Y - \left(\boldsymbol{\kappa}^Y \mathbf{1}\right) \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T = & \left(\bar{\boldsymbol{\mathcal{P}}}_{CL} - \bar{\mathcal{P}}_{CL} \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right) \bar{\Phi} + \bar{\mathcal{P}}_{CL} \left[\left(\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T \Phi - \bar{\Phi} \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right) + \left(\bar{\boldsymbol{\beta}}^T \kappa - \left(\bar{\boldsymbol{\beta}}^T \kappa \mathbf{1}\right) \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\right] \\
& + \left(\bar{\boldsymbol{\mathcal{P}}}_{CL} - \bar{\mathcal{P}}_{CL} \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right) \left[\left(\Phi - \bar{\Phi} I\right) \left(I - \mathbf{1} \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right) + \beta^T \left(\kappa - (\kappa \mathbf{1}) \bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\right]
\end{aligned}
$$

The terms in the first line are explained in the statement of the proposition. The second line is a covariance term, capturing the notion that factors which are both inelastically supplied and have a high pass-through $\bar{\mathcal{P}}_f$ have an even higher incidence on the GDP deflator. Likewise, factors f' which have a higher incidence on the price deflators of factors f with high passthrough $\bar{\mathcal{P}}_f$, also have an even larger incidence on the GDP deflator. $\square$

Proof of Proposition 4.1. Start from the system

$$
\begin{cases}
\boldsymbol{p}_t = & \kappa \left(\boldsymbol{\ell}_t - \Phi^{-1} \Gamma \boldsymbol{\xi}_t\right) + (I - \mathcal{V}) \left(\boldsymbol{p}_{t-1} + \rho \mathbb{E} \boldsymbol{\pi}_{+1}\right) \\
\boldsymbol{\ell}_t = & \mathbf{1} \bar{y} - \mathcal{D}_p \boldsymbol{p}_t + \mathcal{D}_\xi \boldsymbol{\xi}_t
\end{cases}
$$

Combining the two equations above allows us to express employment and inflation responses as a function of aggregate real GDP ($\bar{y}$) and income shifts $\boldsymbol{\xi}$. Further imposing the budget constraint (11) allows us to derive the response of consumption:

$$
\begin{aligned}
\boldsymbol{\pi} &= (I+\kappa\mathcal{D}_p)^{-1}\left[\kappa\left[\mathbf{1}\bar{y}+\left(\mathcal{D}_\xi+\Phi^{-1}\Gamma\right)\boldsymbol{\xi}\right]-(\mathcal{V}+\kappa\mathcal{D}_p)\,\boldsymbol{p}_{t-1}+(I-\mathcal{V})\,\rho\mathbb{E}\boldsymbol{\pi}_{+1}\right]\\
\boldsymbol{\ell} &= (I+\mathcal{D}_p\kappa)^{-1}\left[\mathbf{1}\bar{y}+\left(\mathcal{D}_\xi-\mathcal{D}_p\kappa\Phi^{-1}\Gamma\right)\boldsymbol{\xi}+\mathcal{D}_p\,(I-\mathcal{V})\left(\boldsymbol{p}_{t-1}+\rho\mathbb{E}\boldsymbol{\pi}_{+1}\right)\right]\\
\boldsymbol{c} &= \left[\Xi_{:L}+\left(\Xi\Psi^{-1}-\Omega_{C:}\right)\kappa\right](I+\mathcal{D}_p\kappa)^{-1}\left(\mathbf{1}\bar{y}+\left(\mathcal{D}_\xi+\Phi^{-1}\Gamma\right)\boldsymbol{\xi}\right)+\left(I+\Xi_{:L}\Phi^{-1}\Gamma\right)\boldsymbol{\xi}\\
&\quad+\left[\Xi\Psi^{-1}-\Omega_{C:}-\Xi_{:L}\mathcal{D}_p\right](I+\kappa\mathcal{D}_p)^{-1}(I-\mathcal{V})\left(\boldsymbol{p}_{t-1}+\rho\mathbb{E}\boldsymbol{\pi}_{+1}\right).
\end{aligned}
$$

Neutral fiscal policies

Proof of Proposition 5.1. Consider a setting where supply elasticities are such that $\Phi\mathbf{1}=\hat{\Phi}\mathbf{1}$. As a first step, re-write the equilibrium equations so as to allow for proportional cost subsidies ($\boldsymbol{\tau}_c$) and sales taxes ($\boldsymbol{\tau}_s$), financed through a lump-sum tax $\Xi\left(\boldsymbol{\tau}_c-\boldsymbol{\tau}_s\right)$ proportional to the relevant ownership shares. The equilibrium system becomes

$$
\begin{cases}
\boldsymbol{w}_t=\Phi\boldsymbol{\ell}_t+\beta^T\boldsymbol{p}_t+\Gamma\boldsymbol{\xi}\\
\boldsymbol{p}_t=\mathcal{P}_{:L}\boldsymbol{w}_t+\mathcal{P}\left(\Delta^{-1}\boldsymbol{\tau}_{s,t}-\boldsymbol{\tau}_{c,t}\right)+\left(I-\mathcal{P}\Psi^{-1}\right)\left(\boldsymbol{p}_{t-1}+\rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)-\left(I-\mathcal{P}\Psi^{-1}\right)\left[\boldsymbol{\tau}_{s,t-1}+\rho\mathbb{E}\left(\boldsymbol{\tau}_{s,t+1}-\boldsymbol{\tau}_{s,t}\right)\right].\\
\boldsymbol{\ell}_t=\mathbf{1}\bar{y}_t-\mathcal{D}_p\mathbf{p}_t+\mathcal{D}_{\boldsymbol{\xi}}\boldsymbol{\xi}+\mathcal{D}_{\boldsymbol{\tau}}\left(\boldsymbol{\tau}_{c,t}-\boldsymbol{\tau}_{s,t}\right)
\end{cases}
\tag{35}
$$

where we defined

$$
\mathcal{D}_\tau\equiv -diag\left(\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\right)^{-1}(I-\mathcal{D}_\ell)^{-1}\left(\Psi_{:L}^T-\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\mathbf{1}^T\right)diag\left(\bar{\boldsymbol{\Psi}}\right).
$$

Noting that

$$
\Psi_{:L}^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Omega_{:\mathcal{F}}\mathbf{1}=\Psi_{:L}^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Psi^{-1}\mathbf{1}=\left(\bar{\boldsymbol{\Omega}}_C\Psi_{:L}\right)^T=\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\mathbf{1}^T\bar{\boldsymbol{\Psi}}_{\mathcal{F}}=\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\mathbf{1}^T diag\left(\bar{\boldsymbol{\Psi}}\right)\Omega_{:\mathcal{F}}\mathbf{1},
$$

we can then set

$$
\boldsymbol{\tau}_c-\boldsymbol{\tau}_s\propto\Omega_{:\mathcal{F}}\mathbf{1}
$$

$$
\boldsymbol{\xi}=\mathbf{0}
$$

to obtain

$$
\begin{cases}
\boldsymbol{w}_t=\Phi\boldsymbol{\ell}_t+\beta^T\boldsymbol{p}_t\\
\boldsymbol{p}_t=\mathcal{P}_{:L}\boldsymbol{w}_t+\mathcal{P}\left(\Delta^{-1}\boldsymbol{\tau}_{st}-\boldsymbol{\tau}_{ct}\right)+\left(I-\mathcal{P}\Psi^{-1}\right)\left(\boldsymbol{p}_{t-1}+\rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right)-\left(I-\mathcal{P}\Psi^{-1}\right)\left[\boldsymbol{\tau}_{s,t-1}+\rho\mathbb{E}\left(\boldsymbol{\tau}_{s,t+1}-\boldsymbol{\tau}_{s,t}\right)\right]\\
\boldsymbol{\ell}_t=\mathbf{1}\bar{y}_t-\mathcal{D}_p\mathbf{p}_t.
\end{cases}
$$

Combining the first two equations in the system above yields

$$\begin{cases} \boldsymbol{p}_t = \left(I - \mathcal{P}_{:L}\beta^T\right)^{-1} & \left\{\mathcal{P}\left(\Omega_{:,\mathcal{F}}\Phi\boldsymbol{\ell}_t + \Delta^{-1}\boldsymbol{\tau}_{st} - \boldsymbol{\tau}_{ct}\right) + \left(I - \mathcal{P}\Psi^{-1}\right)\left(\boldsymbol{p}_{t-1} + \rho\mathbb{E}\boldsymbol{\pi}_{t+1}\right) - \right. \\ & \left.\left(I - \mathcal{P}\Psi^{-1}\right)\left[\boldsymbol{\tau}_{s,t-1} + \rho\mathbb{E}\left(\boldsymbol{\tau}_{s,t+1} - \boldsymbol{\tau}_{s,t}\right)\right]\right\} \\ \boldsymbol{\ell}_t = \mathbf{1}\bar{y}_t - \mathcal{D}_p\mathbf{p}_t. \end{cases}$$

Hence imposing

$$\boldsymbol{\tau}_{ct} - \Delta^{-1}\boldsymbol{\tau}_{st} = \Omega_{:\mathcal{F}}\mathbf{1}\hat{\Phi}\bar{y}_t$$

is consistent with $\boldsymbol{\ell}_t = \mathbf{1}\bar{y}_t$, $\boldsymbol{c}_t = \mathbf{1}\bar{y}_t$, and $\boldsymbol{p}_t = \mathbf{0}$. In particular, we can set $\boldsymbol{\tau}_s = \mathbf{0}$ and

$$\boldsymbol{\tau}_{ct} = \Omega_{:\mathcal{F}}\mathbf{1}\hat{\Phi}\bar{y}_t,$$

implying an aggregate cost subsidy equal to $\hat{\Phi}\bar{y}_t$. The path of consumption and prices satisfies all Euler equations, given an appropriately set common nominal rate.

Proof of Proposition 5.2. Let's now allow for $\Phi\mathbf{1} \neq \hat{\Phi}\mathbf{1}$, and set up a planner's problem which maximizes equally-weighted household utility subject to a given path of aggregate real GDP:

$$\begin{aligned} \max\sum_{t=0}^{\infty} e^{\rho t}\left[\sum_{h\in\mathcal{H}}\left[\frac{C_{ht}^{1-\gamma}}{1-\gamma} - \sum_{o\in\mathcal{L}}\frac{L_{hot}^{1+\varphi_o}}{1+\varphi_o}\right]\right] \text{ s.t } & \sum_{h\in\mathcal{Z}} C_{hit} + \sum_{j\in\mathcal{N}} X_{jit} = F_i\left(\{L_{ift}\}_{f\in\mathcal{F}}, \{X_{ijt}\}_{j\in\mathcal{N}}\right) \ \forall i \in \mathcal{N} \\ & \sum_i L_{ift} = \left[(1+\varphi_f) C_{ft}\right]^{\frac{1}{1+\varphi_f}} \bar{L}_f \ \forall f \in \mathcal{K} \\ & \sum_i L_{iot} = \sum_{h\in\mathcal{H}} L_{hot} \ \forall o \in \mathcal{L} \\ & \sum_{h\in\mathcal{Z}} \left(P_h^C\right)^* C_{ht} = \bar{P}^*\bar{Y}_t \end{aligned}$$

Denote the multipliers on the four sets of constraints by $\{\lambda_t\zeta_{1it}\}_{i\in\mathcal{N}}$, $\{\lambda_t\zeta_{2ft}\}_{f\in\mathcal{K}}$, $\{\lambda_t\zeta_{2ot}\}_{o\in\mathcal{L}}$, and $\lambda_t\zeta_{3t}$. The first order conditions for this problem are

$$\begin{aligned} \left(e^{\rho t}C_{ht}^{-\gamma} + \left(P_h^C\right)^*\lambda_t\zeta_{3t}\right)\frac{\partial C_h}{\partial C_{hi}} &= \lambda_t\zeta_{1it} \qquad (36) \\ \zeta_{1jt}\frac{\partial F_j}{\partial X_{ji}} &= \zeta_{1it} \\ \zeta_{1jt}\frac{\partial F_j}{\partial L_{jf}} &= \zeta_{2ft} \\ e^{\rho t}L_{hot}^{\varphi_o} &= \lambda_t\zeta_{2ot}, \ h \in \mathcal{H}, o \in \mathcal{L} \\ \frac{\zeta_{1jt}}{\frac{\partial C_f}{\partial C_{fj}}}U_{ft}^{\varphi_f} + \left(P_f^C\right)^*\zeta_{3t} &= \zeta_{2ft}, \ f \in \mathcal{K} \end{aligned}$$

Log-linearizing and rearranging the first order conditions yields

$$\begin{aligned}\gamma\left(\boldsymbol{c}_{\mathcal{H},t+1}-\boldsymbol{c}_{\mathcal{H}t}\right) &= \mathbf{1}\left(\hat{i}_t-\rho\right)-\Omega_{\mathcal{H},:}\hat{\boldsymbol{\pi}}_{t+1} \\ \hat{\boldsymbol{p}} &= \Psi_{:L}\hat{\boldsymbol{w}}-\mathbf{1}d\zeta_{3t} \\ \boldsymbol{\ell}_t &= \mathbf{1}\bar{y}_t-\mathcal{D}_p\Psi_{:L}\hat{\boldsymbol{w}} \\ \hat{\boldsymbol{w}} &= \Phi\boldsymbol{\ell}_t+\beta^T\hat{\boldsymbol{p}}\end{aligned} \tag{37}$$

and

$$\hat{\boldsymbol{\xi}} \equiv \hat{\boldsymbol{p}}_{\mathcal{H}t}+\boldsymbol{c}_{\mathcal{H}t}-\mathbf{1}\left(\bar{c}_{\mathcal{H}t}+\hat{\bar{p}}_{\mathcal{H}t}\right)-\left(\Gamma^{-1}-I\right)\left(\Omega_{\mathcal{H}:}-\mathbf{1}\bar{\boldsymbol{\Omega}}_{\mathcal{H}}\right)\hat{\boldsymbol{p}}_t=\mathbf{0}\ \forall t$$

where we used the result $\Omega_{ij}\equiv\frac{\zeta_{1j}^*X_{ij}^*}{Y_i^*}$ and defined

$$\begin{aligned}\hat{\boldsymbol{p}}_t &\equiv d\log\boldsymbol{\zeta}_{1t} \\ \hat{\boldsymbol{w}}_t &\equiv d\log\boldsymbol{\zeta}_{2t}+\mathbf{1}d\zeta_{3t} \\ \hat{i}_t &\equiv \left(\lambda_t-\lambda_{t+1}\right)-\left(\zeta_{3t}-\zeta_{3t+1}\right).\end{aligned}$$

Denoting by

$$\mathcal{M}\equiv\left(I-\left(\beta^T\Psi_{:L}-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)+\Phi\mathcal{D}_p\Psi_{:L}\right)^{-1}\Phi,$$

factor suppy and relative factor demand imply

$$d\zeta_{3t}=\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\mathcal{M}\mathbf{1}\bar{y}_t\equiv\mathcal{C}\bar{y}_t.$$

The conditions (37) tell us that, in the planner's solution, households are on their Euler equations, good prices and relative demand are undistorted given factor prices, and there is a uniform subsidy to factor prices. Furthermore, a set of Backus-Smith conditions holds (the first equation in (36)), implying that the planner fully insures households against the policy change. Implementing the planner's allocation still requires a proportional subsidy to factor prices, like in proposition 5.1. In addition, in order to keep producer prices constant and purchaser prices undistorted given factor prices, the planner must implement adjustments to the cost subsidies $\boldsymbol{\tau}_c$ and additional sales taxes $\boldsymbol{\tau}_s$. The optimal fiscal policies are given by cost subsidies $\boldsymbol{\tau}_{ct}^{\mathcal{F}}=\mathcal{C}\bar{y}_t$ on factor marketplaces, and cost subsidies $\boldsymbol{\tau}_{ct}^{\mathcal{N}}=\left(\Psi_{:L}-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\mathcal{M}\mathbf{1}\bar{y}_t$ paired with offsetting sales taxes $\boldsymbol{\tau}_{st}^{\mathcal{N}}=\left(\Psi_{:L}-\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\right)\mathcal{M}\mathbf{1}\bar{y}_t$ on good producing industries. The fiscal scheme must be financed via lump-sum taxes $\mathcal{C}\bar{y}_t$ on the profits of factor marketplaces (or equivalently on households, in proportion to their ownership shares in primary factors).

Finally, let's consider how to implement changes in factor supply proportional to factor-specific supply elasticities ($\Phi\boldsymbol{\ell}=\frac{\mathbf{1}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\Phi^{-1}\mathbf{1}}\bar{y}$). One possible policy choice is a factor cost subsidy $\boldsymbol{\tau}_c$. The employment changes would imply wage and price changes

$$\boldsymbol{w} = \frac{\mathbf{1}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\Phi^{-1}1}\bar{y} + \beta^T\boldsymbol{p}$$
$$\boldsymbol{p} = \left(I - \mathcal{P}_{:\mathcal{F}}\beta^T\right)\left(\frac{\mathcal{P}_{:\mathcal{F}}\mathbf{1}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\Phi^{-1}\mathbf{1}}\bar{y} + \mathcal{P}_{:\mathcal{F}}\boldsymbol{\tau}_c\right).$$

Through the relative demand equation, price and employment changes are consistent if and only if

$$\Theta\kappa\Phi^{-1}\boldsymbol{\tau}_c = \left(I - \frac{(I + \Theta\kappa)\,\boldsymbol{\Phi}^{\mathbf{-1}}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\Phi^{-1}1}\right)\mathbf{1}\bar{y},$$

implying that relative subsidies must satisfy

$$\left(I - \mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T\Phi^{-1}\right)\boldsymbol{\tau}_c = \Phi\left(\mathbf{1}\bar{\boldsymbol{\Psi}}_{\mathcal{F}}^T + \Theta\kappa\right)^{-1}\left(I - \frac{(I + \Theta\kappa)\,\boldsymbol{\Phi}^{\mathbf{-1}}}{\bar{\boldsymbol{\Psi}}_{\mathcal{F}}\Phi^{-1}1}\right)\mathbf{1}\bar{y}.$$

B Examples

Example B.1. Aggregating substitution across stages of production

This example generalizes the substitution elasticity from the two-factor model in Section 2 to show how micro-level substitution elasticities in production and consumption are combined to determine employment responses to relative price changes in a multi-sector economy.

Consider the economy depicted in Figure 8, with two sectors (1 and 2) and two primary factors (K and L).

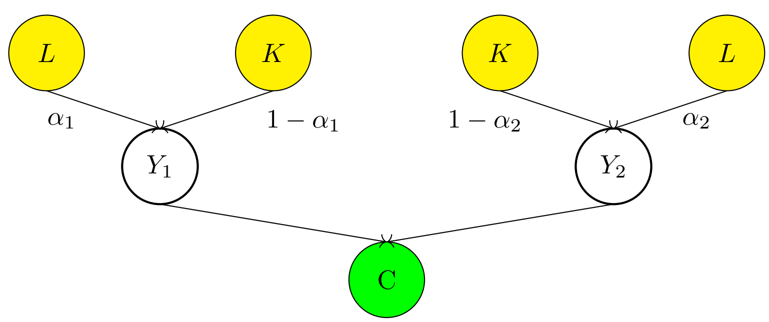

Figure 8: Two-good, two-workers economy

Both sectors use both primary factors but with different expenditure shares on factor L ($\alpha_1 \neq \alpha_2$). Both sectors have CES production functions, with elasticity of substitution θ. All final users have the same expenditure shares β_1 and $1 - \beta_1$ on goods 1 and 2, with substitution elasticity σ.

The relative demand equation allows us to map micro-level substitution elasticities θ (in production) and σ (in consumption) into an aggregate elasticity of substitution between L and K. Following equation (15c), the relative demand for L and K depends on relative factor prices and relative final good prices:

$$\ell_L - \ell_K = -\frac{\beta_1\alpha_1(1-\alpha_1) + (1-\beta_1)\alpha_2(1-\alpha_2)}{s_H(1-s_H)}\theta, \qquad (w_L - w_K) - \frac{\beta_1(1-\beta_1)(\alpha_1-\alpha_2)}{s_H(1-s_H)}\sigma(p_1 - p_2).$$

Using the pricing equation (15b) to express good prices as a function of factor prices, we can further rewrite[12]

$$\ell_L - \ell_K = -\bar{\theta}\left(w_L - w_K\right),$$

where the aggregate elasticity is

$$\bar{\theta} \equiv \left[1 - \frac{\beta_1\left(1-\beta_1\right)\left(\alpha_1-\alpha_2\right)^2}{s_L s_K}\right]\theta + \frac{\beta_1\left(1-\beta_1\right)\left(\alpha_1-\alpha_2\right)^2}{s_L s_K}\sigma\delta.$$

This generalizes the simple substitution elasticity from equation equation (1c) in Section 2, by showing how the aggregate elasticity $\bar{\theta}$ averages substitution elasticities in production and consumption, weighted by the extent to which factors are used differentially across sectors and the degree of price stickiness. As in the simple model, this substitution elasticity governs how aggregate demand fluctuations are transmitted into relative employment and relative labor income.

Example B.2. Home bias in consumption and relative demand

This example extends the demand analysis from Section 2 to demonstrate how heterogeneous consumption patterns across households affect the transmission of monetary policy through factor demand.

Consider the same production structure as Example B.1 but now set $\alpha_1 = 1$ and $\alpha_2 = 0$, label the two households and factors as H(ome) and F(oreign), and allow households to have different final expenditure shares β_H and β_F on good 1. Assume for simplicity that $\beta_H = 1 - \beta_F$, so that the households have equal income shares. There is home bias in consumption whenever $\beta_H > \frac{1}{2}$. Both households have the same elasticity of substitution σ between goods.

Following equation (15c), the relative demand for H and F is given by[13]

$$\ell_H - \ell_F = 2\left[\left(\beta_H - \frac{1}{2}\right)\left(p_H - p_F\right) - \sigma\beta_H\left(p_1 - p_2\right)\right].$$

Using the pricing equation (15b) to express good prices as a function of factor prices, we can further rewrite

$$\ell_H - \ell_F = -\bar{\theta}\left(p_H - p_F\right),$$

where the aggregate elasticity becomes:

$$\bar{\theta} \equiv 2\beta_H\sigma\delta - \left(2\beta_H - 1\right).$$

This generalizes the simple model from section 2, by introducing home bias in consumption ($\beta_H > \frac{1}{2}$). This partially offsets expenditure switching, reducing the responsiveness of relative factor demand to relative price changes. Like in section 2, the substitution elasticity $\bar{\theta}$ determines how employment and factor prices respond to changes in

[12]For simplicity, we assumed that the slope of the real marginal cost Phillips curve is the same for both consumption goods, denoted by δ.
[13]For simplicity, we assumed that each household owns the final sectors in proportion to their consumption shares.

aggregate demand.

Example B.3. Upstream workers have flatter supply curves

This example extends the supply slope analysis from Section 2 to demonstrate how the position of different labor occupations along production chains affects their supply slopes and, consequently, their employment responses to monetary policy.

Consider an economy with two sectors (I and F, standing for intermediate and final goods) and two households, as depicted in Figure 9. Household 1 works in the intermediate good sector, while household 2 works in the final good sector. Wages are flexible, and both sectors have the same price adjustment probability δ. The intermediate good sector uses only labor in production, while the final good sector uses labor and intermediate goods, with expenditure shares α_F and $1 - \alpha_F$ and elasticity of substitution θ. Both households have GHH preferences ($\gamma = 0$), with the same inverse Frisch elasticity of labor supply φ.

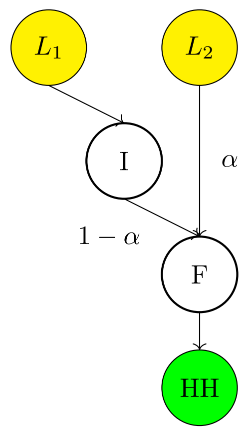

Figure 9: Two-stage vertical chain

Following equations (1c) and (15b), a proportional change in employment has a larger incidence on the wages of final good workers than on intermediate good prices, because intermediate good prices are sticky and do not fully pass through changes in the wage of intermediate good workers:

$$w_2 - p_I \,|_{\ell_1=\ell_2} \propto \varphi \frac{1-\delta}{1-\bar{\bar{\delta}}},$$
$$\bar{\bar{\delta}} \equiv \delta \left[(1-\alpha_F)\,\delta + \alpha_F \right].$$

This generalizes the supply slope concept from Section 2 by showing how input-output linkages affect supply slopes along production chains. The upstream worker (household 1) has a flatter supply slope because the sticky intermediate good prices prevent full pass-through of wage changes. Recalling the insights from section 2, this means that upstream workers experience larger employment increases relative to downstream workers following an aggregate demand expansion, because changes in their wage are not fully passed through into intermediate good prices, making their products relatively cheaper.

Example B.4. Non-labor shares in value added

This example extends section 2, to illustrate how complementarity with non-labor factors affects supply slopes and employment responses to monetary policy.

Consider the economy in Figure 10. The economy has two industries – manufacturing and services – which pull workers from separate labor markets. Both industries combine labor with an industry-specific non-labor factor, with substitution elasticity θ between the two. Services are more labor intensive ($\alpha_M < \alpha_S$). Households consume both goods, with identical preferences across labor occupation and substitution elasticity σ. Labor is more elastically supplied than non-labor assets, with inverse elasticity $\varphi_L < \varphi_K$. All factor prices are flexible, while final good prices are sticky, with real marginal cost Phillips curve slope δ in both industries.

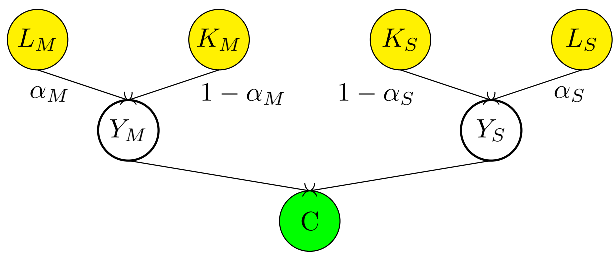

Figure 10: Manufacturing and Services

Following equation (18), a uniform increase in the employment of all factors would raise manufacturing prices by more:

$$(\boldsymbol{\kappa}_{M:} - \boldsymbol{\kappa}_{S:})\,\mathbf{1} = \frac{\delta}{1-\delta}\left(\alpha_S - \alpha_M\right)\left(\varphi_K - \varphi_L\right) > 0$$

Industries with higher non-labor shares (like manufacturing) have steeper supply curves. Recalling our discussion in section 2, an aggregate demand expansion raises the relative price and lowers the relative output of these industries. Employment responses depend on the differece between substitution elasticities in production and in consumption ($\theta - \sigma\delta$):

$$\ell_M - \ell_S \propto \left(\theta - \sigma\delta\right)\left(\alpha_S - \alpha_M\right)\left(\varphi_K - \varphi_L\right).$$

When labor and non-labor assets are more substitutable than manufacturing and services, an aggregate demand expansion raises manufacturing employment relatively more, as manufacturing producers substitute away from non-labor assets and toward labor. If instead labor and non-labor assets are more complementary than manufacturing and services, an aggregate demand expansion reduces relative manufacturing employment, as consumers substitute toward services.

Example B.5. Cross-sectional and Aggregate Non-Neutrality

This example below specializes equation (21) to the simple economy introduced in section 2. Like in the other examples, I assume $\rho = 0$ and $\mathbf{p}_{t-1} = \mathbf{0}$.

Consider the two-worker economy in section 2, with sticky wages and identical labor supply elasticities. Compared to an economy where both workers have the same wage rigidity, equal to the average $\bar{\delta}$, expenditure switching toward the worker with a stickier wage increases the aggregate effects of monetary policy. Formally, the slope $\bar{\boldsymbol{\kappa}}$ is given by

$$\bar{\kappa} = \frac{\bar{\delta}}{1-\bar{\delta}}\varphi\left[1 - \frac{1}{4}\frac{\varphi\theta\bar{\delta}}{1+\varphi\theta\bar{\delta}}\left(\frac{\delta_{flex} - \delta_{sticky}}{\bar{\delta}}\right)^2\right].$$

In turn, the relative response of aggregate real GDP vs aggregate nominal GDP is given by

$$\frac{1}{1+\varphi\frac{\bar{\delta}}{1-\bar{\delta}}\left[1 - \frac{1}{4}\frac{\varphi\theta\bar{\delta}}{1+\varphi\theta\bar{\delta}}\left(\frac{\delta_{flex}-\delta_{sticky}}{\bar{\delta}}\right)^2\right]} > \frac{1}{1+\varphi\frac{\bar{\delta}}{1-\bar{\delta}}}.$$

Real GDP is increasing in the elasticity of substitution θ, which governs the ability to reallocate demand toward the sticky-wage worker. Heterogeneity becomes irrelevant only when the two workers are perfect complements ($\theta = 0$).

Assume now that the two workers face the same wage adjustment probability but different labor supply elasticities $\varphi_E < \varphi_I$. The response of the GDP deflator to changes in real GDP $\bar{y}$ is given by

$$\bar{\kappa} = \frac{\delta}{1-\delta}\bar{\varphi}\left[1 - \frac{1}{4}\frac{\theta\delta\bar{\varphi}}{1+\delta\theta\bar{\varphi}}\left(\frac{\varphi_E - \varphi_I}{\bar{\varphi}}\right)^2\right],$$

while the relative response of aggregate real GDP vs aggregate nominal GDP is given by:

$$\frac{1}{1+\bar{\varphi}\frac{\delta}{1-\delta}\left[1 - \frac{\bar{\varphi}\theta\delta}{1+\bar{\varphi}\theta\delta}\left(\frac{\varphi_E-\varphi_I}{\bar{\varphi}}\right)^2\right]} > \frac{1}{1+\bar{\varphi}\frac{\delta}{1-\delta}}.$$

Similar to example B.5, the response of real GDP is larger when factors are more substitutable, because consumers can shift expenditure toward the cheaper factor. Again, heterogeneity becomes irrelevant only when factors are perfectly complementary ($\theta = 0$).

C Data Construction and Calibration Details

This appendix provides additional detail on the data sources and methodology used to calibrate the model. Section C.1 describes the input-output data. Section C.2 describes the construction of non-labor factor expenditure shares. Section C.3 documents the construction of occupation-by-industry compensation shares from OEWS data, including the top-tail adjustment based on WID income distributions. Section C.4 documents the mapping of job tenure and self-employment data to custom occupation groups. Section C.5 describes the power law fit used to map labor income shares to factor income shares. Section C.6 documents household consumption shares.

C.1 Input-Output Tables and Industry Classification

The production network is constructed from the Bureau of Economic Analysis (BEA) annual input-output tables for the year 2019. These tables provide the expenditure shares of each industry on intermediate inputs from other industries (Ω), as well as value added by industry. I use the 71-industry classification.

C.2 Non-Labor Factors

Expenditure shares on non-labor factors follow the methodology of Vom Lehn and Winberry (2021). The flow expenditure for each asset-industry pair is computed as:

$$RK_{i,a,t} = r_{a,t} \cdot k_{i,a,t} \tag{38}$$

where $r_{a,t}$ is the rental rate for asset a (from Vom Lehn and Winberry 2021) and $k_{i,a,t}$ is the capital stock of industry i in asset a.

Private capital stocks are obtained from the NIPA data on the Net Stock of Private Nonresidential Fixed Assets, which provides estimates for 96 asset categories across BLS industries. These are aggregated to 31 asset categories using a crosswalk.

Government capital stock data come from BEA Table 7.1, which reports stocks for 17 asset types. Since these data do not distinguish between general government and government enterprises, I allocate the total stock based on the share of output from general government versus government enterprises within each government subsector.

The model includes asset utilization producers who combine fixed asset endowments with variable utilization inputs to supply asset services. The composition of utilization bundles—describing which industries' goods are used to maintain and operate each asset type—is constructed from the BEA Capital Flow Tables. These tables record investment flows by asset type and purchasing industry, which I interpret as reflecting the industry composition of goods required for asset utilization. Specifically, the Capital Flow Tables provide the share of each asset type purchased from each producing industry, which determines the expenditure shares Ω^f_{fi} in the utilization bundle for asset $f \in \mathcal{K}$. This ensures that the price index for asset utilization P^C_f appropriately reflects the cost of goods used to operate each type of capital.

C.3 Occupation × Industry Compensation Shares

Data Sources. Occupation-level employment and wage data are obtained from the Occupational Employment and Wage Statistics (OEWS) survey, May 2019 release, published annually by the Bureau of Labor Statistics.[14] The OEWS provides annual earnings per-capita and employment headcount by detailed industry-occupation pairs

[14] Industry-occupation data available at https://www.bls.gov/oes/special-requests/oesm19in4.zip; national data at https://www.bls.gov/oes/special-requests/oesm19nat.zip

across all nonfarm industries. I aggregate detailed SOC occupation codes into 30 custom occupation groups and map 4-digit NAICS industries to 71 BEA industry codes.

OEWS has three important limitations: (i) wages are top-coded at approximately $200,000, compressing the upper tail of the wage distribution; (ii) OEWS captures base wages and salaries but excludes bonuses, stock options, equity compensation, and partnership income—sources that are particularly important for high earners; (iii) farm employment is largely excluded. To address the first two limitations, I use data from the World Inequality Database (WID) to adjust for income that OEWS fails to capture.

Custom Occupation Groups. Rather than using the 22 standard SOC major groups directly, I classify detailed occupations into income tiers based on their average wages. The default thresholds for annual wages are $< \$35,000$ (low tier), $\$35,000$–$\$80,000$ (mid tier), $> \$80,000$ (high tier). Sales occupations use a custom $45,000 low/mid threshold to better separate retail workers from professional sales roles. The 22 base SOC major groups $\times$ 3 income tiers are then aggregated into 30 custom occupation groups based on economic similarity and industry concentration patterns.

Top-Tail Adjustment Methodology. To correct for missing top-tails, I adjust occupation-level wages using data on the true income distribution from the World Inequality Database (WID). The adjustment follows the steps below.

First, for each industry j, I compute the raw share of labor compensation going to occupation o:

$$s_{oj} = \frac{C_{oj}}{\sum_{o'} C_{o'j}}$$

where $C_{oj} = E_{oj} \cdot W_{oj}$ is total compensation (employment $\times$ average wage) for occupation o in industry j. The raw occupation $\times$ industry cost share matrix is then $\alpha_{oj}^{\text{raw}} = s_{oj} \cdot \alpha_j^L$, where α_j^L is industry j's labor share of total cost from the BEA input-output tables.

Second, I rank occupations by their average wage and compute each occupation's percentile position in the employment distribution. For occupation o with employment share η_o:

$$p_o = \sum_{o' : W_{o'} < W_o} \eta_{o'} + \frac{\eta_o}{2}$$

Third, I partition the income distribution into percentile bins matching the WID data (p0–p50, p50–p90, p90–p95, p95–p99, p99–p100) and compute the bin-level adjustment ratio:

$$r_b = \frac{\psi_b^{\text{WID}}}{\psi_b^{\text{OEWS}}}$$

where ψ_b^{WID} and ψ_b^{OEWS} are the labor income shares in bin b according to WID and OEWS respectively. For the top tail (p90+), I pool all top bins and compute a single adjustment ratio to avoid noise from small cell sizes.

Fourth, I build a continuous adjustment function by interpolating between bin midpoints, enforcing monotonicity (higher-earning occupations receive higher adjustments). Each occupation receives a multiplier $m_o = m(p_o)$, normalized so that the income-weighted average equals 1.

Finally, the adjusted compensation for occupation o in industry j is $C_{oj}^{\text{adj}} = C_{oj}^{\text{OEWS}} \cdot \tilde{m}_o$, yielding adjusted within-industry shares:

$$s_{oj}^{\text{adj}} = \frac{C_{oj}^{\text{adj}}}{\sum_{o'} C_{o'j}^{\text{adj}}}$$

By construction, $\sum_o \alpha_{oj}^{\text{adj}} = \alpha_j^L$ for each industry j.

C.4 Job Tenure and Self-Employment Data

BLS Tenure Data. I use median tenure by occupation from BLS Table 6, "Median years of tenure with current employer for employed wage and salary workers 16 years and over by occupation and sex," from the January 2018 Employee Tenure supplement to the CPS.[15] BLS publishes median tenure for the 22 SOC major groups. Table 8 reports the published values.

Table 8: BLS Median Tenure by SOC Major Group (January 2018)

SOC	Occupation Group	Median Tenure (years)
11	Management	6.4
13	Business and financial operations	4.5
15	Computer and mathematical	4.3
17	Architecture and engineering	5.7
19	Life, physical, and social science	4.8
21	Community and social service	4.6
23	Legal	5.1
25	Education, training, and library	5.1
27	Arts, design, entertainment, sports, media	3.9
29	Healthcare practitioners and technical	4.3
31	Healthcare support	3.0
33	Protective service	5.0
35	Food preparation and serving	1.9
37	Building and grounds cleaning/maintenance	4.1
39	Personal care and service	3.0
41	Sales and related	3.2
43	Office and administrative support	3.8
45	Farming, fishing, and forestry	4.0
47	Construction and extraction	4.2
49	Installation, maintenance, and repair	5.1
51	Production	4.4
53	Transportation and material moving	3.5

For each custom occupation group g, median tenure is computed as the employment-weighted average across constituent base occupations:

$$\text{tenure}_g = \frac{\sum_{o \in \mathcal{O}_g} \text{emp}_o \times \text{tenure}_o}{\sum_{o \in \mathcal{O}_g} \text{emp}_o}$$

[15] Available at https://www.bls.gov/news.release/tenure.t06.htm.

where $\mathcal{O}_g$ is the set of base occupations in custom group g. Groups defined via detailed SOC code filters rather than base occupation aggregation default to 4.5 years (the tenure for SOC 13, Business and Financial Operations).

CPS Self-Employment Analysis. I compute self-employment rates using Current Population Survey (CPS) microdata from IPUMS. The analysis uses a CPS extract containing basic monthly data for self-employment rates, with key variables OCC2010, CLASSWKR, and WTFIN.

Self-employment indicators are constructed using CLASSWKR codes: 10 for self-employed unincorporated, 13–14 for self-employed incorporated, and 20–28 for wage/salary workers.

Each CPS observation is matched to custom occupation groups via OCC2010 codes using a crosswalk constructed from SOC-to-OCC2010 mappings. Weighted statistics are computed using CPS sample weights. The resulting self-employment rates by occupation group inform the calibration of wage adjustment frequencies, as described in table 5.

C.5 WID Labor Income Distribution and Power Law Fit

Data Source and Construction. The WID data is accessed using US 2019 distributional national accounts.[16] I download labor income shares (ψ_g^L) and factor income shares (ψ_g^F) for each percentile group using adults aged 20+ with equal-split income between spouses. Factor income includes only labor income (wages, salaries, self-employment income) and capital income (interest, dividends, rents, capital gains), excluding government transfers.

For each percentile group g, I compute the labor share of that group's factor income:

$$\lambda_g = \frac{\psi_g^L}{\psi_g^F}$$

This ratio tells us what fraction of percentile group g's factor income comes from labor versus capital.

Power Law Fit for Ownership Shares. To map occupation-level labor income shares to factor income shares for the calibration of ownership shares, I estimate a power law relationship using WID data. Using per-capita labor income shares (ψ_g) and per-capita factor income shares (s_g) for non-overlapping percentile groups, I fit:

$$s_g = A \cdot \psi_g^B$$

via log-log OLS regression. The exponent B captures how factor income concentration differs from labor income concentration. When $B > 1$, factor income is more concentrated than labor income, reflecting that high labor-income groups have even higher factor-income shares due to capital income.

[16] Available at `https://wid.world/`

The estimated power law is used to construct ownership shares Ξ in the model calibration, mapping observed labor income shares by occupation group to factor income shares.

C.6 Consumption Shares

Household consumption shares by income quintile are constructed by combining BEA input-output tables with the Consumer Expenditure Survey (CEX). The CEX provides detailed expenditure patterns by occupation, which are then mapped to BEA commodity categories to construct occupation-specific consumption bundles.