

Security Issuance with Undersubscription Risk

David M. Frankel (Melbourne Business School)*

December 28, 2023

Abstract

A successful security offering raises more revenue for investments that will be used to pay distributions on the securities. This can give rise to a coordination problem among investors, which the issuer can overcome by underpricing the security. Debt minimizes the required underpricing and is thus optimal. Underpricing of equity can be mitigated by share rationing or a minimum sales requirement. The theory also yields several testable predictions and explains why the inclusion of secondary shares lessens underpricing.

JEL: G12, G14, G32.

Keywords: Security Design, Optimality of Debt, Equity, Initial Public Offerings, Security Issuances, Underpricing, Undersubscription, Oversubscription, Share Rationing, Minimum Sales Requirements, Strategic Uncertainty.

*200 Leicester Street, Carlton, VIC 3053, Australia. Email: d.frankel@mbs.edu. I thank Philip Bond, Peter DeMarzo, Darrell Duffie, Sivan Frenkel, Nisan Langberg, Deborah J. Lucas, Gideon Saar, Keke Song, Jeroen Swinkels, and participants at seminars at AMES China 2022, ESAM 2022, Hebrew U., Hitotsubashi U., Kyoto U., LACEA LAMES 2022, Seoul National U., Tel Aviv U., U. Melbourne, U. Monash, U. Miami, U. Washington, the 17th Annual WEAI Conference, and Yonsei U. for helpful comments.

1 Introduction

“Sometimes [an issuer in a public offering] ... is a comparatively new company, is making the ... offering to raise the capital necessary to begin or expand its activities, and the failure to receive it will substantially impair its ability to continue in business or to conduct necessary operations.” (U.S. Security and Exchange Commission [82])¹

Firms raise substantial capital for investments from issuing securities.² A successful offering will thus let an issuer invest more, leading to a higher future cash flow from which to pay distributions on the securities. This can give rise to a coordination problem among investors, in which it is worthwhile to subscribe only if enough others are expected to do so. We present a new theory of underpricing and security design based on this idea.

Our first result is that standard debt is optimal as it is the least underpriced security. Pecking order behavior à la Myers [64] thus results from an issuer’s desire to mitigate underpricing. This is consistent with the empirical evidence that debt is more common and less underpriced than equity.³

We next study equity offerings. These are often rationed, which leads to oversubscription. They may also have a minimum sales requirement (MSR): a subscription rate below which the issuance is withdrawn. Both features can mitigate underpricing in our setting, which helps explain their prevalence.

Our theory suggests a new explanation for why the inclusion of secondary shares in an IPO mitigates underpricing (Ang and Brau [6, Table 7]; Habib and Ljungqvist [46]).

¹This quote appears in the SEC’s announcement of rule 15c2-4, which penalizes intermediaries who fail to distribute the proceeds of an offering to the issuer.

²Kim and Weisbach [56] find that a dollar raised in an IPO leads to increases of 78¢ in R&D and 20¢ in capital expenditures over the succeeding four years. Firms that undergo successful IPOs also hire more workers in subsequent years (Babina, Ouimet, and Zarutskie [8]; Borisov, Ellul, and Sevilir [14]). IPOs raise substantial revenue, ranging from 29.3% to 52.5% of pre-IPO firm value (Spiess and Pettway [80, Table 2]; Brennan and Franks [16, Table 2]). Similarly, initial public debt offerings - a firm’s first public debt offering after an IPO - raised 19.7% of common equity in the sample of Datta, Iskandar-Datta, and Patel [27, Table II] and 42% in that of Burnie and Ogden [18, Table 1].

³Some relevant studies are discussed in section 5.2.

Intuitively, if the issuer receives a smaller proportion of the IPO proceeds, investors face less strategic risk: the subscription rate has a smaller effect on the value of their shares. A smaller discount thus suffices to induce them to subscribe.⁴

Our results also provide a new explanation of why underpricing is greater in best-effort IPOs than in firm-commitment offerings. In a firm-commitment IPO, the underwriter commits to buy all unsold shares. In our setting, such a commitment raises investors' willingness to pay by making issuance revenue less sensitive to the decisions of other investors.⁵ Indeed, Ritter [70, Table 4, p, 273] finds that underpricing in firm-commitment IPOs is 14.8%, versus 47.8% in best-effort IPOs.⁶

We study a setting in which the dependence of a firm's cash flow on issuance revenue gives rise to multiple equilibria. In this setting, we require a criterion to predict which equilibrium the investors will select. A common approach is to assume that the agents rely on a particular criterion the researcher chooses. We instead let the agents use any criterion from a large set, using a framework recently developed by Frankel [37]. Several robust empirical predictions emerge. We find, e.g., that a firm with better alternative funding sources will optimally choose a lower Minimum Sales Requirement in an IPO - a finding that has indirect support (Welch [84, Table 5]).⁷

⁴In practice, secondary shares make up a small portion of shares sold in IPOs: 14.5% in Habib and Ljungqvist [46, Table 1], 16.4% in Spiess and Pettway [80, Table 2], 29% in Ang and Brau [6, Table 1], and 36.2% in Huyghebaert and Van Hulle [51, Table 2]. This may be because their inclusion is interpreted as a negative signal (Ang and Brau [6]).

⁵In firm-commitment offerings, the offer price is typically chosen on the eve of the offering, well after most investors have indicated their degree of interest (Lowry, Michaely, and Volkova [60, pp. 219 ff.]). In this way, weak interest can reduce the offer price and thus the issuance revenue which, in our theory, would make the shares less valuable. Hence, investors in firm-commitment offerings may also face some uncertainty about issuance revenue, which could help explain why such offerings are also underpriced (though to a lesser extent).

⁶Booth and Chua [13, p. 299] argue that best-effort offerings tend to be smaller and thus require more underpricing to entice investors to gather costly information about the firm. However, the underpricing differential is evident *within* issuance size groups as well (Ritter [70, Table 4, p, 273]).

⁷For this and other empirical predictions, see the discussions around Claims 17 and 34.

1.1 Related Literature

Our finding that firms issue debt to minimize underpricing gives a new foundation for Myers' [64] pecking order hypothesis. The dominant foundation is based on asymmetric information; see, e.g., Biais and Mariotti [11], DeMarzo [31], DeMarzo and Duffie [32], DeMarzo, Frankel, and Jin [33], Leland and Pyle [59], Myers and Majluf [65], and Nachman and Noe [66]. However, these models also predict that any security sold in equilibrium will be fairly priced. They thus do not explain why, in practice, underpricing is larger for equity than for debt.

In our theory, underpricing results from the risk that undersubscription will deprive the issuer of needed capital. Relatedly, in Plantin [68], underpricing results from the risk that undersubscription will lead to an illiquid secondary market, while in Welch [85] the issuer underprices to encourage herding à la Scharfstein and Stein [77]. These studies also rely on a single selection criterion and do not consider security design.⁸

In Allen and Faulhaber [3], Grinblatt and Hwang [44], and Welch [83], a privately informed issuer may underprice to signal optimism, thus obtaining more favorable terms in subsequent SEOs.⁹ Two other theories of underpricing (Rock [75] and Benveniste and Spindt [10]) are discussed below in our section on share rationing (section 6.2).

We study the problem of raising capital for a project whose returns are used to repay investors. Moreover, we rely on equilibrium selection criteria to predict investor behavior. Both ingredients are also present in Allen *et al* [2], Chakraborty, Gervais, and Yilmaz [21] (CGY), Goldstein and Pauzner [43] (GP), and Halac, Kremer, and Winter [47] (HKW). However, these prior papers each use a single selection criterion and do not study security design.¹⁰

⁸More precisely, Plantin restricts to bonds and uses the Laplace criterion (defined in section 3). Welch studies stocks and obtains a unique equilibrium.

⁹Jegadeesh, Weinstein, and Welch [53] find weak support for signaling, while Michaely and Shaw [63] return a negative result.

¹⁰CGY use the Pareto criterion and restrict to portfolios of equity and warrants; HKW use the Unique Implementation criterion and study a setting (two cash flow realizations) in which debt and equity are

2 Base Model

We focus throughout on fixed-price, best-effort issuance mechanisms. Best-effort mechanisms are common in private placement offerings (Robbins [74, p. 2]) and were used in 35% of the 1,028 IPOs in Ritter’s dataset [70, p. 270]. Dunbar [35, p. 63] finds that fewer than 8% of best-effort IPOs have price revisions.¹¹

There are two periods: $t = 1, 2$. The players consist of a single issuer and a set I of *ex-ante* identical agents. The agents may be either discrete ($I = \{1, \dots, n\}$) or infinitesimal ($I = [0, 1]$). They collectively have some high but finite amount $\bar{p} > 0$ of wealth to invest, of which each agent has an equal share.¹² All players are risk-neutral and fully rational; there is no discounting.

The issuer owns a random cash flow $Y \geq 0$ that is realized in period 2. She can raise the distribution of this cash flow by selling claims to it in period 1 in return for capital $K \in [0, \bar{p}]$. The cash flow Y_κ that results from raising capital $K = \kappa$ is bounded and has the atomless distribution function¹³

$$H(y|\kappa) \stackrel{d}{=} \Pr(Y_\kappa \leq y) \tag{1}$$

with associated survivor function

$$\bar{H}(y|\kappa) \stackrel{d}{=} 1 - H(y|\kappa) = \Pr(Y_\kappa > y). \tag{2}$$

equivalent; and Allen *et al* and GP use the Laplace criterion and focus on demand deposit contracts. These criteria are defined below in section 3.

¹¹Moreover, in our setting the issuer knows *ex ante* how much the agents are willing to pay so no price revision is needed to ensure full subscription.

¹²The wealth constraint is assumed to be so high as to be nonbinding, except in section 6.2 where it may bind at the optimal scheme.

¹³The notation “ $\stackrel{d}{=}$ ” denotes a definition. Capital K is random since it depends on the agents’ subscription rate. The parameter κ is a generic realization of K .

Let

$$\underline{y}_\kappa = \max \left\{ \begin{array}{l} y \geq 0 : \\ H(y|\kappa) = 0 \end{array} \right\} \text{ and } \bar{y}_\kappa = \min \left\{ \begin{array}{l} y \geq 0 : \\ H(y|\kappa) = 1 \end{array} \right\} \quad (3)$$

denote the lower and upper bound, resp., on the support of Y_κ .¹⁴ As $H(\cdot|\kappa)$ is bounded and atomless,

$$0 \leq \underline{y}_\kappa \leq \bar{y}_\kappa < \infty. \quad (4)$$

We assume higher capital κ raises the cash flow distribution in the following sense.

Hazard Rate Ordering (HRO)

1. For any capital levels $\kappa' > \kappa$ both in $[0, \bar{p}]$, the ratio $\frac{\bar{H}(y|\kappa)}{\bar{H}(y|\kappa')}$ is decreasing in $y \in [\underline{y}_\kappa, \bar{y}_\kappa]$.
2. The bounds \underline{y}_κ and \bar{y}_κ on the conditional cash flow Y_κ are continuous and nondecreasing in $\kappa \in [0, \bar{p}]$.
3. The function $\bar{H}(y|\kappa)$ is Lipschitz-continuous in κ , uniformly in y : there is a finite constant λ_H such that for all $y \in [0, \bar{y}_{\bar{p}}]$ and $\kappa, \kappa' \geq 0$, $|\bar{H}(y|\kappa') - \bar{H}(y|\kappa)| \leq \lambda_H |\kappa' - \kappa|$.

Part 1 states that a rise in issuance revenue lowers the cash flow hazard rate: the probability that $Y = y$ conditional on $Y \geq y$.¹⁵ Part 2 states that a rise in issuance revenue does not *lower* the highest and lowest possible cash flows. Part 3 is a technical continuity property.

¹⁴Since Y_κ is bounded and atomless, $H(y|\kappa)$ is continuous in y and satisfies $H(0|\kappa) = 0$ and, for high enough y , $H(y|\kappa) = 1$. Thus, the definitions in (3) are well-defined.

¹⁵More precisely, if the density $\frac{\partial}{\partial y} H(y|\kappa)$ exists, then part 1 of HRO implies that for $\kappa' > \kappa$,

$$0 > \frac{\partial}{\partial y} \frac{\bar{H}(y|\kappa)}{\bar{H}(y|\kappa')} = \frac{\bar{H}(y|\kappa)}{\bar{H}(y|\kappa')} \left[\frac{\frac{\partial}{\partial y} H(y|\kappa')}{\bar{H}(y|\kappa')} - \frac{\frac{\partial}{\partial y} H(y|\kappa)}{\bar{H}(y|\kappa)} \right]. \quad (5)$$

That is, the cash flow hazard rate $\frac{\partial}{\partial y} H(y|\kappa) / \bar{H}(y|\kappa)$ is decreasing in issuance revenue κ . However, HRO is more general than (5) as it does not assume the density exists.

HRO is weaker than the monotone likelihood ratio property¹⁶ but stronger than another well-known property:

First Order Stochastic Dominance (FOSD) For any $\kappa' > \kappa$ both in $[0, \bar{p}]$ and $y \geq 0$, $\bar{H}(y|\kappa) \leq \bar{H}(y|\kappa')$ where the inequality is strict for $y \in (\underline{y}_\kappa, \bar{y}_{\kappa'})$.

Claim 1. *HRO implies FOSD.*

In period 1, the issuer may stay out (offer no security), raising zero capital but retaining full rights to her cash flow: her payoff is then $E[Y_0]$. Or she can offer a scheme $s = (p, S)$ where $p \in [0, \bar{p}]$ is the price per share and S is a monotone security:¹⁷

Definition 2. A security S is *monotone* if both $S(y)$ and $y - S(y)$ are nonnegative and nondecreasing in y .

Since a monotone security is nonnegative, the issuer never strictly prefers to offer a scheme with a zero price.¹⁸ We thus restrict attention to schemes (p, S) with positive prices:

$$p > 0. \tag{6}$$

On seeing the scheme (p, S) , the agents simultaneously decide whether or not to *subscribe*:

¹⁶See Shaked and Shanthikumar [78, theorem 1.C.1].

¹⁷A monotone security is one for which both the portion of the cash flow paid to investors and the portion retained by the firm are nonnegative and nondecreasing in the cash flow. Monotonicity is a common assumption in the security design literature; see, e.g., DeMarzo [31], DeMarzo and Duffie [32], DeMarzo, Frankel, and Jin [33], Frankel and Jin [39], Hart and Moore [49], and Nachman and Noe [66]. Examples of monotone securities include equity, standard debt, and warrants (call options). Monotonicity is justified by supposing the issuer has free disposal over her cash flow Y and can also contribute cash to inflate it. Hence, if her payout $Y - S(Y)$ were decreasing in Y , she would freely dispose of some cash in order to raise this payout. And if, alternatively, the payout $S(Y)$ to the agents were falling in Y , the issuer would contribute cash to inflate Y , thus paying the agents less and raising her payout $Y - S(Y)$ by more than the amount contributed.

¹⁸Her payoff is $E[Y_0 - S(Y_0)]$ from offering $(0, S)$ versus $E[Y_0]$ from staying out. (The notation Y_κ is defined in (1).) As $S \geq 0$, the former payoff is never higher than the latter.

to pay p for a share of the security. Subscribing incurs a fixed cost^{19,20}

$$c \geq 0, \tag{7}$$

which captures the time costs of due diligence, paperwork, and so on. Let $L \in [0, 1]$ denote the *aggregate subscription rate*: the proportion of agents who subscribe. If at least one agent subscribes but the fraction $1 - L$ who don't is positive, the firm sells the remaining $1 - L$ shares at a fixed discount $\rho \in (0, 1)$.^{21,22} Accordingly, total capital raised in the issuance is

$$K = Lp + (1 - L)(1 - \rho)p \in [0, p] \tag{8}$$

which equals the price p if all agents subscribe (if $L = 1$).

We will write a subscriber's payoff in terms of the *other-agent subscription rate* $\ell \in [0, 1]$: the proportion of other agents who subscribe. The relation between the aggregate and other-agent subscription rates is given by the function

$$L = \mathbf{L}(\ell) \stackrel{d}{=} \begin{cases} \ell & \text{if } I = [0, 1] \\ \frac{1+\ell(n-1)}{n} & \text{if } I = \{1, \dots, n\} \end{cases} \tag{9}$$

Using (8) and (9), we can express capital K as a function of ℓ :

$$K = p\mathbf{r}(\ell) \in [0, \bar{p}] \tag{10}$$

¹⁹If the agents are discrete, each subscriber incurs a cost c/n and pays the issuer p/n for $1/n$ shares, which entitle her to $S(Y)/n$. If they are infinitesimal ($I = [0, 1]$), any set of agents of measure $\varepsilon > 0$ that subscribes incurs the fixed cost $c\varepsilon$ and pays the issuer $p\varepsilon$, for which they receive ε shares which pay them $S(Y)\varepsilon$ in aggregate. (Each agent decides independently whether or not to subscribe.)

²⁰We show in section 6.3 that a positive cost c prevents MSR schemes from eliminating underpricing. In the two other variants of our model, the fixed cost c plays no essential role.

²¹Without the condition that at least one agent be willing to subscribe, an issuance would raise at least $(1 - \rho)p$ in revenue no matter how attractive it is. The issuer could thus raise infinite revenue by letting p go to infinity, which is absurd.

²²For instance, the unsold shares might be placed with a large investor who demands a discount based on a rule of thumb or who subsequently meddles in the firm's decisions, lowering its cash flow à la Burkart, Gromb, and Panunzi [17]. Alternatively, the firm may retain the unsold shares in its treasury and plan to offer them later to a different set of investors, with the discount capturing uncertainty about the price and timing of this sale. Importantly, no shares are sold at a discount *in equilibrium* as the issuer will either stay out or offer a scheme (p, S) that induces all agents to subscribe.

where we refer to

$$\mathbf{r}(\ell) \stackrel{d}{=} 1 - \rho + \rho \mathbf{L}(\ell) \quad (11)$$

as the *effective subscription rate*: the number of shares that would have to be sold at the price p to yield issuance revenue K . By inspection, $\mathbf{r}(\cdot)$ is an increasing function from $[0, 1]$ to $[1 - \rho + \iota, 1]$ where

$$\iota = \begin{cases} 0 & \text{if } I = [0, 1] \\ \frac{1}{n} & \text{if } I = \{1, \dots, n\} \end{cases} \quad (12)$$

is the measure of a single agent. By (10), an agent's expected payoff from subscribing to the scheme $s = (p, S)$, given the other-agent subscription rate ℓ , is²³

$$\pi_s(\ell) = E[S(Y_{p\mathbf{r}(\ell)})] - \theta(p + c) \quad (13)$$

where

$$\theta > 0 \quad (14)$$

is the gross risk-free interest rate. We assume

$$E[Y_0] > \theta c, \quad (15)$$

so the agents are willing to subscribe if offered a 100% equity stake at a zero price. By (6), (7), and (14), the agents' opportunity cost of subscribing is positive:

$$\theta(p + c) > 0. \quad (16)$$

3 Resolving Indeterminacy

The monotonicity of S combined with FOSD implies that the payoff function defined in (13) may be positive if and only if the other-agent subscription rate ℓ exceeds some threshold. This can give rise to multiple equilibria: if all other agents are expected (not) to subscribe,

²³By (1), $Y_{p\mathbf{r}(\ell)}$ is the random cash flow Y_κ that results from issuance revenue $\kappa = p\mathbf{r}(\ell)$.

it is a best response (not) to subscribe. For the issuer to choose an optimal scheme, she must therefore predict how the agents will select an equilibria: she must have a theory of equilibrium selection.

Frankel [37] shows that under mild single crossing properties, seven well-known equilibrium selection theories from the literature all lead to criteria of a common form:²⁴

$$\text{an agent (does not) subscribe if } \int_{\ell=0}^1 \pi_s(\ell) d\Gamma(\ell) > (<) 0 \quad (17)$$

for some distribution Γ that depends on the theory but not on π_s . Rather than true beliefs, the fictional beliefs Γ capture the intensity with which the choice of equilibrium depends on different segments of the payoff function π_s .

The selection theories and their associated fictional beliefs are as follows.

The Pareto theory is a heuristic argument that the agents will choose “all subscribe” if it is a strict Nash equilibrium. This theory gives rise to the criterion (17) with fictional beliefs Γ^* that put all of their weight on all others subscribing: $\Gamma^*(\ell)$ equals zero if $\ell < 1$ and one if $\ell = 1$.

The Unique-Implementation (UI) theory is a heuristic argument that the agents will choose “none subscribe” if it is a strict Nash equilibrium. This theory gives rise to the criterion (17) with fictional beliefs Γ^{UI} that put all of their weight on no others subscribing: $\Gamma^{\text{UI}}(\ell)$ equals one for all ℓ .

The MM theory (Matsui and Matsuyama [62]) is a continuous-time model in which the subscription game is played repeatedly by randomly chosen groups of n rational agents. The MM theory gives rise to the criterion (17) with fictional beliefs Γ^{Laplace} under which each other-agent subscription rate is equally likely.²⁵

²⁴This section presents an abridged version of Frankel [37], explaining only the concepts and findings that we will need. Complete results, intuitions, and proofs appear in Frankel [37].

²⁵That is, $\Gamma^{\text{Laplace}}(\ell)$ equals ℓ when agents are infinitesimal and $\sum_{i=0}^{\lfloor (n-1)\ell \rfloor} \frac{1}{n}$ when they are discrete. The role of the floor function is to ensure that the beliefs are defined for all $\ell \in [0, 1]$ and thus that the integral

The Global-Games (GG) theory is a static model in which each agent sees a slightly noisy signal of a common unobserved state. The GG theory gives rise to the criterion (17) with the same beliefs Γ^{Laplace} as the MM theory.²⁶

The KMR theory (Kandori, Mailath, and Rob [55]) is a discrete-time model in which random groups of n boundedly rational agents are selected, in each period, to play the subscription game. Agents are more likely to play actions that had higher payoffs in the prior period and each agent also has a small chance of “trembling”: of choosing the unintended action. The KMR theory gives rise to the criterion (17) with fictional beliefs $\Gamma_n^{\text{KMR}}(\ell)$ equal to $\sum_{i=0}^{\lfloor (n-1)\ell \rfloor} \binom{n-1}{i} \left(\frac{1}{2}\right)^{n-1}$. As the number n of agents goes to infinity, Γ_n^{KMR} converges to a step function that jumps from zero to one at $\ell = 1/2$.

The FY theory (Foster and Young [36]) is a continuous-time model in which random groups of n boundedly rational agents are continually selected to play the subscription game and small shocks are added to the population subscription rate. The FY theory gives rise to the criterion (17) with beliefs $\Gamma^{\text{FY}}(\ell)$ equal to $3\ell^2 - 2\ell^3$ when agents are infinitesimal and $\sum_{i=0}^{\lfloor (n-1)\ell \rfloor} \frac{6(i+1)(n-i)}{n(n+1)(n+2)}$ when they are discrete.

The FH theory (Fudenberg and Harris [41]) is like FY but the shocks are added to the *measures* of agents playing the two actions and agents tremble as in KMR. The FH theory gives rise to the criterion (17) with fictional beliefs Γ^{FH} that put one half weight each on other-agent subscription rates of zero and one: $\Gamma^{\text{FH}}(\ell)$ equals one half if $\ell < 1$ and one if $\ell = 1$.

Sufficient conditions for each theory to apply its associated criterion are given in Table 1 from Frankel [37]. These conditions refer to the following single crossing properties from

in (17) is well-defined.

²⁶The GG theory originates with Carlsson and van Damme [20].

Athey [7]:

Weak SC1: For all pairs $\ell' > \ell$, $\pi(\ell) > 0$ implies $\pi(\ell') \geq 0$.

SC1: For all pairs $\ell' > \ell$, $\pi(\ell) \geq (>) 0$ implies $\pi(\ell') \geq (>) 0$.

| | | Sufficient Conditions on π for Theory to Imply Criterion | |
|---------------------------|-----------|--|--|
| Theory | Criterion | Discrete Agents | Infinitesimal Agents |
| A. Heuristic Arguments | | | |
| Pareto | Pareto | Weak SC1 | Weak SC1 |
| UI | UI | Weak SC1 | Weak SC1 |
| B. Rational-Player Models | | | |
| MM | Laplace | Weak SC1 | Continuous on $[0, 1]$ & SC1 on $(0, 1)$ |
| GG | Laplace | Weak SC1 | Weak SC1 |
| C. Evolutionary Models | | | |
| KMR | KMR | Weak SC1 | Theory requires discrete agents |
| FY | FY | Weak SC1 | Lipschitz-continuous on $[0, 1]$ & SC1 on $(0, 1)$ |
| FH | FH | Weak SC1 | Lipschitz-continuous on $[0, 1]$ & SC1 on $(0, 1)$ |

Table 1: Sufficient conditions for selection theories to imply associated fictional beliefs (Frankel [37]).

We study two variants of our base model. In each, the issuer selects a scheme s from some set Σ .²⁷ For any scheme $s \in \Sigma$, let

$$\varphi_{\Gamma}(s) = \int_{\ell=0}^1 \pi_s(\ell) d\Gamma(\ell) \quad (18)$$

be an agent's expected payoff from subscribing under the fictional beliefs Γ . We will say that s is *successful* if $\varphi_{\Gamma}(s) > 0$: if the scheme induces the agents to subscribe under (17). As in Frankel [37], we assume the issuer will either propose a successful scheme s , getting some payoff U_s , or stay out, getting $E[Y_0]$.²⁸ However, an optimal successful scheme may

²⁷E.g., in the base model, Σ will be the set of price-security pairs (p, S) for monotone securities S and a fixed price $p \in (0, \bar{p}]$.

²⁸There are three cases by (17) and (18). If $\varphi_{\Gamma}(s)$ is positive, all agents subscribe. If $\varphi_{\Gamma}(s)$ is negative, no agents subscribe. As failed offerings are costly in practice (Dunbar [35]), we assume the issuer will not

not exist as the set $O = \{s \in \Sigma : \varphi_\Gamma(s) > 0\}$ of successful schemes is not closed. Thus, we follow Frankel [37] in looking for a scheme with the following property.

Definition 3. (Frankel [37]) A scheme $s \in \Sigma$ is *approximately optimal* if (a) there is no successful scheme $s' \in \Sigma$ that satisfies $U_{s'} > U_s$ and (b) for any $\varepsilon > 0$ there is a successful scheme $s' \in \Sigma$ within ε of s , such that $|U_s - U_{s'}| < \varepsilon$.

An approximately optimal scheme s yields a tight upper bound U_s on the issuer's payoff $U_{s'}$ from any successful scheme s' . Moreover, there are nearby successful schemes s' that give the principal payoffs $U_{s'}$ near U_s . Finally, an approximately optimal scheme always exists.²⁹ To find such a scheme, we use the following procedure.

Heuristic Search Procedure (HSP) (Frankel [37]) (a) Specify an agent type (discrete or infinitesimal) and let R denote the set of feasible other-agent subscription rates ℓ ; it is $[0, 1]$ when agents are infinitesimal and

$$\lambda = \left\{ \frac{i}{n-1} : i = 0, \dots, n-1 \right\} \quad (19)$$

when they are discrete. (b) Specify a nonempty set Σ of feasible schemes s and, for each scheme s in Σ , a payoff function $\pi_s : R \rightarrow \mathfrak{R}$ for the agents and a payoff $U_s \in \mathfrak{R}$ that the issuer receives if all agents subscribe. (c) Specify a metric μ on Σ and verify that the set Σ is compact and the maps $s \rightarrow U_s$ and, for all $\ell \in R$, $s \rightarrow \pi_s(\ell)$ are continuous with respect to μ . (d) Show that the sufficient conditions in Table 1 for the chosen agent type (discrete or infinitesimal) hold for any scheme $s \in \Sigma$. (e) If the set $O = \{s \in \Sigma : \varphi_\Gamma(s) > 0\}$ of successful schemes is empty, abort as the issuer cannot

propose a scheme for which $\varphi_\Gamma(s) < 0$, preferring to stay out instead. If $\varphi_\Gamma(s)$ is zero, the agents' response is indeterminate. In two of the three versions of our model, the issuer can induce the agents to subscribe by sweetening the proposal infinitesimally (see Claims 11 and 23). Plausibly, she will do so. In the third version (minimum sales constraints), there are schemes s satisfying $\varphi_\Gamma(s) = 0$ that cannot be sweetened in this way. As letting the issuer propose such schemes leads to absurd results (n. 72), we rule them out.

²⁹One can show that a scheme is approximately optimal if and only if it maximizes U_s on the closure of O . If, moreover, Σ is compact and U_s is continuous (which we will assume), the given maximum exists by the extreme value theorem. See Frankel [37] for details.

induce the agents to subscribe. (f) Let Σ' be the result of removing from Σ an arbitrary (and possibly empty) set of schemes that are not near any successful schemes.³⁰ (g)

Find a scheme s^* that maximizes U_s on Σ' subject to

$$\varphi_{\Gamma}(s) = \int_{\ell=0}^1 \pi_s(\ell) d\Gamma(\ell) \geq 0. \quad (20)$$

(h) Show that for every $\delta > 0$ there is a successful scheme $s' \in \Sigma$ that is within δ of s^* .

Claim 4. (Frankel [37]) *Assume steps (a)-(d) of HSP are satisfied. (A) If a scheme $s^* \in \Sigma$ solves steps (e)-(h), it is approximately optimal in Σ . (B) If s^* is approximately optimal in Σ , then there is a way to delete schemes in step (f) such that s^* satisfies steps (e), (g), and (h).*

We apply HSP to our base model and two extensions. In each case, we simplify by showing that step (g) implies step (h). For one variant, schemes must be deleted in step (f) for HSP to have a solution.³¹

4 Underpricing: An Intuition

An intuition for underpricing is as follows. If the issuer chooses to go ahead with an offering, she will offer her security at a price that the agents are willing to pay given their fictional beliefs.³² Hence the agents will all subscribe. On the other hand, all of the fictional beliefs except Pareto assign a positive probability to undersubscription. If the agents rely on such “pessimistic” fictional beliefs, the issuance must be underpriced to induce them to subscribe.³³ While the price may appear to be too low *ex post*, it is correct *ex ante*: any higher price would lead the agents to choose the bad equilibrium in which no one subscribes.

³⁰A scheme s is *not near any successful schemes* if for some $\delta > 0$ there is no scheme $s' \in O$ that is a distance less than δ away from s under the metric μ .

³¹See section 6.3.

³²This relies on our assumption of symmetric information.

³³An exception can occur in the case of debt with a very low face value; see Theorem 16.

This result is illustrated in Figure 1. An issuer offers a single unit of an equity security to a unit measure of agents.³⁴ The security entitles an agent to a proportion $\alpha \in [0, 1]$ of a future cash flow Y whose expectation is rising in the amount of capital raised. We hold the security price p fixed and let the issuer vary the equity share α , which appears on the horizontal axis. The ray ABC gives the agents' valuation of the security for each share α if all subscribe: if the firm raises capital p . The ray AFG gives their valuation if none subscribe: if the firm raises zero capital. Finally, the ray ADE gives their valuation when the subscription rate is distributed according to the fictional beliefs Γ , which we assume to lie between the first two cases.³⁵

An agent subscribes if doing so is optimal under her fictional beliefs: if her willingness to pay ADE is not less than the price p . The issuer naturally chooses the lowest such equity share, denoted α^* .³⁶ As this share induces all agents to subscribe, the security's *ex-post* market value is given by the height of point B. Since this height exceeds p , the security is underpriced by the length of segment BD.

Now suppose the issuer tries to leave less money on the table by choosing an equity share slightly below α^* . The agents' willingness to pay ADE is now below p : they will not subscribe, so the *ex-post* market value of the security is the height of point F. As this height is less than the price p , the security is now *overpriced* by the length of segment DF. In the model, overpricing is not observed in equilibrium as it follows a deviation. In practice, however, negative information may emerge during the first trading day. If this information

³⁴The equity assumption is for illustration only; the argument applies to any monotone security.

³⁵Why do the agents' valuations have the form of a ray? An agent's valuation is simply the equity share α times the present value of the firm's expected cash flow. This expectation, in turn, depends only on the distribution of the amount pL of capital raised where $L \in [0, 1]$ denotes the proportion of agents who subscribe. As the distribution of L is held fixed within each case, the expected cash flow does not vary with α (although it varies across cases). Hence, an agent's valuation is a ray whose slope is the expected cash flow in the given case.

³⁶This example assumes the agents subscribe if the issuer chooses $\alpha = \alpha^*$ despite being indifferent under their fictional beliefs. As noted in section 3, our results do not rely on this assumption.

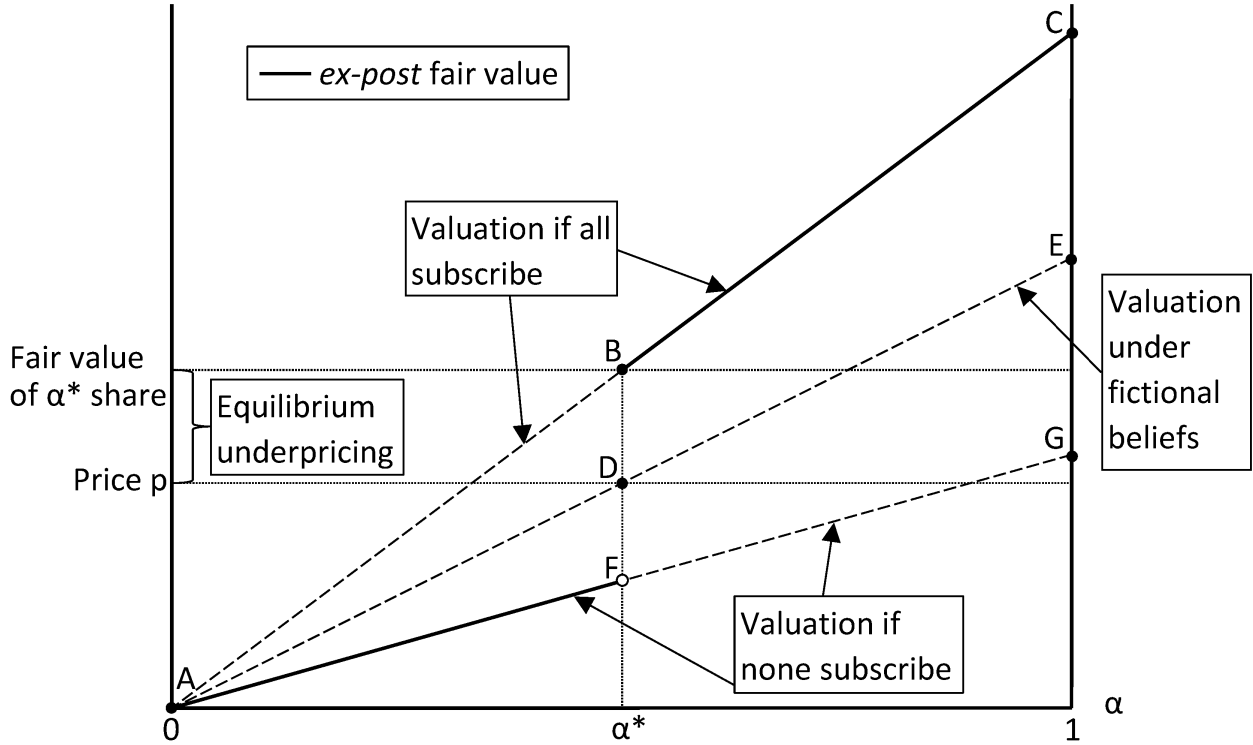


Figure 1: illustration of equilibrium underpricing result.

is large enough, it may push a share's fair value below its offer price: the issuance appears to have been overpriced. This fits the findings of Ritter and Welch [72, p. 1806], in which both negative and positive first-day returns occur while the latter are more prevalent.

5 Solving the Base Model

As noted above, we assume the issuer will either stay out, getting $E[Y_0]$, or propose a successful scheme $s = (p, S)$. Since such a scheme s induces all agents to subscribe ($L = 1$), it raises capital $K = p$ by (8). The issuer's payoff from a successful scheme s is the expectation

$$U_s = E[Y_p - S(Y_p)] \quad (21)$$

of the resulting cash flow Y_p less the security payout $S(Y_p)$. By (13), a subscriber's payoff $\pi_s(1)$ if all others subscribe ($\ell = 1$) equals the expected security payout $E[S(Y_p)]$ less the

opportunity cost $\theta(p + c)$ of subscribing. We will say that the scheme is *fairly priced* if this payoff is zero: if

$$p = V_s \stackrel{d}{=} \frac{1}{\theta} E[S(Y_p)] - c. \quad (22)$$

We refer to V_s as the *fair price* of s and to

$$V_s - p \quad (23)$$

as the *underpricing* of s : the gap between the scheme's fair price and its offer price p . For a fixed capital target p , maximizing the issuer's payoff is equivalent to minimizing underpricing:

Claim 5. *Let $s = (p, S)$ and $s' = (p, S')$ be two schemes with the same price p . Then $U_s > U_{s'}$ if and only if $V_s - p < V_{s'} - p$. Moreover, if s and s' are both successful, then the issuer prefers the scheme that is less underpriced.*

Proof. Immediate from (21) and (22). □

For any fictional beliefs Γ , let the *fictional cash flow* Y_p^Γ be the random cash flow under the counterfactual belief that the other-agent subscription rate ℓ has the distribution Γ . It is defined by

$$\Pr(Y_p^\Gamma \leq y) = G_\Gamma(y|p) \stackrel{d}{=} \int_{\ell=0}^1 \Pr(Y_{p\mathbf{r}(\ell)} \leq y) d\Gamma(\ell) = \int_{\ell=0}^1 H(y|p\mathbf{r}(\ell)) d\Gamma(\ell) \quad (24)$$

by (1). Intuitively, $G_\Gamma(y|p)$ is the expectation, conditional on $\ell \sim \Gamma$, of the distribution function of the cash flow $Y_{p\mathbf{r}(\ell)}$ that results when capital $p\mathbf{r}(\ell)$ is raised. By (24), the survival function of Y_p^Γ is given by

$$\bar{G}_\Gamma(y|p) \stackrel{d}{=} 1 - G_\Gamma(y|p) = \int_{\ell=0}^1 \bar{H}(y|p\mathbf{r}(\ell)) d\Gamma(\ell) \quad (25)$$

where \bar{H} is defined in (2). More optimistic fictional beliefs weakly raise this function:

Claim 6. *Fix a capital target $p \in (0, \bar{p}]$. If Γ' first-order stochastically dominates Γ , then $Y_p^{\Gamma'}$ first-order stochastically dominates Y_p^Γ : if $\Gamma'(\ell) \leq \Gamma(\ell)$ for all $\ell \in [0, 1]$, then $\bar{G}_{\Gamma'}(y|p) \geq \bar{G}_\Gamma(y|p)$ for all y .*

Finally, the fictional cash flow Y_p^Γ that corresponds to Pareto beliefs $\Gamma = \Gamma^*$ is simply the cash flow that results if capital p is raised for sure:³⁷

$$Y_p^{\Gamma^*} = Y_p. \quad (26)$$

We now write a security's expected payout $E[S(Y_p^\Gamma)]$ under the fictional beliefs in terms of the survival function \overline{G}_Γ and show that more optimistic fictional beliefs Γ weakly raise this expected payout:

Claim 7. *For any price p and monotone security S ,*

1. *the security's expected security payout $E[S(Y_p^\Gamma)]$ under the fictional beliefs Γ can be written as $\int_{y=0}^{\overline{y}_p} \overline{G}_\Gamma(y|p) dS(y)$, which equals $\int_{y=0}^{\overline{y}_p} \overline{H}(y|p) dS(y)$ under Pareto beliefs $\Gamma = \Gamma^*$; and*
2. *if Γ' first-order stochastically dominates Γ , then $E[S(Y_p^{\Gamma'})]$ is not less than $E[S(Y_p^\Gamma)]$.*

We now carry out the steps of HSP. For HSP(a), we allow both discrete or infinitesimal agents so $R = [0, 1]$. For HSP(b), we fix a price $p \in (0, \overline{p}]$ and let Σ denote the set $\Sigma_0^p = \{p\} \times M$ of schemes with price p where

$$M \stackrel{d}{=} \{S : [0, \overline{y}_p] \rightarrow [0, \overline{y}_p] : S(y) \text{ \& } y - S(y) \text{ are nonnegative \& nondecreasing}\} \quad (27)$$

is the set of monotone securities. Further, let π_s and U_s be as given in (13) and (21). For HSP(c), we define the distance between any two schemes $s = (p, S)$ and $s' = (p, S')$ to be

$$\mu(s, s') = \sup_{y \in [0, \overline{y}_p]} |S(y) - S'(y)|. \quad (28)$$

We now verify the conditions of HSP(c):

Claim 8. *The set Σ_0^p is compact and the functions U_s and, for all $\ell \in R$, $s \rightarrow \pi_s(\ell)$ are continuous with respect to μ .*

Turning to HSP(d), the conditions of Table 1 hold for both agent types:

³⁷This is because Pareto beliefs Γ^* put all of their weight on $\ell = 1$ and $pr(1)$ equals p by (9) and (11).

Claim 9. For any scheme s in Σ_0^p , π_s is Lipschitz-continuous and satisfies SC1 on $[0, 1]$.

For HSP(e), let O be the set $O_0^p \subseteq \Sigma_0^p$ of schemes s for which $\varphi_\Gamma(s) > 0$. By (13) and (24), for any scheme $s = (p, S) \in \Sigma_0^p$ we can rewrite (18) as

$$\varphi_\Gamma(s) = E[S(Y_p^\Gamma)] - \theta(p + c). \quad (29)$$

By monotonicity, $S(y) \leq y$. Thus, by (29), O_0^p is nonempty if and only if the agents would pay p for a 100% equity stake:

Claim 10. O_0^p is nonempty if and only if

$$p < \frac{1}{\theta} E[Y_p^\Gamma] - c. \quad (30)$$

To satisfy HSP(e), we assume (30).

We remove no schemes in HSP(f). Next we show that HSP(g) implies HSP(h).

Claim 11. If the scheme $s = (p, S)$ satisfies (20) then for any $\delta > 0$ there is a successful scheme s' within δ of s .

To find an approximately optimal scheme, it thus suffices to solve HSP(g). We can rewrite the constraint (20) in HSP(g) as

$$p \leq \frac{1}{\theta} E[S(Y_p^\Gamma)] - c \quad (31)$$

by (29): the price p does not exceed the security's value under the fictional beliefs. Since, moreover, the issuer prefers less underpriced schemes, HSP(g) is equivalent to minimizing underpricing subject to (31):

Claim 12. A scheme s satisfies HSP(g) if and only if there is no alternative scheme s' that satisfies (31) and has lower underpricing (23).

Furthermore, the best one can hope for is fair pricing:³⁸

³⁸Intuitively, (31) states that the price p does not exceed the security's value under the fictional beliefs which, in turn, does not exceed the scheme's fair price by (22) and part 2 of Claim 7.

Claim 13. *For any scheme s that satisfies (31), underpricing (23) is nonnegative.*

Finally, the constraint (31) must bind at any approximately optimal scheme:

Claim 14. *For any scheme $s = (p, S)$ that solves $HSP(g)$, the constraint (31) binds:*

$$p = \frac{1}{\theta} E [S (Y_p^\Gamma)] - c. \quad (32)$$

Intuitively, if (31) holds strictly at (p, S) then the issuer can replace S with $(1 - \varepsilon)S$. For small enough $\varepsilon > 0$, (31) will still hold but the issuer's payoff U_s will be higher.

5.1 The Pareto Case

We will say that a scheme $s = (p, S)$ is *fairly priced* if underpricing $V_s - p$ is zero. By (22), this means that the expected payout of the security under full subscription equals the agents' opportunity cost of subscribing:

$$E [S (Y_p)] = \theta (p + c). \quad (33)$$

Fair pricing suffices for approximate optimality in the Pareto case:³⁹

Claim 15. *Assume $\Gamma = \Gamma^*$. Then $s = (p, S)$ is approximately optimal if and only if it is fairly priced.*

Intuitively, agents who rely on the Pareto criterion play a best response to the correct belief that all others will subscribe. Thus, they will buy any fairly priced security. And, by Claim 5, the issuer is indifferent among all fairly priced securities as well.

5.2 The Non-Pareto Case

We now turn to the non-Pareto case: $\Gamma \neq \Gamma^*$. We show that standard debt is approximately optimal and thus minimizes underpricing among schemes that satisfy (31).⁴⁰ Intuitively,

³⁹The proof relies on the fact that, in light of our prior results, approximate optimality is equivalent to $HSP(g)$ by Claim 4.

⁴⁰By Claim 12, solving $HSP(g)$ is equivalent to minimizing underpricing subject to (31).

debt minimizes the investors' strategic uncertainty as its payout is constant except in the rare case of default. Hence, it is the least underpriced security. We show, further, that a debt issuance can be fairly priced if it is not too large. These predictions are consistent with the literature: stock IPOs are underpriced (Ritter and Welch [72]), while the average IPDO is more or less fairly priced (Datta, Iskandar-Datta, and Patel [27]). Moreover, debt is indeed more prevalent than equity: nonconvertible debt made up 82% of all new capital raised worldwide from 1990 to 2001 (Henderson, Jegadeesh, and Weisbach [50, Table 2, p. 69]).^{41,42}

Let \underline{y}_p^Γ denote the lower bound of the cash flow support under the fictional beliefs Γ .⁴³ We will say that a debt security $S(y) = \min\{y, D\}$ is *risk-free* if it will surely be repaid if all subscribe: if $D \leq \underline{y}_p$. Otherwise we will say that it is *risky*. With this terminology, we can now state the main result of this section:⁴⁴

Theorem 16. *Fix a price $p \in (0, \bar{p}]$ that satisfies (30). Let*

$$S(y) = \min\{D, y\} \tag{34}$$

be the standard debt contract with face value D given implicitly by

$$p = \frac{1}{\theta} E[\min\{D, Y_p^\Gamma\}] - c. \tag{35}$$

The scheme (p, S) is approximately optimal and minimizes underpricing. Moreover:

1. *If $D \leq \underline{y}_p^\Gamma$, the security S is risk-free and fairly priced.*

⁴¹Debt also rises during investment spikes, suggesting that firms prefer to use debt to fund new investments (Bargeron, Denis, and Lehn [9]; DeAngelo, DeAngelo, and Whited [29, pp. 255-7]; DeAngelo and Roll [30, p. 405]; Denis and McKeon [34]; Im, Mayer, and Sussman [52, Table 4]).

⁴²The prevalence of debt supports not only our model, but also prior explanations such as adverse selection (section 1.1) and costly state verification (Townsend [81]). Further research is needed to determine the relative importance of these various theories.

⁴³It equals the lower bound $\underline{y}_{pr(\underline{\ell}_\Gamma)}$ of the cash flow when revenue $pr(\underline{\ell}_\Gamma)$ is raised, where $\underline{\ell}_\Gamma$ is the lower bound of the support of Γ . Under the five non-Pareto beliefs Γ surveyed in section 3, $\underline{\ell}_\Gamma$ equals zero so \underline{y}_p^Γ equals \underline{y}_0 .

⁴⁴The comment in n. 39 applies also to Theorem 16.

2. If $D \in (\underline{y}_p^\Gamma, \underline{y}_p]$, the security S is risk-free and underpriced.

3. If $D > \underline{y}_p$, the security S is risky and underpriced.

Intuitively, in case 1 the face value D lies below the lowest cash flow \underline{y}_p that can occur under the true beliefs Γ^* : the security is risk-free. Moreover, the face value also lies below the lowest cash flow \underline{y}_p^Γ that can occur under the fictional beliefs Γ . Hence, the agents' selection criterion *also* treats the security as risk-free: the agents fairly price the security. In case 2, the security is still risk-free since $D < \underline{y}_p$. However, as D now exceeds \underline{y}_p^Γ , the agents' fictional beliefs place positive weight on default, leading them to value the security *as if* it were risky and thus to undervalue it. Finally, in case 3, D exceeds \underline{y}_p so the security is risky. Moreover, while default is possible under both Γ and Γ^* (as $D > \underline{y}_p \geq \underline{y}_p^\Gamma$), it is more likely under Γ : the security is undervalued as in case 2.

We conclude with two empirical predictions regarding the yield spread $\frac{D}{p} - \theta$ in new bond issuances. We show that this spread is increasing in due diligence costs, as proxied by the time cost c , but decreasing in the availability of alternative funding sources - which we proxy by a rise in the sensitivity of the future cash flow to revenue shortfalls in the issuance.

More precisely, let $H'(y|\kappa)$ be an alternative distribution that satisfies HRO and let $\overline{H}' = 1 - H'$ be the corresponding survival function. We focus on distributions H' that are more sensitive to the cash flow in the following sense. First, the hazard ratio is weakly more sensitive to issuance revenue under H' than H :

$$\overline{H}'(y|\kappa') \overline{H}(y|\kappa) \leq \overline{H}(y|\kappa') \overline{H}'(y|\kappa) \text{ for any } \kappa' > \kappa \text{ and } y \geq 0, \quad (36)$$

which is equivalent to $\frac{\overline{H}'(y|\kappa)}{\overline{H}'(y|\kappa')} \leq \frac{\overline{H}(y|\kappa)}{\overline{H}(y|\kappa')}$ when both ratios are well-defined. Second, the shift from H to H' is not “good news” (in a FOSD sense) for the cash flow conditional on full subscription:

$$\overline{H}'(y|p) \leq \overline{H}(y|p) \text{ for all } y \geq 0. \quad (37)$$

Claim 17. *Fix the capital target p . Let $c' \geq c$ and let H and H' be two distributions that satisfy HRO, (37), and (36). Then the yield spread $\frac{D}{p} - \theta$ given by the face value D in (35) is weakly higher under (c', H') than under (c, H) , and is strictly so if $c' > c$.*

The empirical literature indeed suggests that bond yields in new issuances are rising in the cost of due diligence (Andres, Betzer, and Limbach [5]; Datta, Iskandar-Datta, and Patel [28]; Fridson and Garman [40]). However, we can find no test of the effect of a higher need for issuance revenue.

6 How to Sell Equity if You Must

Many firms do sell equity, despite the fact (both empirically and in our model) that equity leads to more underpricing.⁴⁵ Reasons may include diversification (Zingales [86]; Chemmanur and Fulghieri [23]), inducing information production (Chemmanur [22]), exploiting stock overvaluation (Lucas and McDonald [61]; Ritter [71]), creating public shares to pay for acquisitions (Brau and Fawcett [15]), and preserving borrowing capacity (DeAngelo, DeAngelo, and Whited [29]). For issuers to leave so much money on the table, they must have a strong incentive to issue equity that is unrelated to issuance revenue. Thus, we now study two common devices that can mitigate underpricing in equity issuances. As equity is fairly priced in the Pareto case (Claim 15), we restrict to the non-Pareto case.

In the first device, the issuer can raise the number of shares an investor can request so that shares must be rationed once the subscription rate surpasses a given threshold.⁴⁶ Once this occurs, a further rise in the subscription rate entails a fall in each subscriber's allotment such that issuance revenue is unchanged. This insurance effect mitigates underpricing. However,

⁴⁵See Datta, Iskandar-Datta, and Patel [27] and Ritter and Welch [72] for empirical support.

⁴⁶In our base model, (a) there is a unit number of shares and investors and (b) each investor demands either zero or one shares: oversubscription cannot occur. In the extension, we relax (b): an investor demands either zero or $z > 1$ shares, so oversubscription occurs if more than $1/z$ investors subscribe.

there is an offsetting effect: when the subscription rate is low enough to avoid rationing, a subscriber receives her full share request but - as the issuance raises less capital in this event - the shares are not worth as much. While this effect worsens underpricing, it is smaller than the first effect for all but one of the selection criteria we consider.⁴⁷ This may help explain the empirical prevalence of ex-post share rationing.⁴⁸

In the second device, the issuer specifies a minimum sales requirement (MSR): a minimum subscription rate below which the issuance is withdrawn. By ensuring that subscribers are not forced to buy shares when issuance revenue is low, such a scheme can mitigate underpricing. In the extreme case of a zero time cost ($c = 0$), the issuer can eliminate underpricing by setting the MSR to 100%.⁴⁹ With a positive time cost c , in contrast, a rise in the MSR has a second effect: it leads an investor to defray her fixed, positive time cost over a shrinking set of subscription rates at which the issuance goes through. Because of this effect, underpricing remains under an MSR. Moreover, an MSR below 100% is typically optimal as the time cost effect grows as the MSR rises. This is consistent with the empirical literature: most MSRs are positive but below 100% (Cho [24, Table 3]; Welch [84, Figure 1]).

For the second device, we also show two comparative statics results: if the issuer's cash flow is less sensitive to issuance revenue and/or the agents' time cost is higher, a lower MSR is optimal. These predictions are supported by two findings in Welch [84]. The first is that issuers with higher sales revenue tend to choose lower MSRs (Welch [84, Table 5]). As Welch notes, an issuer with higher sales revenue depends less on issuance revenue to fund its projects. In our setting, this means that the cash flow is less sensitive to issuance revenue, so the insurance effect of a higher MSR is smaller - leading the issuer to choose a lower MSR.

⁴⁷Even for the criterion that is an exception, share rationing mitigates underpricing if the fixed subscription cost is small enough; see section 6.2.

⁴⁸See Amihud, Hauser, and Kirsh [4, p. 146] and Cornelli and Goldreich [26, p. 1419].

⁴⁹This assumes that the upper bound of the support of the agents' fictional beliefs is one, which holds for all of the criteria of section 3 except Unique Implementation.

The second finding is a positive association between the issuer’s MSR and the compensation paid to the underwriter (Welch [84, Table 6]). This can be reconciled with our model via the omitted variable of underwriter reputation. Intuitively, reputable underwriters both charge high fees and certify the issuer, lowering an investor’s time costs (which include due diligence).⁵⁰ And as noted, a lower due time cost leads the issuer to raise her MSR. Our model thus predicts that underwriter compensation and MSRs will be positively related.

6.1 Preliminaries

In the base model, the issuer proposes a price p and a monotone security S . The “base model with equity” will refer to the restriction of this model to equity securities:

$$S(y) = \alpha y \text{ for some constant } \alpha \in (0, 1). \quad (38)$$

In this restricted version, the agents’ participation constraint in step (g) of HSP can be written as

$$p \leq \frac{\alpha}{\theta} E[Y_p^\Gamma] - c. \quad (39)$$

If the agents do not rely on the Pareto criterion, equity is always underpriced - in contrast to debt (Theorem 16):⁵¹

Claim 18. *Assume $\Gamma \neq \Gamma^*$ and $p > 0$. 1. The true expected cash flow $E[Y_p]$ exceeds the cash flow $E[Y_p^\Gamma]$ that is expected under the fictional beliefs. 2. Any equity security S that solves HSP(g) is underpriced.*

To mitigate the underpricing of Claim 18, we modify the base model with equity in two ways. In each extension, the model is changed if all agents subscribe: each agent simply

⁵⁰Evidence for certification comes from Lee and Masulis [58], who find that underwriter reputation is negatively associated with earnings management by the issuer.

⁵¹Intuitively, the increment $dS(y) = \alpha dy$ in the payout of equity is positive at all realizations y as $\alpha > 0$. For y above the lower bound \underline{y}_p^Γ on the cash flow given Γ , the agents require a discount to purchase such increments since (by FOSD) their fictional beliefs Γ place a lower probability on cash flows above such y than do the correct beliefs Γ^* . (The notation \underline{y}_p^Γ is defined in section 5.2.)

pays p for one share of the security. Reasoning as at the start of section 2 and using (38), each extension thus has the following properties.

1. The issuer's realized payoff U_s from proposing a successful scheme s is

$$U_s = (1 - \alpha) E[Y_p]. \quad (40)$$

2. A subscriber's payoff if all others subscribe is $\pi_s(1) = \alpha E[Y_p] - \theta(p + c)$.
3. The scheme s is *fairly priced* if $\pi_s(1)$ is zero or, equivalently, if underpricing $V_s - p$ is zero where

$$V_s = \frac{\alpha}{\theta} E[Y_p] - c \quad (41)$$

is the security's fair price.

As U_s and V_s depend only on p and α and U_s (resp., V_s) is increasing (decreasing) in α , an optimal scheme minimizes underpricing:

Claim 19. *In the base model with equity and each extension thereof, a scheme s maximizes the issuer's payoff U_s over a set if and only if it minimizes underpricing $V_s - p$ over the same set.*

We will also make use of the notation

$$\Upsilon_\ell^p = E[Y_{pr(\ell)}], \quad (42)$$

which denotes the expected cash flow conditional on the price p and the other-agent subscription rate ℓ . A higher subscription rate is good news for this quantity and thus for the fair value of any equity security:

Claim 20. *For any price $p > 0$, the conditional expected cash flow Υ_ℓ^p is positive and continuous, and is increasing in ℓ .*

We now turn to the two extensions. In each, a 100% equity stake for the price p will be feasible and will induce subscription under (30), which we henceforth assume. Thus, HSP(e) is satisfied in each extension.

6.2 First Device: Ex-Post Share Rationing

Many IPOs are oversubscribed, leading to share rationing (n. 48). Underwriters seem to view rationing as desirable:

“Discussions with investment bankers indicate that they perceive that an offer should be two to three times oversubscribed to create a ‘good IPO.’” (Lowry, Michaely, and Volkova [60, p. 223])

What is the advantage of rationing? Our model suggests an answer: the prospect of *ex-post* rationing reduces *ex-ante* strategic uncertainty among the investors. Intuitively, once the subscription rate is high enough that shares must be rationed, a further rise in this rate has no effect on the amount of capital raised by the issuance. This can mitigate underpricing, helping the issuer. This theory supplements two answers from the prior literature:⁵²

- In Rock’s [75] winner’s-curse theory, the anticipation that high-quality issuances will be rationed lowers the willingness to pay of uninformed traders. In order to elicit the participation of these traders, issuers must underprice. This theory has empirical support⁵³ but raises the question of why issuers do not take steps to mitigate the winner’s curse - e.g., by capping share requests *ex ante*.⁵⁴
- In Benveniste and Spindt [10] and Sherman [79], issuers use underpricing to induce investors to reveal private information about the firm, rewarding those who do so

⁵²We focus here on models of share rationing. While these overlap with models of underpricing, they do not coincide. For instance, the security in our base model is underpriced but not rationed. Other underpricing theories are discussed below in section 1.1.

⁵³See Aggarwal, Prabhala, and Puri [1], Amihud, Hauser, and Kirsh [4], and Michaely and Shaw [63].

⁵⁴Brennan and Franks [16] suggest that issuers use *ex-post* rationing to prevent the formation of large external blockholders and thus to avoid monitoring. Indeed, dispersed ownership may also strengthen incentives for managerial initiative (Burkart, Gromb, and Panunzi [17]) and lead to greater liquidity in the secondary market (Plantin [68]). However, *ex-ante* caps would also give these benefits while avoiding the winner’s curse. Moreover, external blockholders can act as a useful check on managers (Bolton and von Thadden [12]), so a founder might welcome them as a way to commit to good management.

with larger share allocations. While this theory has empirical support,⁵⁵ there is also evidence that underwriters allocate underpriced shares to their own profitable clients and mutual funds.⁵⁶ Moreover, there is an alternative mechanism - auctions - that aggregates information efficiently without the need for underpricing (Pesendorfer and Swinkels [67]).⁵⁷

In the base model, each subscriber requests a single share; since the numbers of shares and agents are equal, rationing cannot occur. To permit rationing, we now let each investor ask for $z \geq 1$ shares, where the cap z is announced by the issuer prior to the issuance. If more than $1/z$ investors subscribe, demand exceeds supply: shares must be rationed. Since issuance proceeds in this range are fixed, strategic uncertainty is reduced. However, rationing also introduces a winner's curse: a subscriber gets more shares when issuance revenue is lower. The net effect of such a scheme is to worsen underpricing under the FH criterion but to lessen it under the four other non-Pareto criteria of section 3.

In addition to a price p and equity stake α , the issuer now chooses a number $z \in [1, \bar{p}/p]$ of shares to offer to each agent.^{58,59} We focus on the case of infinitesimal agents: $L = \ell$.⁶⁰ If total share demand ℓz does not exceed the unit supply of shares, each subscriber gets z shares. Else shares are rationed: each subscriber gets $\frac{1}{\ell}$ shares, so the total number $\ell \times \frac{1}{\ell}$ of

⁵⁵For instance, Hanley [48] finds that offer price rises are followed by greater underpricing. Cornelli and Goldreich [25, 26] find that bidders who include limit prices receive more shares and that these limit prices affect the issue price.

⁵⁶Goldstein, Irvine, and Puckett [42], Jenkinson, Jones, and Suntheim [54], and Reuter [69] find that issuers sell underpriced shares to their profitable clients, while Ritter and Zhang [73] find that they place the shares with their own mutual funds.

⁵⁷This point is made also by Ritter and Welch [72, n. 10 (p. 1810)] and Lowry, Michaely, and Volkova [60, p. 241].

⁵⁸The upper bound on z is from the agents' wealth constraint: a subscriber's wealth \bar{p} must suffice to pay p per share for z shares.

⁵⁹An equivalent way to model rationing is to reduce the number of shares below one, while continuing to let each investor request one share; details available on request.

⁶⁰The assumption merely simplifies notation; the results hold qualitatively in the discrete case as well.

shares sold is one. Let

$$q = \begin{cases} \ell z & \text{if } \ell \leq 1/z \\ 1 & \text{if } \ell \geq 1/z \end{cases} \quad (43)$$

denote the aggregate number of shares that the subscribers get. As in the base model, the remaining shares are sold at a discount ρ : revenue K equals $p\mathbf{r}(q)$ where $\mathbf{r}(q)$ is defined in (11). Thus by (42), the expected cash flow conditional on q is Υ_q^p . Let

$$\psi_s(q) = \alpha \Upsilon_q^p - p\theta \quad (44)$$

denote the expected payoff per share purchased, gross of the time cost. The payoff $\pi_s(\ell)$ from subscribing to a scheme s is simply the payoff $\psi_s(q)$ per share, evaluated at the number q of shares sold given in (43), times the number of shares

$$\frac{q}{\ell} = \begin{cases} z & \text{if } \ell \leq 1/z \\ \frac{1}{\ell} & \text{if } \ell \geq 1/z \end{cases} \quad (45)$$

that a subscriber actually receives, minus the opportunity cost $c\theta$ of her time:

$$\pi_s(\ell) = \begin{cases} z\psi_s(\ell z) - c\theta & \text{if } \ell \leq 1/z \\ \frac{1}{\ell}\psi_s(1) - c\theta & \text{if } \ell \geq 1/z, \end{cases} \quad (46)$$

which can be written succinctly as

$$\pi_s(\ell) = \left(z \wedge \frac{1}{\ell} \right) \psi_s(\ell z \wedge 1) - c\theta \quad (47)$$

where “ $a \wedge b$ ” denotes $\min\{a, b\}$. The base model corresponds to a cap z of one, which in (47) yields

$$\pi_{(p,\alpha,1)}(\ell) = \psi_s(\ell) - c\theta. \quad (48)$$

Comparing (47) to (48) reveals the three effects of an *ex-post* rationing scheme. The first two mitigate underpricing while the third worsens it:

1. The *expected payoff per share* is higher for each $\ell < 1$: it is $\psi_s(\ell z \wedge 1)$ in (47) versus $\psi_s(\ell)$ in (48). Intuitively, the higher cap z allows the issuer to reach its capital target

p more rapidly as ℓ rises, so the expected cash flow is higher for all $\ell < 1$ (and is the same at $\ell = 1$). Thus, an agent with fictional beliefs Γ relies on a higher valuation to make her subscription decision.

2. The *number of shares purchased* is higher for each $\ell < 1$ under the scheme: it is $z \wedge \frac{1}{\ell}$ in (47) versus one in (48). The scheme thus leads an agent to defray her time cost c over a larger number of shares purchased in making her decision.
3. The number (45) of shares that a subscriber buys is constant on $\ell \leq 1/z$ and declining on $\ell \geq 1/z$: she buys more shares when they are less valuable. This “winner’s curse” is due not to asymmetric information as in Rock [75], but rather to the form (17) of agents’ equilibrium selection criterion.

Effects 1 and 2 raise an agent’s willingness to pay while effect 3 lowers it. Thus, whether the scheme lessens underpricing depends on the relative strengths of the three effects.

We now carry out the steps of HSP. For HSP(a), the agents are infinitesimal so $R = [0, 1]$. For HSP(b), fix a price $p \in (0, \bar{p}]$ that satisfies (30) and let Σ denote the set Σ_1^p of all schemes $s = (p, \alpha, z)$ that satisfy $\alpha \in [0, 1]$, $z \in [1, \bar{p}/p]$, and the following condition:⁶¹

$$\pi_s(1) \geq 0 \text{ or, equivalently, } p \leq \frac{\alpha}{\theta} \Upsilon_1^p - c, \quad (49)$$

which states that all-subscribe is a Nash equilibrium - or, equivalently, that the issuance is not overpriced under full subscription.⁶²

Let the payoff functions U_s and π_s be given by (40) and (46), respectively. For HSP(c), let the distance $\mu(s, s')$ between s and any scheme $s' = (p, \alpha', z')$ be the maximum difference between the parameters, $\max\{|\alpha - \alpha'|, |z - z'|\}$.

Claim 21. *The set Σ_1^p is compact and the functions U_s and, for all $\ell \in R$, $s \rightarrow \pi_s(\ell)$ are continuous with respect to μ .*

⁶¹The equivalence in (49) is by (44) and (47).

⁶²We rely on (49) to show that π_s satisfies SC1. It holds automatically if $c = 0$.

For HSP(d), we show:

Claim 22. For $s \in \Sigma_1^p$, π_s is Lipschitz-continuous on $[0, 1]$ and satisfies SC1 on $(0, 1)$.

For HSP(e), by (30) the 100% equity scheme $(p, 1, 1)$ is successful. We remove no schemes in HSP(f). We next show that HSP(g) implies HSP(h).

Claim 23. Let s solve HSP(g). For any $\delta > 0$ there is a successful scheme s' within δ of s .

To find an approximately optimal scheme, it thus suffices to solve HSP(g). We first transform (20) to ease its interpretation:

Claim 24. The constraint (20) holds if and only if

$$p \leq \frac{\alpha}{\theta} E [Y_p^{\Gamma^z}] - \frac{c}{Q^z} \quad (50)$$

where $Y_p^{\Gamma^z}$ is the fictional cash flow that results when the aggregate number q of shares sold has the distribution Γ^z given by⁶³

$$\Gamma^z(q) = \begin{cases} \frac{z\Gamma(q/z)}{Q^z} & \text{if } q < 1 \\ 1 & \text{if } q = 1 \end{cases} \quad (51)$$

and

$$Q^z = \int_{\ell=0}^1 \min \left\{ z, \frac{1}{\ell} \right\} d\Gamma(\ell) \in [1, z] \quad (52)$$

is the expected number of shares that a subscriber will receive under the fictional beliefs Γ .

We refer to Γ^z as the *effective fictional beliefs* as they play the same role in (50) as the agents' fictional beliefs Γ do in (39). An intuition for (51) is as follows. If $q < 1$, there is no ex-post rationing: each of the ℓ subscribers gets z shares whence $\ell z = q$ or, equivalently, $\ell = z/q$. Hence $\Gamma(z/q)$ is the probability under Γ that at most q shares are sold. But we must also take account that a subscriber receives an above-average number of shares in this

⁶³Importantly, Γ^z is *not* the distribution of the number q of shares sold under Γ . Rather, as discussed after the claim, it is the result of weighting that distribution by the number of shares purchased by an individual subscriber when q shares are sold in aggregate.

case: z rather than the unconditional expected number of shares, $Q^z \in [1, z]$. The weight $d\Gamma^z(q)$ that an agent assigns to the per-share payoff $\psi_s(q)$ for a quantity $q < 1$ is simply equal to the probability $d\Gamma(q/z)$ under Γ that q shares are sold times the relative number z/Q^z of shares that she receives when $q < 1$. This completes the intuition.⁶⁴

A rise in the cap z has three effects on the constraint (50). First, $\Gamma(q/z)$ falls: there is now a higher chance that more than q shares will be sold under Γ . This is effect #1 above; it lessens underpricing. Second, Q^z rises, which shrinks c/Q^z : the time cost c is defrayed over a larger mean number Q^z of shares purchased under the fictional beliefs. This is effect #2 above; it also lessens underpricing. Finally, the weight z/Q^z on quantities $q < 1$ rises: a subscriber now buys more shares when they are not rationed.⁶⁵ This is effect #3 above; it *worsens* underpricing.

If the right hand side of the constraint (50) is monotone in z , a corner solution is optimal:

Claim 25. *If, for fixed p and α , the right hand side of (50) is everywhere rising (falling) in z , then $z = \bar{p}/p$ (resp., $z = 1$) at any solution to HSP(g).*

Intuitively, by (21), HSP(g) involves maximizing $U_s = (1 - \alpha) E[Y_p]$ subject to (50). At a solution s , the constraint (50) must bind since otherwise the issuer could lower α , raising U_s . If, moreover, the right hand side of (50) is rising in z , the issuer can raise her payoff U_s by raising z while lowering α . This also lowers underpricing by Claim 5. Hence, it is optimal to set z at its maximum (minimum) if the right hand side of (50) is everywhere rising (resp., falling) in z .

For each non-Pareto selection criterion of section 3, Figure 2 shows the fictional beliefs

⁶⁴The weight $d\Gamma^z(1)$ that an agent assigns to the per-share payoff $\psi_s(1)$ when all shares are sold is simply the residual $1 - \lim_{q \uparrow 1} \Gamma^z(q)$. It equals the probability that $\ell \geq 1/z$ times the mean relative number of shares received in this case. It can be written explicitly as $\int_{\ell=1/z}^1 \frac{1}{\ell Q^z} d\Gamma(\ell)$: the integral, over subscription rates ℓ at which shares are rationed, of the probability $d\Gamma(\ell)$ that exactly ℓ others will subscribe under the fictional beliefs, times the relative number $(1/\ell)/Q^z$ of shares that a given subscriber gets when ℓ others subscribe.

⁶⁵This holds since, by (52), Q^z/z equals $\int_{\ell=0}^1 \min\{1, \frac{1}{\ell z}\} d\Gamma(\ell)$ which is decreasing in z .

Γ which, by (51) and (52), are equal to the effective fictional beliefs Γ^z with cap $z = 1$.⁶⁶ In contrast, Figure 3 shows Γ^z with the higher cap $z = 1.5$: shares are rationed if over two thirds of the agents subscribe. This comparison suggests that a rise in z lowers the function Γ^z under the Laplace, KMR, and FY criteria, raises it under the FH criterion, and has no effect under the UI criterion. In the proof of Proposition 26, we show that these observations generalize to any rise in z .

Moreover, if a rise in z lowers Γ^z , then it mitigates underpricing as it lowers both terms on the right side of (50). If it raises Γ^z , then it lowers the first term but raises the second. Hence, it mitigates (worsens) underpricing if the time cost c is sufficiently high (low). Finally, if it leaves Γ^z unchanged, then it has no effect on underpricing if $c = 0$ but lowers it if $c > 0$. These intuitions underlie the main result of this section:⁶⁷

Proposition 26. *The effects of a rise in z on underpricing in an approximately optimal scheme are as follows, for each non-Pareto selection criterion.*

1. *UI criterion: mitigates underpricing if $c > 0$; no effect if $c = 0$.*
2. *Laplace criterion: always mitigates underpricing.*
3. *KMR criterion in limit as $n \rightarrow \infty$: underpricing falls to zero as z rises from 1 to 2 and is zero for $z \geq 2$.⁶⁸*
4. *FY criterion: always mitigates underpricing.*
5. *FH criterion: mitigates (worsens) underpricing if c exceeds (is less than) the positive threshold $\frac{\Upsilon_1^p - \Upsilon_0^p}{2\Upsilon_0^p}p$.*

⁶⁶Intuitively, a subscriber's allotment is capped at one share in the base model.

⁶⁷The comment in n. 39 applies here as well.

⁶⁸While this section assumes infinitesimal agents, there is continuity in the limit as $n \rightarrow \infty$. Thus, the results of this section apply also to the case of an arbitrarily large number of discrete agents.

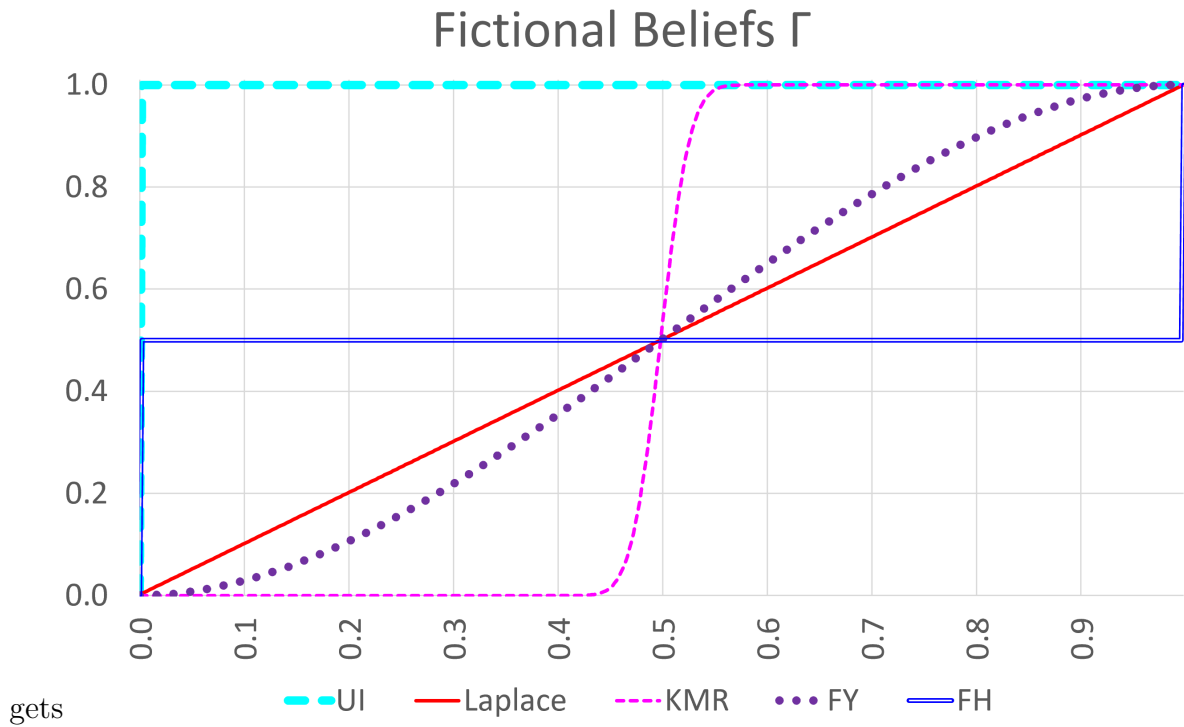


Figure 2: Fictional beliefs Γ in the base model (equal to Γ^z with $z = 1$).

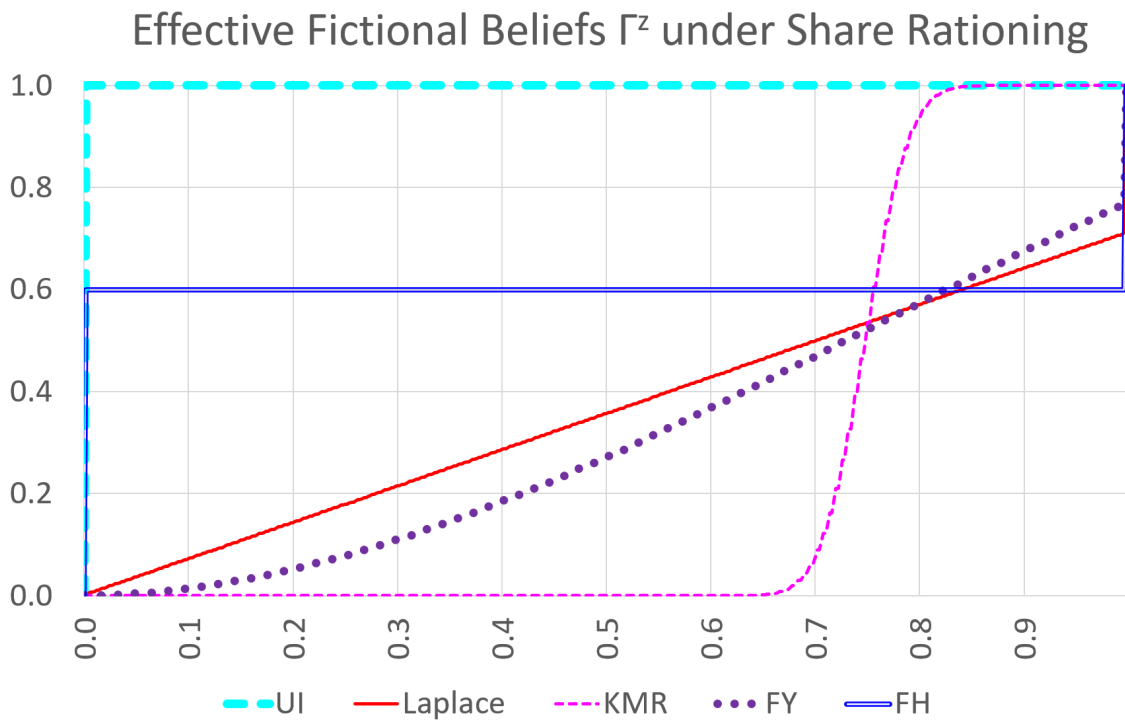


Figure 3: Effective fictional beliefs Γ^z with cap $z = 1.5$.

Claim 27. *The effects on the issuer's payoff are opposite to those on underpricing.*

For most criteria, rationing mitigates underpricing or at least leaves it unchanged. For the FH criterion, it may worsen underpricing but only if the time cost c is small. Thus, rationing usually helps the issuer by letting her sell equity at a higher price. This may help explain the empirical prevalence of this practice.

6.3 Second Device: a Minimum Sales Requirement (MSR)

In the vast majority of best-effort equity offerings, there is a minimum share requirement (MSR): a subscription rate below which the issuance is canceled (Cho [24, Table 3]; Welch [84, Fig. 1]). We now extend the base model with equity to include this feature.

If the time cost c is zero and the upper bound of the support of the fictional beliefs Γ is one,⁶⁹ then underpricing shrinks as the MSR rises and reaches zero at an MSR of 100%. This is not so with a positive time cost c , as this cost must now be defrayed over a shrinking set of subscription rates at which the issuance goes through. In this case, an MSR below 100% is typically optimal and the scheme does not eliminate underpricing. We also derive two comparative statics results: the MSR is decreasing in the time cost c and rising in the sensitivity of the expected cash flow $E[Y_\kappa]$ to issuance revenue κ . The findings of Welch [84] support these two predictions.

As the payoff function π_s in an MSR scheme can be discontinuous, we must assume agents are discrete (see Table 1). In addition to a price p and an equity share α , the issuer now chooses a subscription rate $L_0 \in \Lambda$ below which the issuance is withdrawn where

$$\Lambda \stackrel{d}{=} \left\{ \frac{k}{n} : k = 1, \dots, n \right\} \tag{53}$$

is the set of aggregate subscription rates L that can occur if at least one agent subscribes.⁷⁰

⁶⁹This is so for five of the six criteria of section 3. The exception is UI, for which the upper bound is zero.

⁷⁰If $L_0 = 0$, the issuance is never withdrawn, which is equivalent to the base model with equity.

Let $\ell_0 \in \lambda$ be the other-agent subscription rate that corresponds to L_0 ; it is given by $L_0 = \mathbf{L}(\ell_0)$.

As in (44), let $\psi_s(\ell) = \alpha \Upsilon_\ell^p - p\theta$ denote the expected payoff from subscribing, gross of the time cost, if ℓ others subscribe and the issuance is not withdrawn. This function is increasing in ℓ by Claim 20. If the issuance goes through, a subscriber incurs the time cost $c\theta$ in return for the benefit $\psi_s(\ell)$ of buying a share. If it is canceled, she incurs the time cost for no benefit. Hence, the agents' payoff function is

$$\pi_s(\ell) = \begin{cases} -c\theta & \text{if } \ell < \ell_0; \\ \psi_s(\ell) - c\theta & \text{if } \ell \geq \ell_0. \end{cases} \quad (54)$$

We now step through HSP. For HSP(a), the agents are discrete so $R = \lambda$. For HSP(b), fix a price $p \in (0, \bar{p}]$ and let Σ denote the set Σ_2^p of all schemes $s = (p, \alpha, \ell_0)$ for $(\alpha, \ell_0) \in [0, 1] \times \lambda$.⁷¹ Let the payoff functions U_s and π_s be given by (40) and (54), respectively. For HSP(c), let the distance $\mu(s, s')$ between s and any scheme $s' = (p, \alpha', \ell_0')$ be the maximum distance between parameters, $\max\{|\alpha - \alpha'|, |\ell_0 - \ell_0'|\}$. We have:

Claim 28. *The set Σ_2^p is compact and the functions U_s and, for all $\ell \in R$, $s \rightarrow \pi_s(\ell)$ are continuous with respect to μ .*

For HSP(d) we show:

Claim 29. *For any scheme s in Σ_2^p , the function $\pi_s(\ell)$ satisfies weak SC1.*

For HSP(e), we assume (30) so the set of successful schemes is nonempty: it includes $(p, 1, 0)$. A difference comes in HSP(f) where - unlike in the other two variants of the model - we must delete schemes for HSP to produce a result. Let

$$\bar{\ell}_\Gamma = \min\{\ell' \in [0, 1] : \Gamma(\ell') = 1\} \quad (55)$$

⁷¹We can let the issuer choose ℓ_0 rather than L_0 since, by (9), the map $L_0 = \mathbf{L}(\ell_0)$ is a bijection between $\ell_0 \in \lambda$ and $L_0 \in \Lambda$.

be the highest other-agent subscription rate that can occur under the fictional beliefs Γ . Since the agents are discrete, Γ puts positive weight on $\bar{\ell}_\Gamma$. In HSP(f), we eliminate any scheme $s = (p, \alpha, \ell_0)$ for which $\ell_0 > \bar{\ell}_\Gamma$.⁷² For any scheme $s = (p, \alpha, \ell_0)$ that survives HSP(f), let

$$Q_{\ell_0} \stackrel{d}{=} 1 - \lim_{\ell' \uparrow \ell_0} \Gamma(\ell') \quad (57)$$

denote the probability, under Γ , that $\ell \geq \ell_0$: that the issuance will not be withdrawn or, equivalently, the expected number of shares that a subscriber receives under Γ . It is positive:

Claim 30. *If $s = (p, \alpha, \ell_0)$ survives HSP(f), then $Q_{\ell_0} > 0$.*

As a result of the deletions in HSP(f), HSP(g) implies HSP(h):

Claim 31. *Let s solve HSP(g). For any $\delta > 0$ there is a successful scheme s' within δ of s .*

To find an approximately optimal scheme, it thus suffices to solve HSP(g). Let the truncated beliefs

$$\Gamma_{\ell_0}(\ell) = \begin{cases} 0 & \text{if } \ell < \ell_0 \\ \frac{\Gamma(\ell) - [1 - Q_{\ell_0}]}{Q_{\ell_0}} & \text{if } \ell \geq \ell_0 \end{cases} \quad (58)$$

be the posterior over the subscription rate ℓ for an agent with prior Γ who learns that the issuance is not canceled: that $\ell \in [\ell_0, 1]$.⁷³

We now rewrite the participation constraint in terms of the truncated beliefs:

⁷²Such a scheme s can be eliminated as it is not near any successful scheme: for any $\varepsilon < \ell_0 - \bar{\ell}_\Gamma$, no scheme s' that is within ε of s is successful as the withdrawal threshold of s' exceeds $\bar{\ell}_\Gamma$ and thus

$$\varphi_\Gamma(s') = -c\theta \leq 0 \quad (56)$$

by (18) and (54). To see why we must remove such schemes, suppose the time cost c is zero and let s equal $(p, 0, \ell_0)$ for some $\ell_0 > \bar{\ell}_\Gamma$. The scheme asks the agents to give the issuer p for nothing. It thus maximizes the issuer's full-subscription payoff U_s in Σ_2^p . And since $c = 0$, the scheme satisfies (20) by (56). However, it is absurd that investors would agree to pay $p > 0$ for nothing. Moreover, if not dropped in HSP(f), the scheme s would be selected in HSP(g) but fail HSP(h): the procedure would not produce a solution.

⁷³Equation (58) follows from Bayes's Rule.

Claim 32. *The constraint (20) can be written as*

$$p \leq \frac{\alpha}{\theta} E \left[Y_p^{\Gamma_{\ell_0}} \right] - \frac{c}{Q_{\ell_0}}. \quad (59)$$

Comparing (59) to (39), the benefit of an MSR is to raise the fictional cash flow $Y_p^{\Gamma_{\ell_0}}$ that the agents use to value the shares. The cost of the scheme, captured by c/Q_{ℓ_0} , is the agents' need to defray the time cost c over a dwindling number of shares Q_{ℓ_0} that they will be allowed to purchase under their fictional beliefs.

We now characterize the approximately optimal scheme when the time cost is zero, and derive some properties of underpricing in such a scheme when the time cost is positive.⁷⁴ Let $\eta_{\Gamma}(p)$ denote the expected cash flow $E \left[Y_{pr}(\bar{\ell}_{\Gamma}) \right]$ when the other-agent subscription rate equals the upper bound $\bar{\ell}_{\Gamma}$ of the support of the fictional beliefs.

Proposition 33. *The following holds in any approximately optimal MSR scheme $s = (p, \alpha, \ell_0)$.*

1. *If $c = 0$, then $\ell_0 = \bar{\ell}_{\Gamma}$,*

$$\alpha = \frac{\theta p}{\eta_{\Gamma}(p)} > 0, \quad (60)$$

and underpricing $V_s - p$ equals

$$p \left[\frac{E[Y_p]}{\eta_{\Gamma}(p)} - 1 \right] \quad (61)$$

which is zero (resp., positive) if $\bar{\ell}_{\Gamma} = (<) 1$.

2. *If $c > 0$, then underpricing $V_s - p$ is positive and decreasing in c . It is bounded below by*

$$\underline{u}_p^c = (p + c) \left[\frac{E[Y_p]}{\eta_{\Gamma}(p)} - 1 \right] \geq 0 \quad (62)$$

and above by

$$\bar{u}_p^c = p \left[\frac{E[Y_p]}{\eta_{\Gamma}(p)} - 1 \right] + c \left[\frac{E[Y_p]}{Q_{\bar{\ell}_{\Gamma}} \eta_{\Gamma}(p)} - 1 \right] > 0. \quad (63)$$

⁷⁴The comment in n. 39 applies to Proposition 33 as well.

As c shrinks to zero, both bounds converge to (61).

For all of the non-Pareto fictional beliefs in section 3 *except* UI, the upper bound $\bar{\ell}_\Gamma$ of the support of Γ is one and thus $\eta_\Gamma(p) = E[Y_p]$. By Proposition 33, underpricing is zero for such fictional beliefs if $c = 0$. However, even for such beliefs, underpricing cannot be eliminated if $c > 0$ because the c/Q_{ℓ_0} term in (59) exceeds the corresponding c term in (39). Moreover, as this term can become large as $\ell_0 \rightarrow 1$, a threshold below one will typically be optimal. This is consistent with empirical patterns (Cho [24, Table 3]; Welch [84, Fig. 1]).

Welch [84, Table 5] also finds that in best-effort offerings, firms with higher sales revenue tend to choose lower MSRs. His explanation is that such firms do not rely as much on issuance revenue to fund their projects. This suggests that if the sensitivity of the issuer's cash flow $E[Y_\kappa]$ to the issuance revenue κ is lower, the issuer should choose a lower MSR. We now derive this result formally in our setting. We show, moreover, that a higher cost of due diligence (as proxied by the time cost c) also make a lower MSR optimal and link this to another finding in Welch [84].

Formally, we study the effect of replacing the parameters (c, H) with alternative parameters (\hat{c}, \hat{H}) where the conditional distribution function \hat{H} also satisfies HRO. For each $\kappa \geq 0$, let \hat{Y}_κ be the conditional cash flow corresponding to \hat{H} ; that is, $\Pr(\hat{Y}_\kappa \leq y) = \hat{H}(y|\kappa)$. We assume that under the new parameters, the time cost is weakly higher and the expected cash flow is weakly less sensitive to issuance revenue:

P1 $\hat{c} \geq c \geq 0$ and, for any pair $\kappa' \geq \kappa$ both in $[0, \bar{p}]$, $\frac{E[\hat{Y}_{\kappa'}]}{E[\hat{Y}_\kappa]} \leq \frac{E[Y_{\kappa'}]}{E[Y_\kappa]}$.

Fix a price $p \in (0, \bar{p}]$. For an issuer with parameters (c, H) , let η denote the set of MSRs ℓ_0 that occur in approximately optimal schemes.⁷⁵ Let $\hat{\eta}$ denote the analogous set for an issuer with parameters (\hat{c}, \hat{H}) . If an issuer is willing to choose ℓ_0 over the higher ℓ'_0 with when her parameters are (c, H) , then she is also willing to choose ℓ_0 over ℓ'_0 under (\hat{c}, \hat{H}) :

⁷⁵More precisely, let A denote the set of approximately optimal schemes of an issuer with parameters (c, H) and let η be the set of all MSRs ℓ_0 for which there is an $\alpha \in [0, 1]$ such that (p, α, ℓ_0) is in A .

Claim 34. *Let $\ell_0 \in \eta$ and let $\ell'_0 > \ell_0$. Under P1, if ℓ'_0 is in $\hat{\eta}$ then so is ℓ_0 .*

Intuitively, raising the MSR ℓ_0 has a cost and a benefit for the issuer. The cost is captured by the $\frac{c}{Q_{\ell_0}}$ term in (59): the agents must defray their fixed time cost c over a smaller chance Q_{ℓ_0} that the issuance will go through under the fictional beliefs.⁷⁶ This cost is increasing in the time cost c . On the other hand, a higher MSR ℓ_0 benefits the issuer by raising the cash flow $E \left[Y_p^{\Gamma_{\ell_0}} \right]$ that the agents expect under their fictional beliefs, conditional on the issuance going through. This benefit is increasing in the sensitivity of the cash flow \hat{Y}_κ to issuance revenue κ . A switch from (c, H) to (\hat{c}, \hat{H}) thus raises the cost of a higher MSR and lowers the benefit, leading the issuer to choose a weakly lower MSR.

Claim 34 indicates that a lower time cost c leads an issuer to choose a weakly higher MSR. This can explain Welch's [84] finding that the MSR is positively related to underwriter fees. Intuitively, a higher quality underwriter both certifies issuer quality better (leading to a lower time cost c of subscription) and charges a higher fee. By Claim 34, the lower time cost leads to a higher MSR: the association between underwriter fees and the MSR is positive as in Welch [84, Table 6].

7 Discussion

Firms raise substantial capital from selling primary securities. This capital can be invested to raise the firms' future cash flows which, in turn, leads to higher distributions on the same securities. Thus, investors in a public offering face strategic uncertainty. In this setting, which securities are best for the issuer? And if equity must be sold, how best to structure the offering?

We study these questions in a stylized setting. An issuer owns a stochastic cash flow. She designs and sells a monotone security whose distributions are funded by the cash flow.

⁷⁶This harms the issuer by raising the equity share α needed to satisfy the participation constraint (59).

If the issuance raises more capital, the distribution of the cash flow is higher, which makes the securities more valuable. This leads to strategic complementarities in the decision to subscribe. In order to select among multiple equilibria, investors can use any criterion from a large set.

We show that investors who face strategic uncertainty require a discount in order to be willing to subscribe.⁷⁷ Standard debt is optimal as it minimizes this underpricing. Intuitively, as long as an issuer of debt does not default, the security payout does not depend on her cash flow. Debt thus minimizes the negative impact of strategic uncertainty on the agents' valuation of the security. Indeed, empirical studies have found that public debt offerings are less underpriced than stock IPOs which, in turn, are less underpriced when they contain more secondary shares (whose proceeds accrue to insiders rather than to the firm). We show, moreover, that the optimal yield spread on new bond issuances is increasing in due diligence costs but decreasing in the availability of alternative funding sources.

We also study two common devices in equity offerings. The first is *ex-post* share rationing: if too many investors subscribe, the issuance is deemed “oversubscribed” and each subscriber’s share allotment is reduced proportionally. Rationing reduces uncertainty over issuance revenue but introduces a winner’s curse: a subscriber gets more shares when issuance revenue is lower. The net effect of such a scheme is to mitigate underpricing under all but one of the selection criteria that we study.

The second is the minimum sales requirement (MSR): if the subscription rate falls below some threshold, the issuance is canceled. This device presents a trade off: while a higher MSR lowers an agent’s uncertainty about the revenue she will receive conditional on the sale going through, it shrinks the set of subscription rates over which the agent can defray any time costs of subscription (such as due diligence). Accordingly, such a scheme cannot eliminate underpricing unless the agents’ time costs are zero. In the general case, we show

⁷⁷Under limited conditions, a security may be fairly priced (see Theorem 16).

that the optimal MSR is decreasing in investors' due diligence costs and in the availability of alternative funding sources.

References

- [1] Aggarwal, R., N.R. Prabhala, and M. Puri. 2002. Institutional allocation in initial public offerings: Empirical evidence. *Journal of Finance* 57:1421-1442.
- [2] Allen, F., E. Carletti, I. Goldstein, and A. Leonello. 2018. "Government Guarantees and Financial Stability." *Journal of Economic Theory* 177:518-557.
- [3] Allen, F., and G.R. Faulhaber. 1989. "Signaling by Underpricing in the IPO Market." *Journal of Financial Economics* 23:303-324.
- [4] Amihud, Y., S. Hauser, and A. Kirsh. 2003. "Allocations, Adverse Selection, and Cascades in IPOs: Evidence from the Tel Aviv Stock Exchange." *Journal of Financial Economics* 68:137-158.
- [5] Andres, C., A. Betzer, and P. Limbach. 2014. "Underwriter Reputation and the Quality of Certification: Evidence from High-Yield Bonds." *Journal of Banking and Finance* 40:97-115.
- [6] Ang, J.S., and J.C. Brau. 2003. "Concealing and Confounding Adverse Signals: Insider Wealth-Maximizing Behavior in the IPO process." *Journal of Financial Economics* 67:149-172.
- [7] Athey, S. 1996. "Comparative Statics Under Uncertainty: Single Crossing Properties and Log-Supermodularity." M.I.T. Working Paper no. 96-22.
- [8] Babina, T., P. Ouimet, and R. Zarutskie. 2022. "IPOs, Human Capital, and Labor Reallocation." Mimeo.

- [9] Barger, L., D. Denis, and K. Lehn. 2018. “Financing Investment Spikes in the Years Surrounding World War I.” *Journal of Financial Economics* 130:215-236.
- [10] Benveniste, L.M., and P.A. Spindt. 1989. “How Investment Bankers Determine the Offer Price and Allocation of New Issues.” *Journal of Financial Economics* 24:343–362.
- [11] Biais, B., and T. Mariotti. 2005. Strategic Liquidity Supply and Security Design. *Review of Economic Studies* 72:615–49.
- [12] Bolton, P., and E.L. von Thadden. 1998. “Blocks, Liquidity, and Corporate Control.” *Journal of Finance* 53:1-25.
- [13] Booth, J.R., and L. Chua. 1996. “Ownership Dispersion, Costly Information, and IPO Underpricing.” *Journal of Financial Economics* 41:291-310.
- [14] Borisov, A., A. Ellul, and M. Sevilir. 2021. “Access to Public Capital Markets and Employment Growth.” *Journal of Financial Economics* 141:896-918.
- [15] Brau, J.C., and S.E. Fawcett. 2006. “Initial Public Offerings: An Analysis of Theory and Practice.” *Journal of Finance* 61:399-436.
- [16] Brennan, M.J. and J. Franks. 1997. “Underpricing, Ownership and Control in Initial Public Offerings of Equity Securities in the U.K.” *Journal of Financial Economics* 45:391-413.
- [17] Burkart, M., D. Gromb, and F. Panunzi. 1997. “Large Shareholders, Monitoring, and the Value of the Firm.” *Quarterly Journal of Economics* 112:693–728.
- [18] Burnie, D.A., and J.P. Ogden. 1996. “Initial Corporate Debt Offerings, Certification and Equity Valuation.” *Journal of Business Finance & Accounting* 23:399-417.
- [19] Busaba, W.Y., L.M. Benveniste, and R.-J. Guo. 2001. “The Option to Withdraw IPOs during the Premarket: Empirical Analysis.” *Journal of Financial Economics* 60:73-102.

- [20] Carlsson, H., and E. van Damme. 1993. "Global Games and Equilibrium Selection." *Econometrica* 61:989-1018.
- [21] Chakraborty, A., S. Gervais, and B. Yilmaz. 2011. "Security Design in Initial Public Offerings." *Review of Finance* 15:327-357.
- [22] Chemmanur, T.J. 1993. "The Pricing of Initial Public Offerings: A Dynamic Model with Information Production." *Journal of Finance* 48:285-304.
- [23] Chemmanur, T.J., and P. Fulghieri. 1999. "A Theory of the Going-Public Decision." *Review of Financial Studies* 12:249-279.
- [24] Cho, S-I. 1993. "Who Failed to Go Public with Best Effort Offerings." *Journal of Small Business Finance* 3:63-77.
- [25] Cornelli, F., and D. Goldreich. 2001. "Bookbuilding and Strategic Allocation." *Journal of Finance* 56:2337-2370.
- [26] —. 2003. "Bookbuilding: How Informative is the Order Book?" *Journal of Finance* 58:1415-1443.
- [27] Datta, S., M. Iskandar-Datta, and A. Patel. 1997. "The Pricing of Initial Public Offers of Corporate Straight Debt." *Journal of Finance* 52:379-396.
- [28] —. 1999. "Bank Monitoring and the Pricing of Corporate Public Debt." *Journal of Financial Economics* 51:435-449.
- [29] DeAngelo, H., DeAngelo, L., and T.M. Whited. 2011. "Capital Structure Dynamics and Transitory Debt." *Journal of Financial Economics* 99:235-261.
- [30] DeAngelo, H., and R. Roll. 2015. "How Stable are Corporate Capital Structures?" *Journal of Finance* 70:373-418.

- [31] DeMarzo, P.M. 2005. “The Pooling and Tranching of Securities: a Model of Informed Intermediation.” *Review of Financial Studies* 18:1-35.
- [32] DeMarzo, P.M., and D. Duffie. 1999. “A Liquidity Based Model of Security Design.” *Econometrica* 67:65-99.
- [33] DeMarzo, P.M., D.M. Frankel, and Y. Jin. 2021. “Portfolio Liquidity and Security Design with Private Information.” *Review of Financial Studies* 34:5841–5885.
- [34] Denis, D.J., and S.B. McKeon. 2012. “Debt Financing and Financial Flexibility: Evidence from Proactive Leverage Increases.” *Review of Financial Studies* 25:1897-1929.
- [35] Dunbar, C.G. 1998. “The Choice between Firm-Commitment and Best-Efforts Offering Methods in IPOs: The Effect of Unsuccessful Offers.” *Journal of Financial Intermediation* 7:60-90.
- [36] Foster, D., and P.H. Young. 1990. “Stochastic Evolutionary Game Dynamics.” *Theoretical Population Biology* 38:219–232.
- [37] Frankel, David M. 2023. “Equilibrium Selection in Participation Games: A Unified Framework.” Mimeo, Melbourne Business School.
- [38] —. 2023. “Online Appendix to ‘Security Issuance with Undersubscription Risk.’” Mimeo, Melbourne Business School.
- [39] Frankel, D.M., and Y. Jin. 2015. “Securitization and Lending Competition.” *Review of Economic Studies* 82: 1383-1408.
- [40] Fridson, M.S., and M.C. Garman. 1998. “Determinants of Spreads on New High-Yield Bonds.” *Financial Analysts Journal* 54:28-39.
- [41] Fudenberg, D., and C. Harris. 1992. “Evolutionary Dynamics with Aggregate Shocks.” *Journal of Economic Theory* 57:420-441.

- [42] Goldstein, M.A., P. Irvine, and A. Puckett. 2011. "Purchasing IPOs with Commissions." *Journal of Financial and Quantitative Analysis* 46:1193-1225.
- [43] Goldstein, I., and A. Pauzner. 2005. "Demand-Deposit Contracts and the Probability of Bank Runs." *Journal of Finance* 60:1293-1327.
- [44] Grinblatt, M., and C.Y. Hwang. 1989. "Signaling and the Pricing of New Issues." *Journal of finance* 44:393-420.
- [45] Grinblatt, M.S., R.W. Masulis, and S. Titman. 1984. "The Valuation Effects of Stock Splits and Stock Dividends." *Journal of Financial Economics* 13:461-490.
- [46] Habib, M.A., and A.P. Ljungqvist. 2001. "Underpricing and Entrepreneurial Wealth Losses in IPOs: Theory and Evidence." *Review of Financial studies* 14:433-458.
- [47] Halac, M., I. Kremer, and E. Winter. 2020. "Raising Capital from Heterogeneous Investors." *American Economic Review* 110:889-921.
- [48] Hanley, K.W. 1993. "The Underpricing of Initial Public Offerings and the Partial Adjustment Phenomenon." *Journal of Financial Economics* 34:231-250.
- [49] Hart, O., and J. Moore. 1995. "Debt and Seniority: an Analysis of the Role of Hard Claims in Constraining Management." *American Economic Review* 85:567-585.
- [50] Henderson, B.J., N. Jegadeesh, and M.S. Weisbach. 2006. "World Markets for Raising New Capital." *Journal of Financial Economics* 82:63-101.
- [51] Huyghebaert, N., and C. Van Hulle. 2006. "Structuring the IPO: Empirical Evidence on the Portions of Primary and Secondary Shares." *Journal of Corporate Finance* 12:296-320.
- [52] Im, H.J., C. Mayer, and O. Sussman. 2020. "Heterogeneity in Investment Spike Financing." Mimeo.

- [53] Jegadeesh, N., M. Weinstein, and I. Welch. 1993. "An Empirical Investigation of IPO Returns and Subsequent Equity Offerings." *Journal of Financial Economics* 34:153-175.
- [54] Jenkinson, T., H. Jones, and F. Suntheim. 2018. "Quid Pro Quo? What Factors Influence IPO Allocations to Investors?" *Journal of Finance* 73:2303-2341.
- [55] Kandori, M., G. Mailath, and R. Rob. 1993. "Learning, Mutation and Long Equilibria in Games." *Econometrica* 61:27-56.
- [56] Kim, W., and M.S. Weisbach. 2008. "Motivations for Public Equity Offers: An International Perspective." *Journal of Financial Economics* 87:281-307.
- [57] Lakonishok, J., and B. Lev. 1987. "Stock Splits and Stock Dividends: Why, Who, and When." *Journal of Finance* 42:913-932.
- [58] Lee, G., and R.W. Masulis. 2011. "Do More Reputable Financial Institutions Reduce Earnings Management by IPO Issuers?" *Journal of Corporate Finance* 17:982-1000.
- [59] Leland, H.E., and D.H. Pyle. 1977. "Informational Asymmetries, Financial Structure, and Financial Intermediation." *Journal of Finance* 32:371-387.
- [60] Lowry, M., R. Michaely, and E. Volkova. 2017. "Initial Public Offerings: A Synthesis of the Literature and Directions for Future Research." *Foundations and Trends in Finance*, vol. 11, no. 3-4, pp. 154-320.
- [61] Lucas, D., and R. McDonald. 1990. "Equity Issues and Stock Price Dynamics." *Journal of Finance* 45:1019-1043.
- [62] Matsui, A., and K. Matsuyama. 1995. "An Approach to Equilibrium Selection." *Journal of Economic Theory* 65:415-434.
- [63] Michaely, R., and W.H. Shaw. 1994. "The Pricing of Initial Public Offerings: Tests of Adverse- Selection and Signaling Theories." *Review of Financial Studies* 7:279-319.

- [64] Myers, S.C. 1984. “The Capital Structure Puzzle.” *Journal of Finance* 39:575–592.
- [65] Myers, S., and N. Majluf. 1984. “Corporate Financing and Investment when Firms have Information Shareholders do Not Have.” *Journal of Financial Economics* 13:197–221.
- [66] Nachman, D., and T. Noe. 1994. “Optimal Design of Securities under Asymmetric Information.” *Review of Financial Studies* 7:1-44.
- [67] Pesendorfer, W., and J.M. Swinkels. 2000. “Efficiency and Information Aggregation in Auctions.” *American Economic Review* 90:499-525.
- [68] Plantin, G. 2009. “Learning by Holding and Liquidity.” *Review of Economic Studies* 76:395-412.
- [69] Reuter, J. 2006. “Are IPO Allocations for Sale? Evidence from Mutual Funds”. *Journal of Finance* 61:2289–2324.
- [70] Ritter, J.R. 1987. “The Costs of Going Public.” *Journal of Financial Economics* 19:269-281.
- [71] ——. 1991. “The Long-Run Performance of Initial Public Offerings.” *Journal of Finance* 46:3-27.
- [72] Ritter, J.R., and I. Welch. 2002. “A Review of IPO Activity, Pricing, and Allocations.” *Journal of Finance* 57:1795-1828.
- [73] Ritter, J.R., and D. Zhang. 2007. “Affiliated Mutual Funds and the Allocation of Initial Public Offerings”. *Journal of Financial Economics* 86:337–368.
- [74] Robbins, R.B. 2013. “Structuring Best Efforts Offerings and Closings under Rule 10b-9.” Outline, American Law Institute, Continuing Legal Education.

- [75] Rock, Kevin. 1986. “Why New Issues are Underpriced.” *Journal of Financial Economics* 15:187-212.
- [76] Royden, H.L., and P.M. Fitzpatrick. 2010. *Real Analysis, 4th ed.* New York: Pearson.
- [77] Scharfstein, D.S., and J.C. Stein. 1990. “Herd Behavior and Investment.” *American Economic Review* 80:465-479.
- [78] Shaked, M., and J.G. Shanthikumar. 2007. *Stochastic Orders.* New York: Springer.
- [79] Sherman, A.G. 1992. “The Pricing of Best Efforts New Issues.” *Journal of Finance* 47:781-790.
- [80] Spiess, D. K., and R.H. Pettway. 1997. “The IPO and First Seasoned Equity Sale: Issue Proceeds, Owner/Managers’ Wealth, and the Underpricing Signal.” *Journal of Banking & Finance* 21:967-988.
- [81] Townsend, R.M. 1979. “Optimal Contracts and Competitive Markets with Costly State Verification.” *Journal of Economic Theory* 21:265-293.
- [82] U.S. Security and Exchange Commission, Feb. 21, 1962. “Securities Exchange Act Release No. 6737. [1961-64 Transfer Binder]” *Fed. Sec. L. Rep. (CCH)* ¶ 76,825.
- [83] Welch, I. 1989. “Seasoned Offerings, Imitation Costs, and the Underpricing of Initial Public Offerings.” *Journal of Finance* 44:421-449.
- [84] —. 1991. “An Empirical Examination of Models of Contract Choice in Initial Public Offerings.” *Journal of Financial and Quantitative Analysis* 26:497-518.
- [85] —. 1992. “Sequential Sales, Learning, and Cascades.” *Journal of Finance* 47:695-732.
- [86] Zingales, L. 1995. “Insider Ownership and the Decision to Go Public.” *Review of Economic Studies* 62:425-448.