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JEL Classification: D52, E31, E43, E52, E62, E63

Keywords: Price level, incomplete markets, inflation, Monetary policy, Fiscal policy

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Abstract

In this paper I show that the price level is globally unique in incomplete markets models. I base my argument on the simple idea that the price equates demand with supply in the goods market. Monetary policy works through setting nominal interest rates, e.g. an interest rate peg, while fiscal policy is committed to satisfying the present value budget constraint at all times (in contrast to the FTPL). Together, these determine the unique price level, as well as consumption and employment, jointly. In particular, the model predicts a unique equilibrium in response to a fiscal stimulus if the nominal interest is pegged, whereas there is a continuum of equilibria in the standard New Keynesian model. In contrast to the conventional view the long-run inflation rate is, in the absence of output growth, equal to the growth rate of nominal government spending, which is controlled by fiscal policy. This new theory where nominal government spending anchors aggregate demand, and therefore current and future prices, offers a different perspective on a range of important issues including the fiscal and monetary transmission mechanism, policy coordination, policies at the zero-lower bound, U.S. inflation history and recent attempts to stimulate inflation in the Euro area.

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1 Introduction

This paper proposes a novel and to the best of my knowledge, the first theory that generates a globally determinate price level, with monetary and fiscal policies resembling those actually implemented in practice. The idea is quite simple. Monetary policy works through setting an arbitrary sequence of nominal interest rates, for example through an interest rate peg. Fiscal policy sets sequences of nominal government spending, taxes and government debt, and these sequences satisfy the present value government budget constraint at all times. The price level is then determined such that demand equals supply in the goods market. Both monetary and fiscal policies that aim at increasing or decreasing the price level are effective in this endeavor only if they can stimulate or contract nominal aggregate demand.

The importance of a uniquely determined price level becomes obvious if we consider the problems encountered when prices are indeterminate. The presumably most compelling illustration of these is in Cochrane (2015)'s policy analysis of new-Keynesian models during a liquidity trap. Cochrane (2015) shows that the policy predictions depend heavily on the researcher's choices when selecting a specific equilibrium. These specific choices can cause the fiscal multiplier to be arbitrarily large or negative. The root of this problem is clear. Since the price level is indeterminate (Sargent and Wallace (1975)), the researcher must select one equilibrium. Yet, a continuum of other choices exists and these entail very different policy predictions. The resulting lack of robust policy implications is unsatisfactory.[1]

To overcome these difficulties, I propose a demand theory of the price level, motivated by the findings of a large empirical literature which rejects the permanent income hypothesis (Campbell and Deaton (1989), Attanasio and Davis (1996), Blundell et al. (2008), Attanasio and Pavoni (2011)).[2] One of the striking empirical findings in this literature is that a

[1]Sargent and Wallace (1975) observed that an interest rate target determines only expected inflation but not the price level. The literature since then has proposed various selection mechanism, the most prominent one being an active interest rate rule which implies a locally determined inflation rate, but still an indeterminate price level. In contrast, the theory proposed in this paper implies a determinate price level which automatically implies a determinate inflation rate. The theory also implies a determinate long-run price level which anchors households' inflation expectations and the real interest rate, and thus renders it possible to calculate a well-defined fiscal multiplier.

[2]The same empirical literature motivates Kaplan et al. (2016), Auclert (2016) and Lütticke (2015) to study monetary policy in a model with incomplete markets and pricing frictions, however with a different focus. Whereas these authors emphasize and quantify several redistributive channels of the transmission mechanism of monetary policy which are absent in standard complete market models, the price level is endogenously determinate in equilibrium only in my paper. Earlier contributions are Oh and Reis (2012) and Guerrieri and Lorenzoni (2015), who were among the first to add nominal rigidities to a Bewley-Imrohoroglu-Huggett-Aiyagari model and Gornemann et al. (2012) who were the first to study monetary policy in the same environment. More recent contributions include McKay and Reis (2016) (impact of automatic stabilizers),

permanent income gain - like a permanent tax rebate - does increase household consumption less than one-for-one and thus increases savings too. This simple fact is the key to the determinacy result, as it implies that taking prices as given, a permanent decrease in government spending by one dollar and a simultaneous permanent tax rebate of the same amount to private households lowers real total aggregate demand - the sum of private and government demand. The same logic also establishes why in a steady state, real aggregate demand depends on the price level. Given monetary and fiscal policy, a higher steady-state price level lowers real government consumption since government spending is (partially) fixed in nominal terms. At the same time, it lowers the tax burden for the private sector by the same amount. As private sector demand does not substitute one-for-one for the drop in government consumption but instead saves a fraction of the tax reduction, aggregate real demand falls, establishing a downward sloping aggregate demand-price curve. The unique equilibrium steady-state price level is then such that aggregate real demand equals real supply.[3] It is important to point out that here the price level is determinate, whereas it would remain an unknown variable if only the inflation rate were determinate, as is the typical case in new-Keynesian models with an active monetary policy. Moving beyond steady-state results and establishing price determinacy globally requires us to assume that the above empirical finding holds also outside the steady state; that is, that a precautionary demand never vanishes.[4]

It is important to emphasize, however, that first, it is the presence of precautionary savings that delivers the result and second, prices are not determinate in every model where Ricardian equivalence fails. For example, the price level is not determinate in an economy where a fraction of households are hand-to-mouth consumers while the remaining households act according to the permanent income hypothesis (PIH). The reason for the indeterminacy is the absence of precautionary savings. The PIH consumers increase their consumption one-for-one in response to a permanent tax rebate, since this what PIH households do. The hand-to-mouth consumers do the same, not as a result of optimization but by assumption. In such a model an increase in the price level also lowers real government consumption (since

McKay et al. (2015) (forward guidance), Bayer et al. (2015) (impact of time-varying income risk), Ravn and Sterk (2013) (increase in uncertainty causes a recession) and Den Haan et al. (2015) (increase in precautionary savings magnifies deflationary recessions).

[3]Werning (2015) (using incomplete markets) and Angeletos and Lian (2016) (using incomplete information) also propose theories of aggregate demand, with the important difference that those theories are real whereas I propose a nominal theory, a prerequisite to obtain a determinate price level.

[4]In a standard incomplete markets model (Bewley-Imrohoroglu-Huggett-Aiyagari), the precautionary savings motive arises due to a potentially binding credit constraint and thus the assumption is trivially satisfied without an additional assumption, as some households are always constrained.

it is fixed in nominal terms) and increases private consumption. The higher price level does not affect total consumption though, because the drop in government consumption is offset exactly by the increase in private demand. As a result aggregate demand equals supply for an infinite number of price levels, each of these corresponding to a different size of government.

To keep the model tractable, I use the simplest setup that delivers the empirical finding on individual consumption and precautionary saving behavior discussed above. The key simplifying assumption is that households are members of a family that provides insurance, such that the distribution of asset holdings across agents is degenerate. These families live in an infinite horizon endowment economy without capital which in terms of preferences, technology and trading arrangements is closer to an infinite horizon replication of a Diamond and Dybvig (1983) economy than to conventional macroeconomic models. It is not difficult to integrate this framework into a standard one such as the Aiyagari (1994) incomplete markets model and then to explore policy implications quantitatively, since the price level is determined in a large class of incomplete market models as I show in Section 2.[5] An advantage of using the simpler framework, besides enabling the researcher to better understand the monetary and fiscal transmission mechanism, is that a banking sector can be added to the model. Although beyond the scope of this paper, such an extension will allow the researcher to study the interaction of banking distress, policy, deflation and the real economy in a model where the price level is determinate.

The main result of the theoretical analysis is that the price level is globally uniquely determinate and that it depends on both monetary and fiscal policy. To illustrate the workings of the model I add some key features for a meaningful numerical analysis: labor supply is endogenous, prices are sticky and only a small fraction of government spending is nominally fixed. I then numerically compute impulse responses to monetary and fiscal policy shocks as well as to technology and discount factor shocks. I find that all impulse responses are in line with their empirical counterparts, a finding which is easily explained by the supply/demand logic at the foundation of the determinacy result. An increase in nominal interest rates stimulates saving and therefore lowers consumption demand, implying a drop in prices. An increase in government spending stimulates aggregate demand, implying a rise in prices. An increase in technology raises supply and households' incentives to save, implying a drop in prices. And finally an increase in the discount factor stimulates savings since households are more patient, implying a drop in prices. Quite remarkably, the model delivers these results not only for sticky prices but also when prices are flexible. In particular,

[5]For such an exploration see for example Hagedorn et al. (2016) who study the size of the fiscal multiplier.

price rigidities are not needed for monetary policy to have effects.[6]

In the theory proposed here fiscal policy provides a nominal anchor through setting nominal spending and nominal bonds, making the treasury a key player in determining the price level and *the* key player in determining the long-run inflation rate. The steady-state inflation rate is equal to the growth rate of nominal government spending (minus productivity growth) and therefore is controlled by the treasury. In contrast to conventional wisdom, a tough, independent central bank not only is insufficient to guarantee price stability in the long-run, but also has no direct control over long-run inflation. By controlling the nominal anchor the treasury always wins out when it comes to long-run inflation. However, if the treasury does not exercise its power, for example if government spending is fixed in real and not in nominal terms, the central bank takes over the job of determining the steady-state inflation rate as conventional wisdom suggests. But here monetary policy does so in a model with a determinate price level (since government debt is nominal).

The price level is always controlled jointly by fiscal and monetary policies, as the impulse responses to increases in spending and in the nominal interest rate already suggest. Fiscal policy can raise the price level and stimulate employment through actively increasing spending. But an apparently passive fiscal policy, one that fixes nominal spending, also automatically stabilizes the economy. Consider an increase in the discount factor, which lowers prices and contracts employment. Lower prices automatically lead to higher real government spending since nominal spending is fixed, and thus stimulate aggregate demand which partly offsets the fall in prices and employment. The effectiveness of expansionary fiscal policy in stabilizing the economy depends also on how it is financed, through higher deficits or higher taxes. Not surprisingly, raising taxes is less effective than increasing deficits, because higher taxes lead to lower demand and thus partially offset the stimulative demand effects of higher government spending.

Monetary policy can lower the price level and employment through increasing nominal interest rates and is quite effective in stabilizing the economy. Again as an example, consider an increase in the discount factor. This shock can be fully neutralized through lowering nominal interest rates by the same amount as the increase in the discount factor, which keeps unchanged the effective discounting - given by the product of the discount factor and the nominal interest rate - by households. All variables, including employment, prices and consumption remain at their steady-state values. In contrast, an expansion in government

[6]Garriga et al. (2013), Sterk and Tenreyro (2015) and Buera and Nicolini (2016) among others also find that monetary policy has real effects in an incomplete market models with flexible prices.

spending cannot stabilize employment and thus output and consumption at the same time, suggesting that monetary policy is the more effective option for stabilizing the economy.

However, the zero lower bound (ZLB) makes monetary policy ineffective. This is the case if the increase in the discount factor is so large that stabilizing the economy only through nominal interest rates requires setting these at a negative value, which is impossible (the ZLB binds). In this scenario, expansionary fiscal policy can stabilize aggregate demand and completely offset the contractionary effects of the discount factor increase, such that prices and employment remain at their steady-state values.

A stimulative policy initiated by the treasury to raise employment and prices requires coordination with monetary policy. In the absence of coordination, a central bank committed only to price stability can raise nominal interest rates and cancel out the price and employment effects of the fiscal stimulus. The result of this (attempted) stimulative fiscal policy and the response of monetary policy is: no change in employment and prices but higher government spending, taxes and debt. It is obvious that policy coordination extends beyond this example, simply because monetary and fiscal policy jointly determine demand and prices in this framework. Such political economy questions naturally come up in a framework where the price level is determinate, and they can be answered because the price level is determinate.

While monetary policy can neutralize short-run inflationary fiscal policy, taming inflation in the medium and long runs requires constraining fiscal policy from running an inflationary spending plan. Although the central bank does not set the spending itself, and therefore the treasury can control medium and long-run inflation if it wants to, control of the nominal interest rate is an effective tool for making an inflationary policy quite costly. Raising the nominal interest rate raises the interest payments on government debt, which can sharply constrain government spending. High nominal interest rates may force the treasury into a less expansionary fiscal policy, and thus indirectly lead to a lower inflation rate. Since there is no upper bound on nominal interest rates, there is no limit on the cost the central bank can inflict on the treasury. But the central bank can also support an expansionary fiscal policy through lowering nominal interest rates, if it considers inflation to be too low. However, the ZLB puts a limit on the budgetary support the central bank can provide. The independence of central banks guarantees that those interest rate decisions are taken through monetary and not fiscal policy. As central banks arguably put more weight on price stability than treasuries do (they are more interested in taming deficits), independence leads to a more active interest rate policy to curb inflation than if the treasury were to control the interest rate.

This reasoning and the theory set out in this paper suggest a different perspective on US inflation history. After experiencing high inflation rates in the 1970s, the 1980s saw success in keeping inflation rates low. The standard interpretation is that central banks eventually recognized that keeping inflation low was their primary objective and as a consequence, were successful in doing so. The framework proposed in this paper suggests that it was not the change in the conduct of monetary policy that kept inflation in check but a shift to a less expansionary fiscal policy during the presidency of Ronald Reagan, perhaps so forced by the prolonged high nominal interest rates set by central banks under chairman Paul Volcker and resulting high deficits.

Before presenting the model, I start with a graphical analysis in Section 2 to show that the steady-state price level is determinate in a large class of incomplete market models. I explain the mechanism behind the determinacy result; why market incompleteness is necessary for determinacy and why complete markets lead to indeterminacy; why fiscal policy has to be partially nominal; why this is not the FTPL; and why adding capital or cash to the model does not alter these conclusions. I also use the graphical analysis to show how the price level responds to monetary and fiscal policy, and to changes in technology and in the need for liquidity, and find these responses to be in line with conventional wisdom. For pedagogical purposes, I then move to a baseline model where labor is supplied inelastically and prices are flexible, in Section 3. In Section 4 I prove price level determinacy for arbitrary sequences of monetary and fiscal policy. I show that monetary and fiscal policy together determine the price level and the inflation rate, and how these variables respond to policy changes and shocks. In Section 5, I describe the extension to the case of elastic labor supply and sticky prices. I also present some numerical exercises to illustrate the workings of the model, computing impulse responses, government spending as an automatic stabilizer, coordination of monetary and fiscal policy and policies at the ZLB. Section 6 concludes.

2 The Price Level and Incomplete Markets: A Graphical Analysis

In this Section I provide a graphical analysis to argue that the steady-state price level is determinate in a large class of incomplete market models. I start with an endowment economy with uninsurable idiosyncratic labor income risk, based on Huggett (1993), where only one asset - a nominal government bond - can be traded subject to exogenously imposed borrowing limits. The key features sufficient for determinacy of the steady-state price level are that steady-state savings depend on the real interest rate, and that fiscal policy is partially nominal. The determinacy of the price level outside the steady state is considered in Sections 3 and 4 where I use a simple incomplete markets model which features tractable out of steady-state dynamics.

Equilibrium in a Huggett Economy

I consider a cashless economy (Woodford (2003)) where monetary policy sets the steady-state nominal interest rate $R = 1 + i$. Fiscal policy sets nominal government spending G, nominal taxes T and nominal bonds B such that the steady-state government nominal budget constraint holds, $iB + G = T$.[7] All policies are exogenous. As is well known the Huggett economy is in equilibrium iff aggregate asset supply (households' savings) equals real aggregate asset demand (government bonds), which can be represented by the well-known Figure 1.

Households' savings $S(1 + r, ...)$ is an upward sloping function of the real interest rate $1+r$. Other variables such as taxes, transfers, and properties of the income process shift the savings function. Real asset demand by the government equals B/P, where B is nominal bonds and P is the price level.[8] The equilibrium condition is

$$S(1 + r, ...) = \frac{B}{P}. \tag{1}$$

This is one equation with two unknowns, the real interest rate $1 + r$ and the price level P. I will now argue that this equation nevertheless determines the price level since the real interest rate is determined by monetary and fiscal policy.

How Monetary and Fiscal Policy determine the Steady-State Real Interest Rate:

In both complete and incomplete market models, a Fisher relation between the steady state nominal interest i_{ss}, real interest rate r_{ss} and inflation π_{ss} holds:

$$1 + r_{ss} = \frac{1 + i_{ss}}{1 + \pi_{ss}}. \tag{2}$$

[7]The government budget constraint is assumed to involve nominal variables only to make clear that it holds independent of the price level. Note that this is in contrast to the FTPL where the price level is such that it clears the government budget constraint. As I explain below, price level determinacy does not require that fiscal policy is fully nominal, only partially so.

[8]With positive inflation rate π, bonds in a steady state a time t equal $B(1+\pi)^t$ and the price level equals $P(1+\pi)^t$ for initial values B and P, so that the term $(1+\pi)^t$ term cancels when computing the real value of bonds B/P.

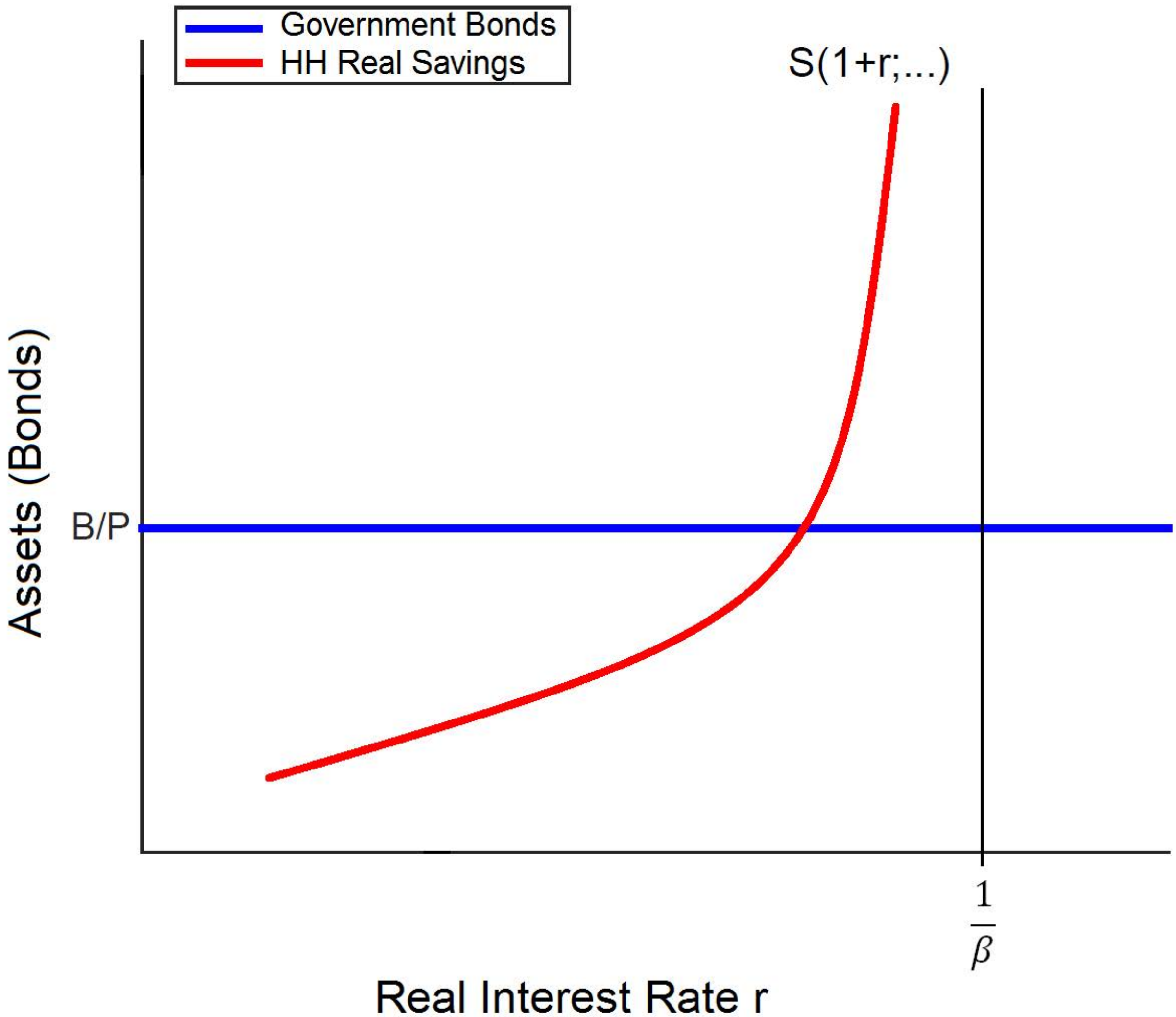


Figure 1: Asset Market in Huggett economy

Monetary policy sets the steady-state nominal interest rate i_{ss}. Fiscal policy sets the growth rate of nominal spending (G), nominal tax revenues (T) and nominal debt (B). In a steady state, real government spending, real tax revenue and real government debt are constant, such that the steady-state condition for fiscal policy is that the growth rates of nominal spending, nominal tax revenues and nominal debt all are equal to the inflation rate (in the absence of economic growth),[9]

$$1 + \pi_{ss} = \frac{G' - G}{G} = \frac{T' - T}{T} = \frac{B' - B}{B}. \tag{3}$$

To be clear about the interpretation of these steady-state conditions: If fiscal policy decides

[9] With real economic growth of rate g, $(1 + \pi_{ss})(1 + g) = \frac{G'-G}{G} = \frac{T'-T}{T} = \frac{B'-B}{B}$.

for a 2% nominal growth rate in government spending, $\frac{G'-G}{G}$, then the steady state condition that steady state real government expenditures are constant requires that the steady-state inflation rate equals 2% as well. The steady-state further requires that nominal tax revenues T and nominal government debt B also grow at 2%. It is important to note that these considerations do not determine the levels of real spending, real taxes and real debt except in the sense that these are unchanging over time in a steady state. In particular, the price level is not yet determined.

Equation (3) means that the inflation rate is set by fiscal policy and is equal to the growth rate of nominal government spending, implying that the equilibrium real interest rate is determined jointly by monetary and fiscal policy.[10]

Price Level Determinacy

I can now use equation (1) to determine the price level. Using the result that $(1+r_{ss}) = \frac{1+i_{ss}}{1+\pi_{ss}}$ is set by policy to eliminate the real interest rate from the list of unknowns, equation (1) now has just one unknown, the price level P^*:

$$S(\frac{1+i_{ss}}{1+\pi_{ss}}, ...) = \frac{B}{P^*}, \tag{4}$$

which serves to determine the unique price level, as illustrated in Figure 2.

There two key assumptions to obtain price level determinacy. First, fiscal policy is nominal. Without this assumption fiscal policy would be specified fully in real terms, and the equilibrium in the asset market would not depend on the price level. Thus this equilibrium condition cannot be used to determine the price level. Second, there is a steady-state aggregate savings function, which depends on the real interest rate. This is a standard result in models with heterogeneous agents and market incompleteness. I explain below in detail why the arguments for the Huggett economy do not apply in complete market environments.

Price Level Determinacy with Fully Indexed Bonds

I now generalize the discussion of price level determinacy in the Huggett economy. For illustrative purposes, I make the extreme assumption that the real value of government bonds is fixed at B^{real} and that government spending and taxes are fully nominal.[11]

[10]Monetary and fiscal policy cannot implement any arbitrary steady-state real interest rate, only one that is consistent with a steady state. In particular, as in any incomplete markets model, $\beta(1+r_{ss}) < 1$ has to hold since otherwise asset demand would become infinite.

[11]This assumption also makes clear that the theory in my paper is different from the Fiscal Theory of the Price Level (FTPL), where the price level is determined such that the real value of bonds clears the government present value budget constraint. Obviously the FTPL has no bite if the real value of bonds is fixed and nominal taxes are set to balance the budget every period.

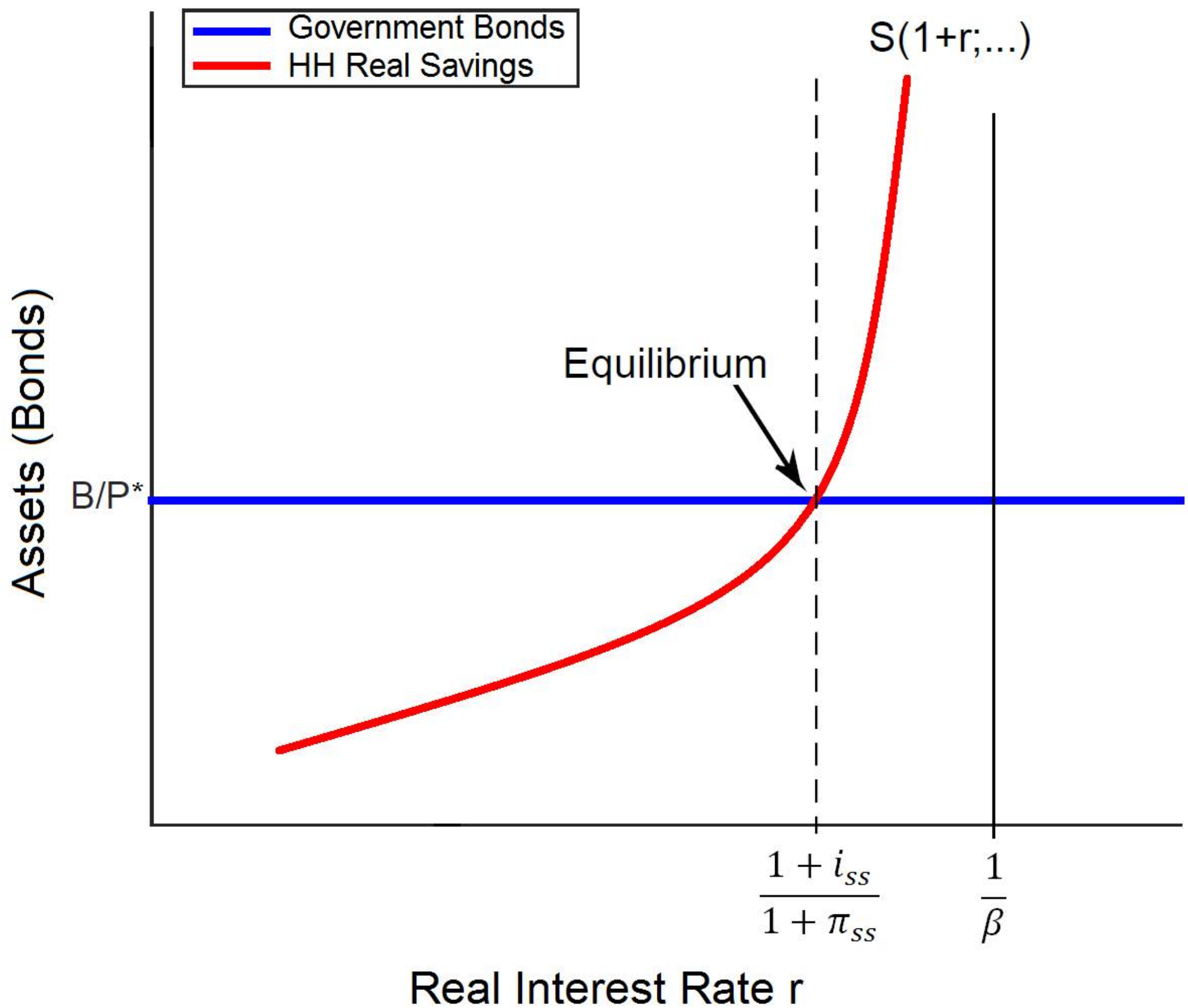


Figure 2: Asset Market in Huggett economy

In the above discussion I assumed that bonds are nominal but did not have households' real savings depend on nominal variables as well,

$$S(1 + r, G/P, T/P, \ldots). \tag{5}$$

The reason why households' real savings depend on the level of real taxes, T/P, for a fixed real interest rate is again explained by heterogeneity and incomplete markets. Those features imply the failure of the permanent income hypothesis and that agents, as a result of this failure, engage in precautionary savings: A lower steady-state level of real taxes increases both demand and (precautionary) savings. This reasoning extends to changes in the price level which translate one-for-one into changes in real taxes, since the nominal level of taxes in fixed.

The intuition is straightforward. A higher-steady-state price level lowers real government consumption since fiscal policy is nominal, and at the same time lowers the tax burden for the private sector by the same amount. Households, however, do not spend all of the tax rebate on consumption but instead use some of the tax rebate to increase their precautionary savings. This less than one-for-one substitution of private sector demand for government consumption implies a drop in aggregate demand (households plus government demand) and an increase in households' savings, what would require an adjustment of the real interest rate to stimulate demand and lower savings such that both the goods and the asset markets clear. As in the scenario considered above, however, the steady-state real interest rate cannot adjust to equate supply and demand because it is pinned down by the nominal interest rate set by monetary policy, and by the inflation rate, which is equal to the growth rate of nominal government spending. Therefore the price level must adjust such that demand equals supply when the real interest rate equals $1 + r_{ss} = \frac{1+i_{ss}}{1+\pi_{ss}}$,

$$S(\frac{1 + i_{ss}}{1 + \pi_{ss}}, G/P^*, T/P^*, ...) = B^{real}, \tag{6}$$

as Figure 3 illustrates.

In the general case that all fiscal variables, T, G and B are nominal the equilibrium condition determining the price level P^* is

$$B/P^* = S(\frac{1 + i_{ss}}{1 + \pi_{ss}}, G/P^*, T/P^*, ...). \tag{7}$$

Instead of writing savings as a function of the real interest rate, one can also write the real interest rate as a function of assets, so that the equilibrium in the asset market can be represented equivalently as

$$\frac{1 + i_{ss}}{1 + \pi_{ss}} = (1 + r_{ss}(T/P^*, B/P^*, \ldots)) \tag{8}$$

which again determines the steady state price level P^*. The same arguments made above imply that the real interest rate r_{ss} depends on B/P and T/P.[12]

Price Level and Aggregate Demand

[12] The interpretation is that this is the real interest rate that makes households willing to hold B/P real assets in steady state.

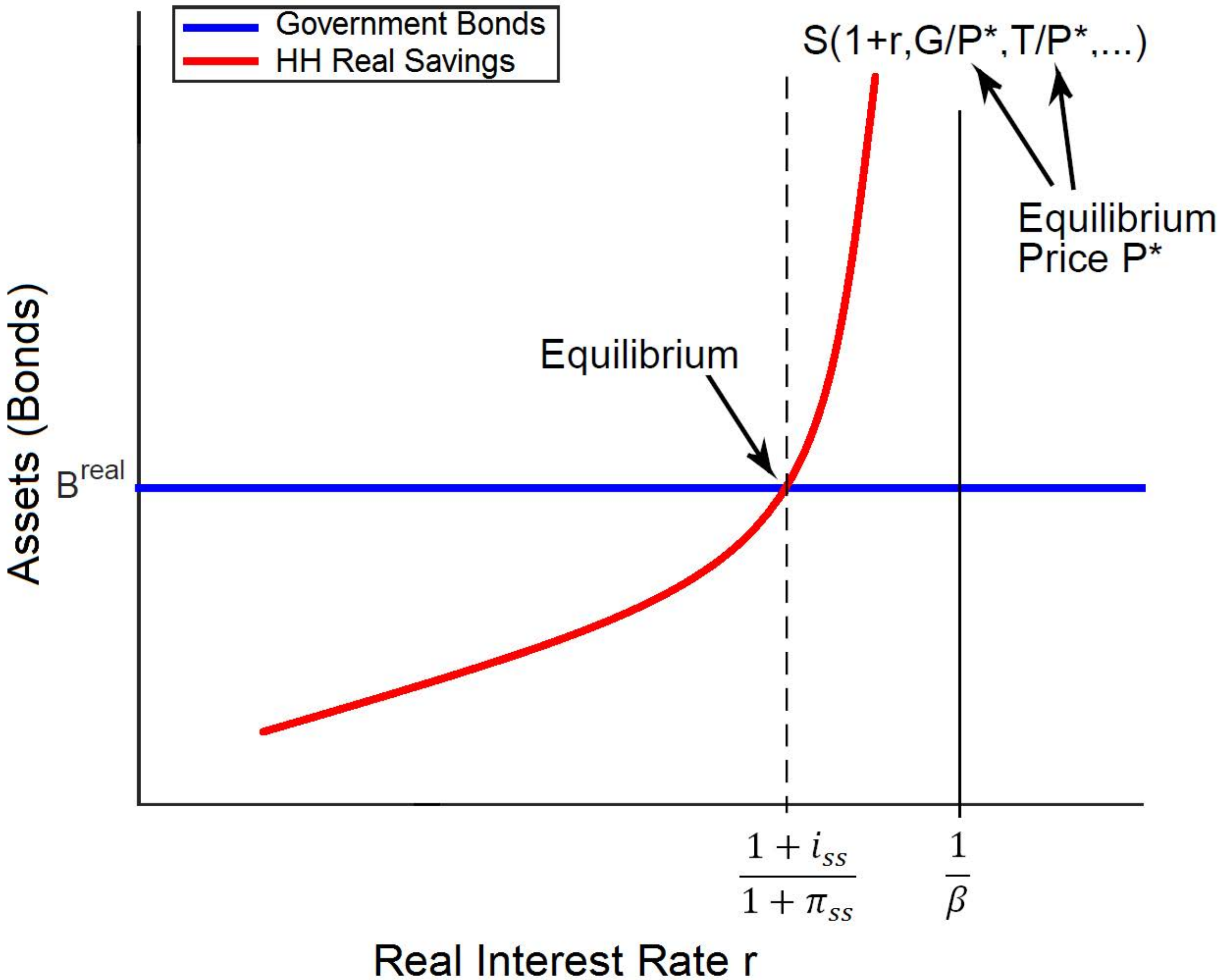


Figure 3: Asset Market in Huggett economy with Price-Indexed Government Debt B^{real}.

There is an equivalent characterization of the price level as clearing the goods market, which not surprisingly mirrors the above characterization as clearing the asset market. Private steady-state real demand equals, where Y is real income,

$$D(1 + r, T/P, ...) = Y + \frac{RB - T}{P} - S(1 + r, T/P, ...), \tag{9}$$

which is a function of the real interest rate and the price level. Demand depends on the real interest rate because savings do. It also depends on the price level, since private and public consumption are not one-for-one substitutes. Using the government budget constraint

$RB - T = B - G$,

$$D(1+r, P, ..) = Y + \frac{B-G}{P} - S(1+r, T/P, ...), \tag{10}$$

so that aggregate demand, the sum of private demand D and government consumption G/P,

$$D(1+r, P, ..) + \frac{G}{P} = Y + \frac{B}{P} - S(1+r, T/P, ...), \tag{11}$$

which, since private and public consumption are not perfect substitutes, is downward sloping in the price level,

$$\frac{\partial D(1+r, P, ..) + \frac{G}{P}}{\partial P} < 0, \tag{12}$$

determining a unique steady-state price level as equating supply and demand,

$$D(\frac{1+i_{ss}}{1+\pi_{ss}}, P^*, ..) + \frac{G}{P^*} = Y. \tag{13}$$

Figure 4 illustrates how the price level is determined as clearing the goods market.

Price Level Determinacy and the Representative Agent

The above reasoning does not extend to representative agent environments so that the price level is indeterminate if markets are complete. The key implication of complete markets is that the steady-state real interest rate is determined by the discount factor only, $(1 + r_{ss})\beta = 1$, whereas in incomplete market models the real interest rate depends on virtually all model primitives. The counterpart to equation (8) in a representative agent model is thus

$$\frac{1+i_{ss}}{1+\pi_{ss}} = 1 + r_{ss} = 1/\beta, \tag{14}$$

which no longer depends on the price level, and the price level is therefore indeterminate. Figure 5 illustrates the indeterminacy, depicting supply and demand in the asset market as before with incomplete markets, but with the difference that now the steady-state savings curve is a vertical line at the steady-state interest rate $1/\beta$. With incomplete markets, it is an upward sloping curve. The vertical savings curve with complete markets reflects the result that the real interest rate is independent of the quantity of real bonds such that a continuum of price levels, e.g. P_1^*, P_2^*, P_3^*, satisfies all equilibrium conditions. The same conclusion is

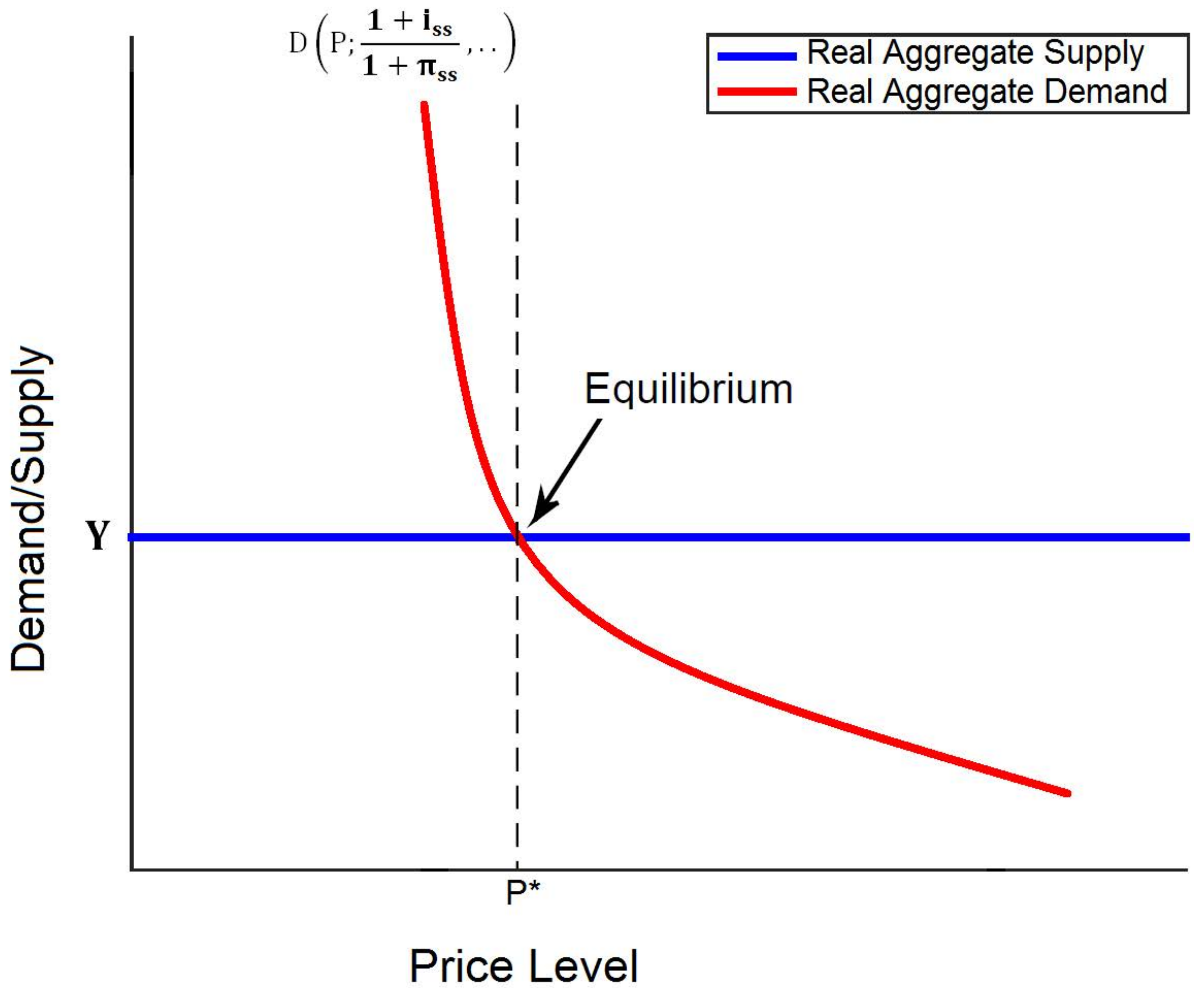


Figure 4: Price Level Determination in Goods Market

reached if government spending G is nominal. Again, a continuum of equilibrium price levels exists. Different price levels P_1 and P_2 lead to different levels of real government consumption, G/P_1 and G/P_2, and different levels of household consumption, $Y - G/P_1$ and $Y - G/P_2$, respectively, where Y is output. Although the division of output between private and government consumption varies for different price levels, all those divisions of output are equilibria. If fiscal policy is specified in real terms then the price level does not matter at all (complete dichotomy) and any price level can be the equilibrium price level.

Price Level Determinacy and Hand-to-Mouth Consumers

The same basic arguments apply to models where a fraction of households is always hand-to-mouth and the remaining ones behave according to the permanent income hypothesis (PIH). Since hand-to-mouth consumers do not participate in the asset market, the real interest rate is determined by the discount factor of PIH households only, $(1 + r_{ss})\beta = 1$,

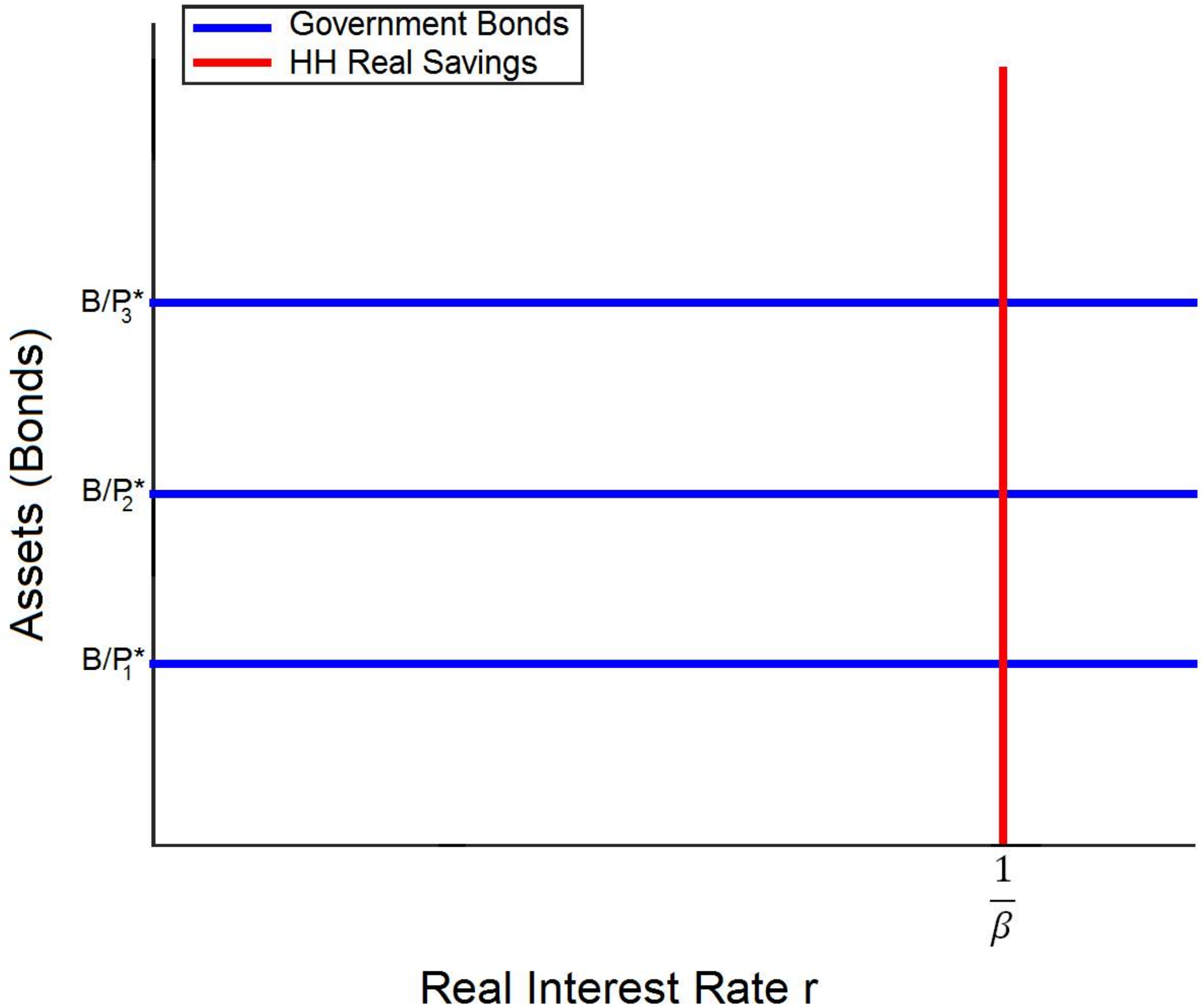


Figure 5: Representative Agent: Indeterminate Price Level

and equilibrium in the asset market is again characterized through

$$\frac{1+i_{ss}}{1+\pi_{ss}} = 1 + r_{ss} = 1/\beta, \tag{15}$$

which does not depend on the price level, implying that the price level is indeterminate. This model shows that it is not heterogeneity alone that delivers the result. Rather it is the combination of heterogeneity and market incompleteness that leads to precautionary savings and a well-defined aggregate savings function, implying price level determinacy. By the same argument, permanent heterogeneity in productivity will not lead to price level determinacy either, since again $(1 + r_{ss})\beta = 1$ in a steady state.

Price Level Determinacy with Capital and Money

The same determinacy result holds in a model with capital K and where households have a non-trivial demand for money. Denote by $L(1+i, ..)$ the aggregate demand for real balances as a function of the nominal interest rate i, and let M be nominal money supply. It is important that M is an endogenous variable, adjusted by the central bank to satisfy whatever money demand households have given the nominal interest rate set by the central bank. The steady-state price level P^* and money M are determined as solutions to

$$\frac{M}{P^*} = L(1+i_{ss}, ..) \tag{16}$$

$$K + \frac{B}{P^*} = S(\frac{1+i_{ss}}{1+\pi_{ss}}, ...), \tag{17}$$

now two equations in two unknowns M and P^*, where B and π_{ss} are set by fiscal policy, i is set by monetary policy and K is such that the marginal product of capital is equal to the real interest rate. Here the assumption is that households exchange consumption goods for money. If one assumes instead that households obtain money through open market operations, then P^* and M solve

$$\frac{M}{P^*} = L(1+i_{ss}, ..) \tag{18}$$

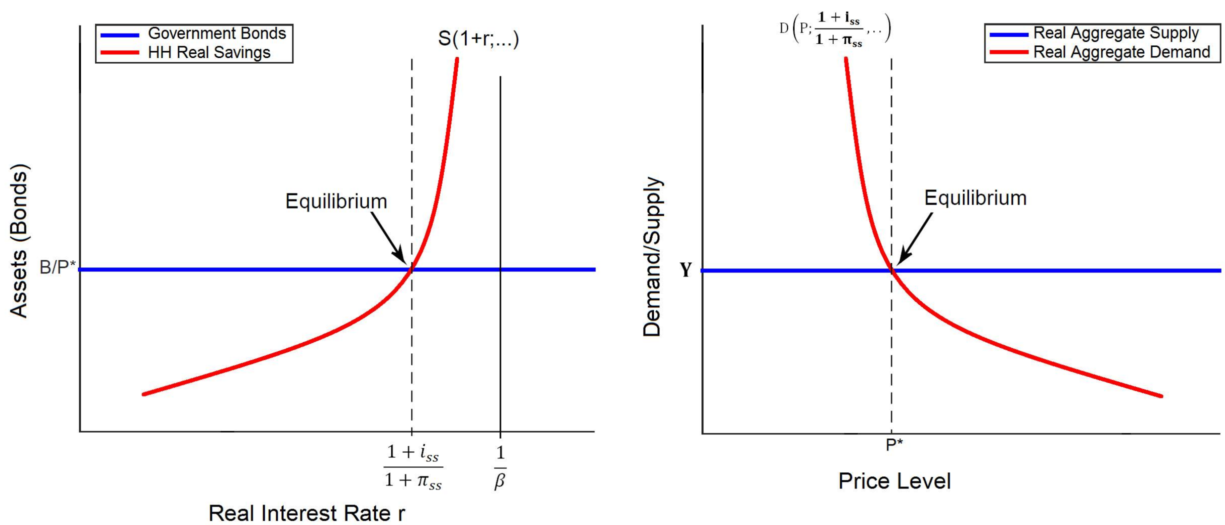
$$K + \frac{B-M}{P^*} = S(\frac{1+i_{ss}}{1+\pi_{ss}}, ...), \tag{19}$$

again two equations in two unknowns. Clearly, equation (18) alone does not determine the price level since the central bank sets i and not M, which adjusts endogenously to satisfy the quantity equation. It is the asset market clearing condition that determines the price level, which depends on fiscal variables G, T and B *and* on i.

Price Level: Monetary and Fiscal Policy, Technology, Liquidity

Finally to illustrate the mechanism of price level determination, I use the graphical analysis to show how the price level responds to monetary and fiscal policy as well as to changes in technology and in the need for liquidity. These responses are in line with conventional wisdom and with their precise characterization in Section 4.3. For each of these four experiments, I will show diagrams both for the asset market and for the goods market to derive how prices move and compare these new steady states to the pre-experiment steady state in the asset market and in the goods market in Figure 6.

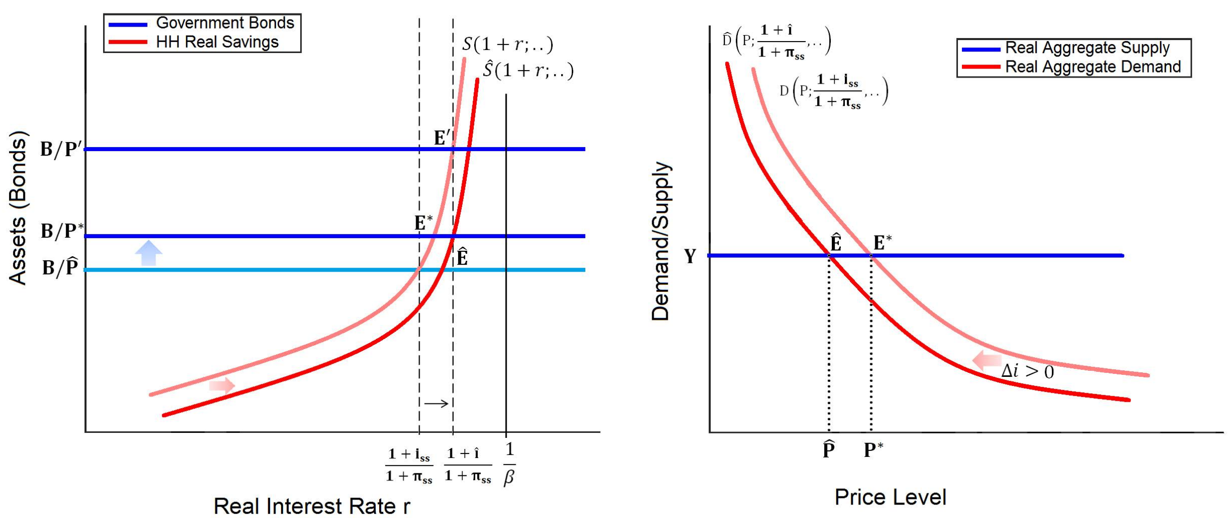
i) Monetary Policy (Figure 7) An increase in the nominal interest rate from i_{ss} to $\hat{i}$ increases savings and moves the economy up the savings curve from E^* to E', leading to a fall in



(a) Asset Market (b) Goods Market

Figure 6: Steady State : a) Asset Market b) Goods Market

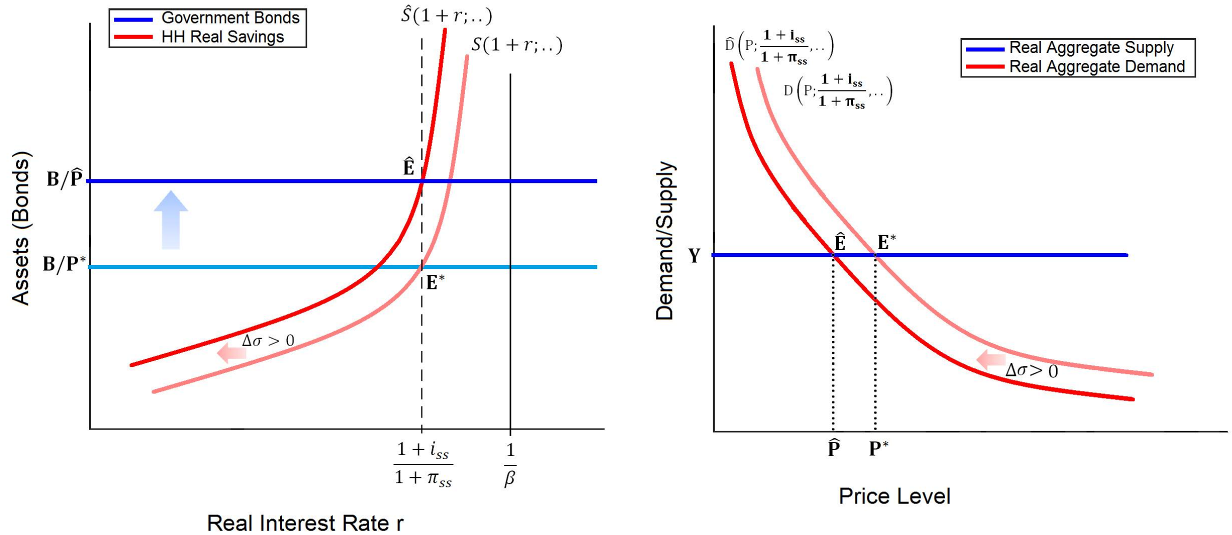
the price level. The lower price level increases the real tax burden and thus shifts the savings curve down so that the new equilibrium is at $\hat{E}$ with a lower price $\hat{P} < P^*$ to restore equilibrium in the asset market. A higher nominal interest rate also contracts demand (shifts down the demand curve) such that the price level has to fall to ensure an equilibrium in the goods market.



(a) Asset Market (b) Goods Market

Figure 7: Steady State : Tighter Monetary Policy $\Delta i > 0$

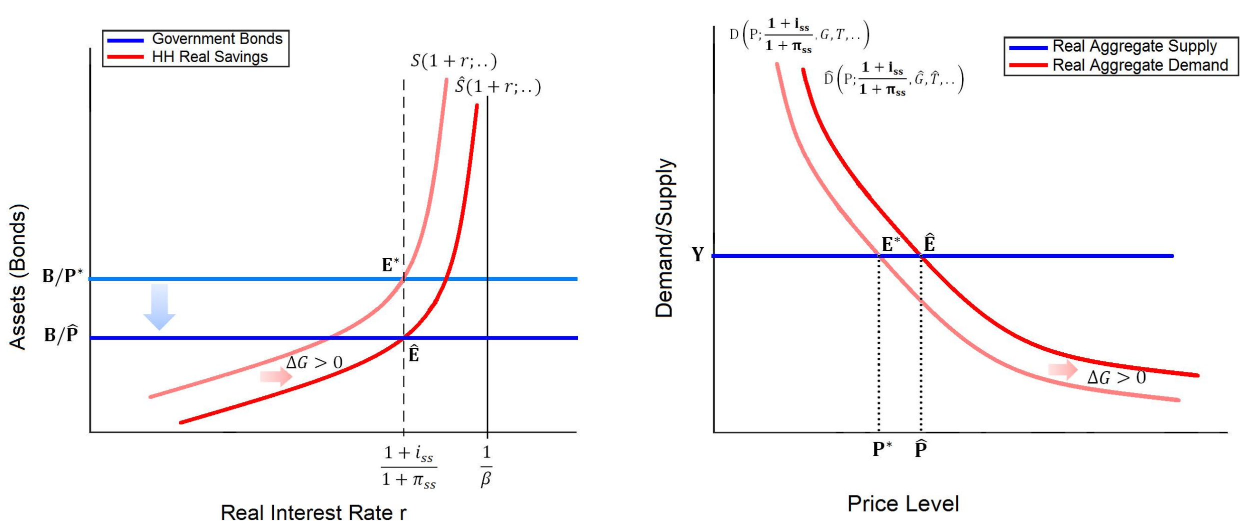
<u>ii) Fiscal Policy (Figure 8)</u> A tax-financed increase in government spending from G to $\hat{G}$ shifts the aggregate demand curve up and leads to a higher price level since private and



(a) Asset Market (b) Goods Market

Figure 9: Steady State : Higher Liquidity Demand $\Delta\sigma > 0$

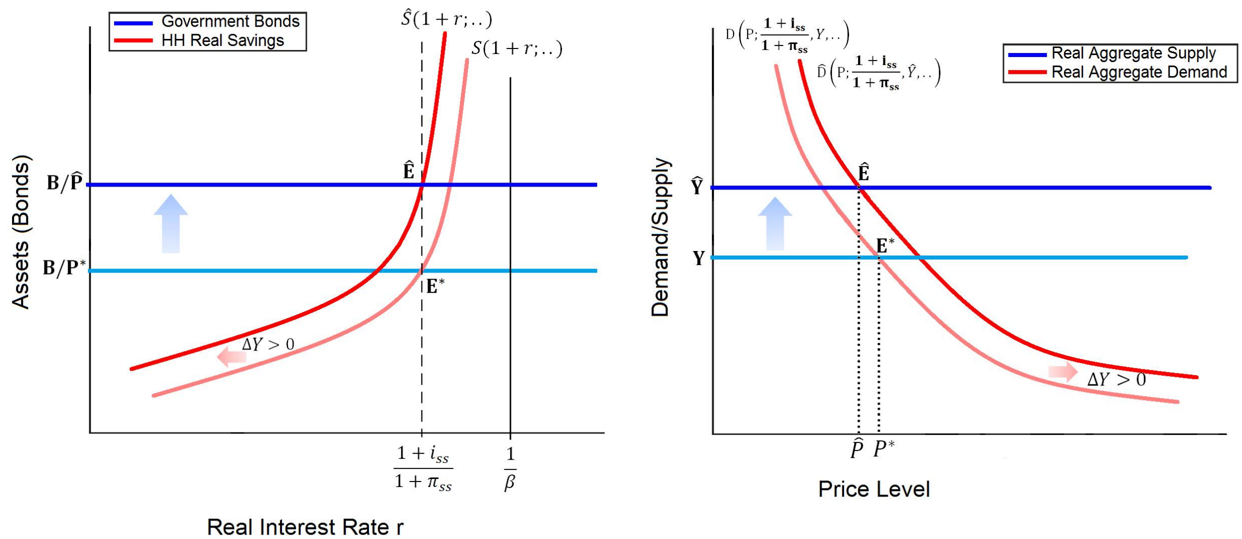
public consumption are not one-for-one substitutes. For the same reason aggregate savings shift down, the economy moves from E^* to $\hat{E}$, and the price increases from P^* to $\hat{P}$.



(a) Asset Market (b) Goods Market

Figure 8: Steady State : Expansionary Fiscal Policy $\Delta G > 0$

iii) Liquidity (Figure 9) A higher liquidity demand ($\Delta\sigma > 0$) increases savings and depresses demand. The savings curve shifts up, the demand curve shifts down. The economy moves from E^* to $\hat{E}$ and the price decreases from P^* to $\hat{P}$.



(a) Asset Market (b) Goods Market

Figure 10: Steady State : Productivity Increase $\Delta Y > 0$

iv) Technology (Figure 10) An increase in productivity leads to an increase in households' income, of which a fraction is spent on consumption and a fraction is saved. As a result, the savings curve shifts up and the price level drops. In the goods market, both the supply and the demand curve shift up, but the latter by less, since a fraction of the additional income is saved.

To summarize, I can obtain price level determinacy when fiscal policy is partially nominal and heterogeneity and incomplete markets lead to precautionary savings and a well-defined aggregate savings function. The determinacy result of the steady-state price level therefore holds in a large class of incomplete markets models. The response of the price level to monetary and fiscal policy and to technology and liquidity shocks is in line with conventional wisdom. In the next Section I develop a simple incomplete markets model which allows me to prove determinacy outside the steady state also. The important economic assumption for global determinacy is that the demand for precautionary savings does not disappear outside the steady-state. This assumption, combined with tractable global dynamics, allows me to prove determinacy.[13]

[13]It is easy to ensure that precautionary savings never disappear in a Huggett economy, e.g. by assuming a state without any income and that the marginal utility of zero consumption is infinite. The dynamics of the Huggett economy outside the steady state however are not tractable at all, such that e.g. cycles etc. cannot be ruled out.

3 Model

Households in this economy are infinitely-lived and heterogenous in their spending needs. In terms of preferences and trading frictions each period resembles a Diamond and Dybvig (1983) economy. However, to keep the heterogeneity analytically tractable, households are members of large families which pool all family assets at the beginning of each period. As in the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete market model households can acquire one interest-bearing liquid asset. I will not distinguish between money and bonds but show in Section 4.5, following the analysis of a cashless economy in Woodford (2003), that a demand for money can be added to the model, so that households can acquire cash and the liquid asset. The demand for money then determines the quantity of money that the central bank will need to supply in order to implement its nominal interest rate target. In the theoretical analysis in this and the next section, prices are flexible and labor supply is inelastic since these assumptions, by themselves, do not imply determinacy or indeterminacy of the price level. To provide numerical illustrations of the workings of the model I relax both assumptions in Section 5 where prices are sticky and labor supply is elastic.

3.1 Households

Time is discrete and extends from $t = 0, \ldots, \infty$. There is a continuum of measure one of households. Each period $t \geq 0$ is divided into two distinct and successive sub-periods t_1 and t_2. The only source of uncertainty is an idiosyncratic i.i.d. emergency expenditure shock in the spirit of Diamond and Dybvig (1983), which realizes only in period t_2.

The timing of events is as follows: In subperiod t_1, before the realization of the the risk household h consumes C_t^h and consumes c_t^h in the second subperiod t_2.

Households are exposed to liquidity risks at t_2, which leads to heterogeneity in consumption and asset holdings. To keep the model tractable, I make the assumption that each household is a family consisting of a continuum of individuals of measure one. Each member of the household has a need for spending in t_2 which is governed by the i.i.d. shock

$$\theta \in [\underline{\theta}, \infty] \sim F, \tag{20}$$

where $\underline{\theta} \geq 0$ and corresponding pdf f. A household that experiences a shock θ and consumes

C_t^h in period t_1 and c_t^h at t_2 derives utility

$$u(C_t^h) + \theta v(c_t^h) \tag{21}$$

in period t. Each individual's demand for consumption at t_2 is increasing in the idiosyncratic value of θ. Because the household has a continuum of members the distribution of θ across the members of a household is given by the distribution F.

As I will later add elastic labor supply, I assume here a linear technology which transforms each individual's inelastically supplied unit of labor $h^i = 1$ into Ah^i units of output y:

$$y = Ah, \tag{22}$$

so that each individual has A consumption goods in period t_1.

As in Diamond and Dybvig (1983), because the expenditures needs at t_2 are sudden, I assume that a liquid asset (bonds) is necessary for these expenditures beyond some level $\overline{b}$. The interpretation is that each member of the household has to acquire period t_2 consumption from the market and cannot obtain it from his or her own family because members are spatially separated. Up to some limit $\overline{b}$ a member can obtain a credit. For all additional expenditures the individual has to use savings (bonds).

In period t_1 each household chooses consumption in period t_1, C_t^h, consumption at t_2 as a function of θ, $c_t^h(\theta)$, and how many nominal bonds to acquire, b_t^h, so that b_t^h/P_t is the value of bonds in terms of consumption goods, where P_t is the price level at time t (same at t_1 and t_2 as goods are produced with the same technology). The return on bonds is R_{t+1}. Since all members of a household are identical, each has the same level of consumption at t_1 and enters period t_2 with the same amount of bonds. During period t_2, each member has access only to his or her own bonds to be spend on consumption $c_t^h(\theta)$,

$$c_t^h(\theta) - \overline{b} \leq b_t^h/P_t. \tag{23}$$

Excess bonds not needed for emergency expenditures, $\min(b_t^h - P_t(c_t^h(\theta) - \overline{b}), b_t^h)$, are returned to the family.

The household's budget constraint at t_1 is:

$$P_t C_t^h + b_t^h = P_t A h_t - T_t + R_t b_{t-1}^h - P_t \overline{C}_t^h, \tag{24}$$

where T_t are the households' nominal tax obligations of to be paid at t_1, and $\overline{C}_t^h$ is the sale of household consumption goods to members of other families who need consumption in period t_2. Since θ is distributed according to F in all families, expected spending on consumption in period t_2 is equal to sales in period t_2 in a symmetric equilibrium

$$E_\theta(c_t^h(\theta)) = \overline{C}_t^h, \tag{25}$$

so that the amount of bonds owned by a household at the end of period t_2 equals

$$b_{t_2}^h = E_\theta(b_t^h - P_t c_t^h(\theta)) + P_t \overline{C}_t^h = b_t^h. \tag{26}$$

The household's flow budget constraints simplifies to

$$P_t C_t^h + E_\theta(P_t c_t^h(\theta)) + b_t^h = P_t A h_t - T_t + R_t b_{t-1}^h. \tag{27}$$

The decision problem of a household with initial period bond holdings b_{t-1}^h is

$$V_t(b_{t-1}^h) = \max_{b_t^h, C_t^h, c_t^h(\theta)} \{u(C_t^h) + E_\theta \theta v(c_t^h(\theta)) + \beta E_t[V_{t+1}(b_t^h)]\} \tag{28}$$

subject to the flow budget constraint (27) and the liquidity constraint (23).
The optimal decision for $c_t(\theta)$ is described through the threshold $\hat{\theta}_t$, which solves

$$\hat{\theta}_t v'(b_t^h/P_t + \overline{b}) = u'(C_t) \tag{29}$$

such that

$$\begin{array}{lll} i) & c_t(\theta) \text{ solves } \theta v'(c_t(\theta)) = u'(C_t) & \text{if } \theta \le \hat{\theta}_t, \text{ i.e. } v'(b_t^h/P_t + \overline{b}) \le u'(C_t), \\ ii) & c_t(\theta) = b_t^h/P_t + \overline{b} & \text{if } \theta > \hat{\theta}_t, \text{ i.e. } v'(b_t^h/P_t + \overline{b}) > u'(C_t). \end{array}$$

In case $i)$, θ is below the threshold where households do not use all their bonds for emergency expenditures at t_2. In case $ii)$, θ is above the threshold where all bonds acquired at t_1 and the full credit line $\overline{b}$ are used for consumption, so that overall consumption spending equals $b_t^h/P_t + \overline{b}$, the sum of the consumption value of bonds, b_t^h/P_t, and credit $\overline{b}$. The credit line expands the consumption possibilities of each individual at t_2. Paying with savings or by credit is equivalent since the family has to pay back the credit in $t+1$ with a real interest rate $\frac{R_{t+1}P_t}{P_{t+1}}$. Note that in the absence of a binding liquidity constraint, households would choose the first best allocation $c_t^{FB}(\theta)$ solving $\theta v'(c_t^{FB}(\theta)) = u'(C_t)$, which is not feasible for values of θ with $c_t^{FB}(\theta) > b_t^h/P_t + \bar{b}$.

To guarantee existence of an equilibrium I impose an Inada condition for bond holdings which ensures that the demand for liquidity is sufficiently strong[14]

$$v'(\bar{b} + \mathbf{0})E[\theta \mid \theta \geq \frac{u'(A - \bar{b})}{v'(\bar{b} + \mathbf{0})}] > u'(A - \bar{b}), \tag{30}$$

that is, the marginal value of acquiring a bond (LHS) exceeds its cost (RHS) when the household has zero bonds.[15]

The remaining decision on the quantity of bonds to acquire is characterized through the first-order condition:

$$u'(C_t) = \int_{\hat{\theta}_t}^{\infty} \theta v'(b_t^h/P_t + \bar{b})dF(\theta) + F(\hat{\theta}_t)E_t[\frac{R_{t+1}P_t}{P_{t+1}}\beta u'(C_{t+1})]. \tag{31}$$

Observe that if an individual's expenditure needs are strong enough, $\theta_t > \hat{\theta}_t$, which is the case with probability $1 - F(\hat{\theta}_t)$, there is a shortage of the liquid asset. The infinite support of θ ensures the existence of a finite $\hat{\theta}_t$ such that there is shortage of the liquid asset with positive, potentially small, probability.

3.2 Fiscal and Monetary Policy

The aim of this paper is to show how monetary policy, fiscal policy, and their interaction determine the price level. A standard way to represent monetary policy is through setting a sequence of nominal interest rates,

$$\mathcal{R} = R_0, R_1, R_2, \ldots, R_t, \ldots. \tag{32}$$

Fiscal policy is represented by a sequence of nominal government spending

$$\mathcal{G} = G_0, G_1, \ldots, G_t, \ldots, \tag{33}$$

[14]This condition is satisfied for example if $F = LogNormal(0,1)$ and $\frac{v'(\bar{b})}{u'(A-\bar{b})} > 1$.

[15]Note that in this case the LHS is not identical but smaller than the marginal value of a bond since the threshold $\hat{\theta} \leq \frac{u'(A-\bar{b})}{v'(\bar{b}+\mathbf{0})}$, ensuring existence of an equilibrium. At the cost of a more involved notation but without any substantive gain the Inada condition could be made exact.

which needs to be financed by levying nominal lump-sum taxes T_t,

$$\mathcal{T} = T_0, T_1, \dots, T_t, \dots. \tag{34}$$

The government's flow budget constraint has to be satisfied at any point in time, which implicitly defines a sequence of nominal bonds

$$B_{t+1} = R_t B_t + G_t - T_t, \tag{35}$$

such that the intertemporal government budget constraint is satisfied:

$$B_0 = \sum_{t=0}^{\infty} (T_t - G_t) \prod_{s=0}^{t} \frac{1}{R_s} \tag{36}$$

and

$$\lim_{t \to \infty} B_t \prod_{s=0}^{t-1} \frac{1}{R_s} = 0. \tag{37}$$

Since fiscal and tax policies are expressed in nominal terms, this constraint holds for *all* sequences of prices,

$$\mathcal{P} = P_0, P_1, \dots, P_t, \dots. \tag{38}$$

In particular the price level is not determined such that the government budget constraint holds. Finally, define the sequence of bonds

$$\mathcal{B} = B_0, B_1, \dots, B_t, \dots. \tag{39}$$

3.3 Competitive Equilibrium

Definition 1. *Given sequences of nominal interest rates $\mathcal{R}$, nominal government spending $\mathcal{G}$, nominal taxes $\mathcal{T}$ and nominal bonds $\mathcal{B}$ a competitive symmetric equilibrium are sequences of consumption spending $\{C_t\}_{t=0}^{\infty}$ at t_1 and $\{c_t(\theta)\}_{t=0}^{\infty}$ at t_2, bonds purchases $\{b_t^h\}_{t=0}^{\infty}$ and prices $\mathcal{P}$, such that for all t, the following holds:*

1. *Households take prices and policies as given and choose* $\{C_t, c_t(\theta), b_t^h\}_{t=0}^{\infty}$ *to maximize utility.*

2. *Given prices, firms choose* $\{h_t\}_{t=0}^{\infty}$ *to maximize profits.*

3. *The government budget constraint holds (36).*

4. *Market clearing and resource constraint:*

   (a) *Bond Market :* $B_t = b_t^h$,

   (b) *Resource Constraint* $C_t + E_\theta c_t(\theta) + G_t = Ah_t$,

   (c) *Labor Market:* $h_t = 1$.

4 Prices, Inflation and Nominal Demand

This section shows that the price level is determined as the unique solution where supply equals demand. The main result delivering determinacy is that aggregate demand - the sum of private and government demand - is decreasing in the price level. Key to this result is that households engage in precautionary savings. A fall in government consumption is then not offset one-for-one by an increase in private consumption of the same amount. Instead, households engage in precautionary savings, implying lower aggregate demand for a given price level.

This line of reasoning will be used throughout this section, first to show the existence and uniqueness of a steady-state price level, then to rule out a vanishing or exploding price level, and finally to prove determinacy globally.

4.1 Price Level Determinacy: Steady State

To establish that the steady-state price level is determined, I proceed through several steps. I first characterize the steady state and prove existence and uniqueness of the steady-state price level. Next, I show that the nominal anchor provided by government spending prevents prices from converging to zero. Furthermore, I establish that an exploding price level would lead to insufficient market demand, as private consumption is not fully substituting for government demand, but that households instead engage in precautionary savings, leading to a downward pressure on prices and thus ruling out such an explosive path. Finally, in

Section 4.2, I use the same arguments ruling out explosive and vanishing price paths to prove price level determinacy outside of steady states in Theorem 1.

In a *steady state* nominal interest rates are constant at R; nominal government spending, taxes and nominal bonds are all strictly positive growing at rate γ,

$$G_t = G(1+\gamma)^t, T_t = T(1+\gamma)^t, B_t = B(1+\gamma)^t, \tag{40}$$

consumption C at t_1 and $c(\theta)$ at t_2 are time-invariant, and prices are growing at a constant rate $\frac{P_{t+1}}{P_t} = \pi_{t+1} = \pi$,

$$P_t = P(1+\pi)^t. \tag{41}$$

Since in a steady state C_t, B_t/P_t, $\hat{\theta}_t$ and $\frac{R_{t+1}}{1+\pi_{t+1}}$ are constant, equation (31) implies that

$$\frac{\beta R}{1+\pi} < 1. \tag{42}$$

Permanently higher nominal interest rates would not be consistent with a steady state and would lead instead to exploding asset demand as in the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model.

In a steady state real government spending is constant, implying that the inflation rate π equals the growth rate of nominal spending γ,

$$\pi = \gamma. \tag{43}$$

It is important to stress that this result is not a tautology. Rather, it implies that the steady-state inflation rate is set by fiscal policy. To see this, compare two steady states with the same fiscal policy (the same γ), but with different nominal interest rates $R^H > R^L$. The previous result implies that inflation is the same in the two steady states and that the real interest rate is different. Similarly, two steady states with different fiscal policies, $\gamma^H > \gamma^L$ feature different inflation rates even if nominal interest rates are identical. Note also that this logic breaks down when markets are complete since in this case the real interest rate is equal to $1/\beta$ across all steady states and, in particular, is independent from policy.

Next, I characterize equilibrium consumption C_t at t_1,

$$C_t = A - \frac{G_t}{P_t} - E_\theta c_t(\theta), \tag{44}$$

and how it depends on the price level before moving on to the main result on the price level. Equilibrium consumption C_t equals output A minus government consumption G_t/P_t and expected family consumption $E_\theta c_t(\theta)$ at t_2. Clearly, G/P is falling in the price level, leaving more resources to the private economy and implying that consumption C at t_1 increases in P. Households' aim to also increase spending at t_2 is partly obstructed by the credit constraint (23), which imposes an upper bound on spending at t_2. As a result more households are credit constrained. This constraint becomes binding at a lower value of θ, so that the equilibrium threshold $\hat{\theta}$ falls in P. In the appendix I prove

Proposition 1. *Given a fixed sequence of government policies, consumption C_t in period t_1 can be written as a function of P_t only, $C_t = C(P_t)$, and is increasing in P_t and the threshold $\hat{\theta}_t$ is a function of P_t only, $\hat{\theta}_t = \hat{\theta}(P_t)$ and is decreasing in P_t.*

The steady-state price level P^* then clears the goods market as the solution to

$$u'(C(P^*)) = \int_{\hat{\theta}}^{\infty} \theta v'(B/P^* + \bar{b}) dF(\theta) + F(\hat{\theta}(P^*)) \frac{R}{(1+\pi)} \beta u'(C(P^*)), \tag{45}$$

so that the price at time t, $P_t = P^*(1+\pi)^t$. The next proposition summarizes the findings for the steady state[16]

Proposition 2. *A steady exists and is unique. In particular, the steady-state price level is determined uniquely. The steady-state inflation rate is equal to the growth rate of nominal government spending,*

$$\pi = \gamma,$$

which is set by fiscal policy only.

[16] I consider only finite price levels since otherwise the whole government sector would disappear, which I assume shuts down down the economy (no legal system and law enforcement, not even basic democratic institutions, ...). Alternatively, without this assumption, imposing instead that $\lim_{x \to 0} v'(1/x)x > 0$ (satisfied by standard log utility) and an upper nominal bound on credit available at t_2 also imply the finiteness of the price level.

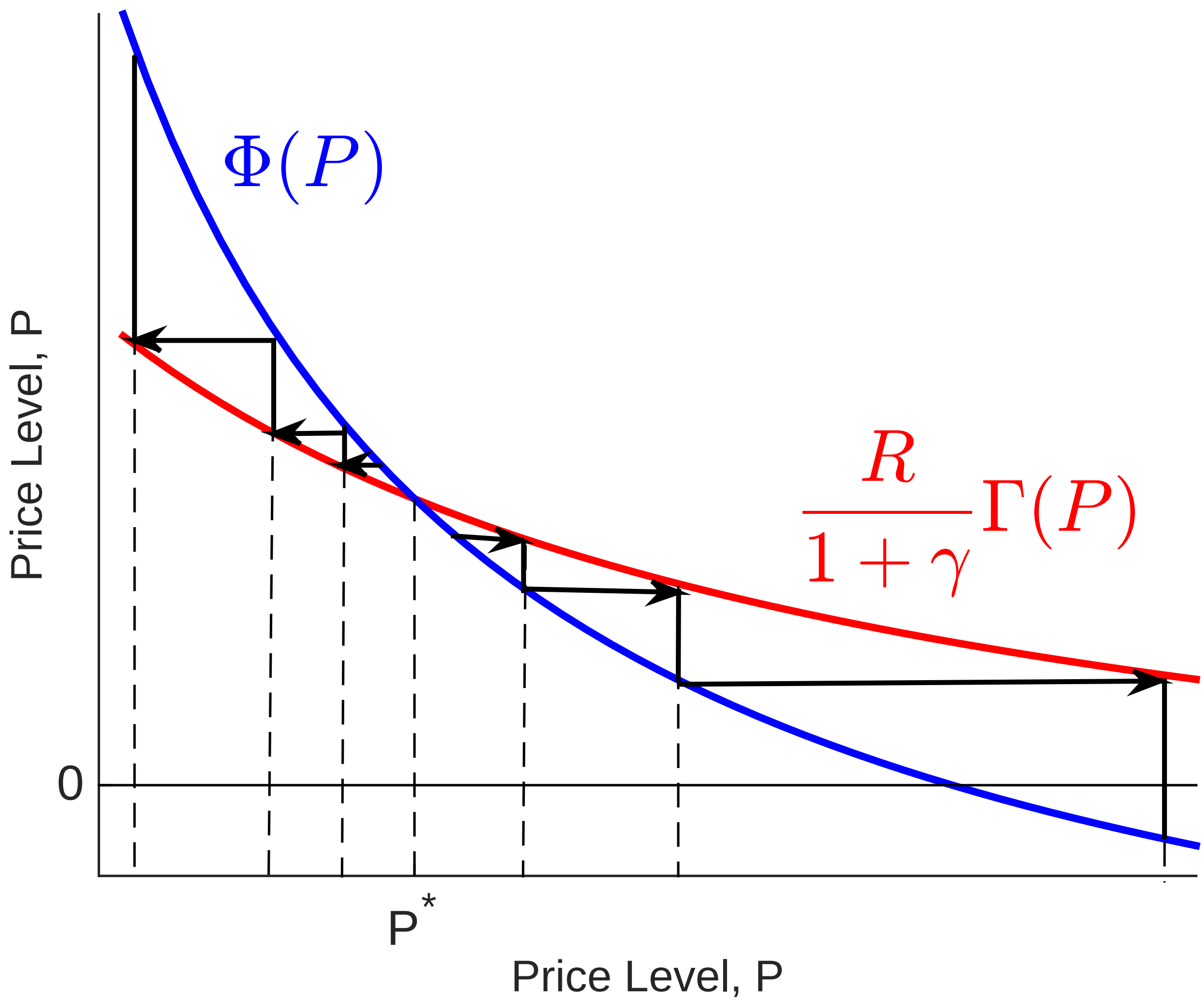


Figure 11: Dynamics of the Price Level

The preceding analysis implies that an equilibrium defines a difference equation in prices, relating prices in periods t and $t+1$,

$$\Phi(\tilde{P}_t) := \frac{u'(C(\tilde{P}_t)) - \int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} = \frac{R_{t+1}}{1+\gamma} \beta \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} =: \frac{R_{t+1}}{1+\gamma} \Gamma(\tilde{P}_{t+1}), \quad (46)$$

where I define the detrended price

$$\tilde{P}_t = \frac{P_t}{(1+\gamma)^t}. \quad (47)$$

In the steady state, $\tilde{P}_t = P^*$. I now show that this is the only solution by ruling out vanishing and explosive price paths. Since both functions Φ and Γ are decreasing in $\tilde{P}_t$ and

$\tilde{P}_{t+1}$ respectively, this equation allows to solve $\tilde{P}_{t+1}$ as an increasing function of $\tilde{P}_t$,

$$\tilde{P}_{t+1} = \Gamma^{-1}(\frac{1+\gamma}{R_{t+1}}\Phi(\tilde{P}_t)). \tag{48}$$

Figure 11 illustrates the dynamics of the price level, $\tilde{P}_t$, that is implied by equations (46) and (48). Both functions Φ and Γ are downward sloping and from the diagram it is apparent that there is a unique steady state at P^* since Φ is steeper than Γ.

A price level $\tilde{P}_t$ higher than P^* implies lower real government spending and therefore that aggregate supply exceeds aggregate demand. Goods market clearing requires an even higher price $\tilde{P}_{t+1} > \tilde{P}_t$, i.e. a lower real interest rate, next period to decrease savings and increase private demand such that aggregate demand increases to match period t aggregate supply, which leads by the same argument again to a higher price level in period $t+2$, $\tilde{P}_{t+2}$, and so on. Eventually the exploding price path will drive real government spending to zero. Since precautionary savings do not disappear, private demand will fall short of aggregate supply, establishing that this price sequence is not an equilibrium. The non-existence becomes apparent in the diagram as Φ eventually becomes negative, and the iteration breaks down since $\Gamma > 0$.

A price level $\tilde{P}_t$ lower than P^* implies higher real government spending and therefore that aggregate demand exceeds aggregate supply. Goods market clearing at time t then requires an even lower price $\tilde{P}_{t+1} < \tilde{P}_t$, i.e. a higher real interest rate, to increase savings and lower private demand such that aggregate demand falls to period t aggregate supply, again leading by the same arguments to a lower price level in period $t+2$, $\tilde{P}_{t+2}$, and so on. Eventually the price level will be so low that government demand exceeds output, which clearly cannot be an equilibrium.

The only price sequence which forms an equilibrium is the one where the price is constant and equal to the steady-state price level P^*, that is, the price at time t equals $P^*(1+\gamma)^t$ and aggregate demand equals aggregate supply.

Proposition 3. *For a constant nominal interest rates R, and strictly positive nominal government spending, taxes and nominal bonds which are growing at rate γ, there is a unique equilibrium,*

$$\begin{aligned} \tilde{P}_t &= P^* \\ P_t &= P^*(1+\gamma)^t \end{aligned}$$

so that $C_t = C(P^*)$ *and* $\hat{\theta}_t = \hat{\theta}(P^*)$.

In particular the nominal anchor set by fiscal policy ensures that both exploding prices and vanishing prices are not an equilibrium.

4.2 Price Level Determinacy: Non Steady State

The analysis so far has considered stationary policies where the nominal interest rate is constant and fiscal policy is characterized by a constant growth rate. Since I also want to consider impulse responses to shocks to the nominal interest rate or to government spending, the analysis has to go beyond steady states. I now consider arbitrary sequences of nominal interest rates R_t, B_t, T_t and G_t which are assumed to be stationary only after time S, $R_t = R$, $G_t = G(1+\gamma)^{t-S}$, $T_t = T(1+\gamma)^{t-S}$ and $B_t = B(1+\gamma)^{t-S}$ for $t \geq S$. The next proposition establishes that the determinacy result extends to these non-stationary policies with the difference that now, the unique price sequence is no longer constant.

Theorem 1. *The price level is determined for arbitrary sequences of nominal interest rates and nominal government spending. In particular there is a unique price sequence.*

The proof uses similar arguments - non vanishing precautionary savings / government demand cannot exceed output - as those used to establish Proposition 3. The only difference is that now I must rule out prices higher or lower than the unique price sequence; above I had to rule out prices above or below the unique steady state. Figure 12 illustrates the key features of the argument. The aggregate demand curve is downward sloping and intersects the aggregate supply curve at price level P^*. Two components, government and private demand, add up to aggregate demand but the first is decreasing while the latter is increasing in the price level. For high price levels government consumption approaches zero, but a non-vanishing precautionary demand prevents private demand from fully substituting for the fall in government consumption such that it always falls short of aggregate supply. This rules out prices higher than P^*. For low prices real government spending explodes and therefore private consumption falls, such that government spending eventually exceeds output, ruling out prices lower than P^* as well.

The economic mechanism which ensures price determinacy in my simple incomplete markets model is the same as the one described in the graphical analysis in Section 2. To see this I derive the aggregate savings schedule from the first-order condition for households savings

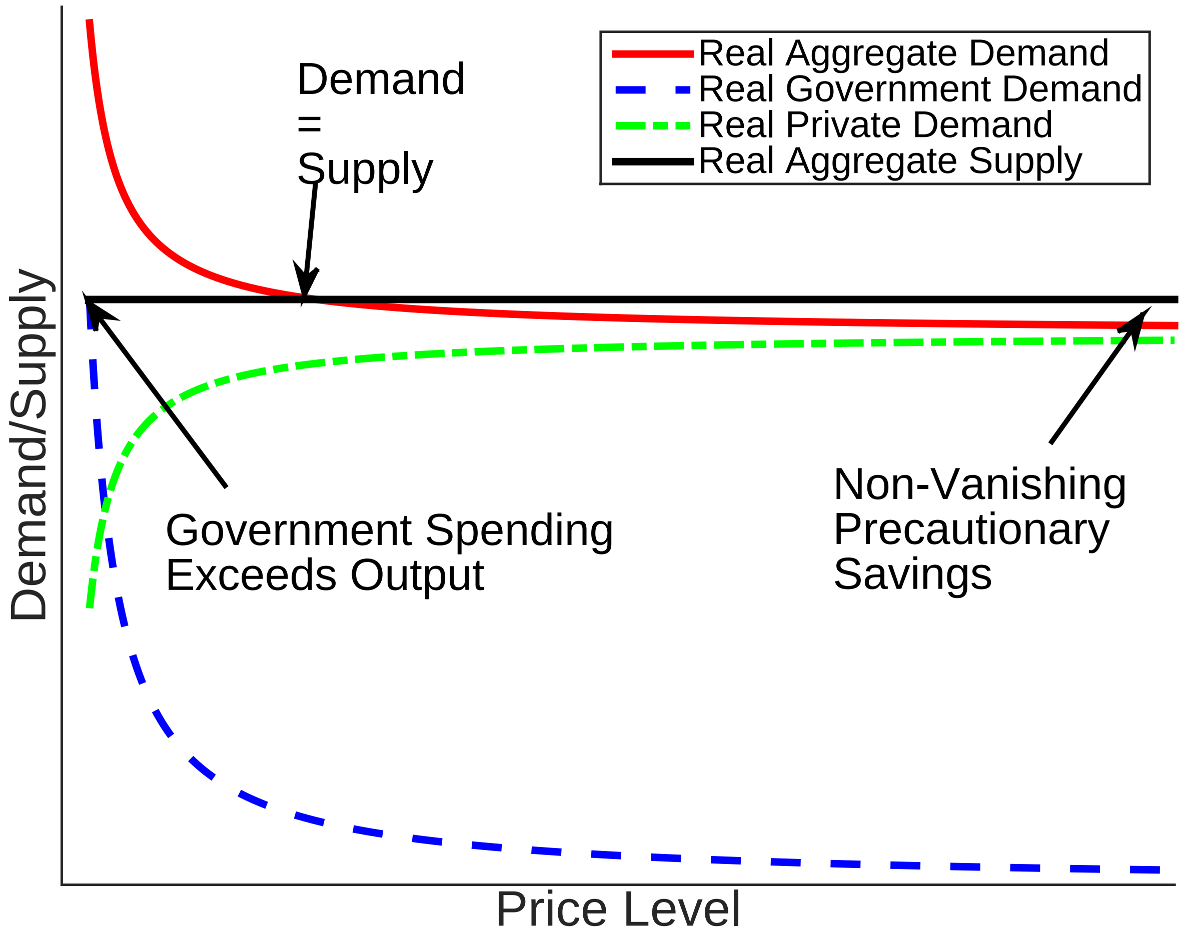


Figure 12: Aggregate Demand = Government Demand + Private Demand = Aggregate Supply

decision,

$$u'(C) = \int_{\hat{\theta}}^{\infty} \theta v'(S + \bar{b}) dF(\theta) + F(\hat{\theta}(\tilde{P})) \frac{R}{(1+\pi)} \beta u'(C). \tag{49}$$

Steady-state consumption C and $c(\theta)$ satisfy

$$C + E_{\theta} c(\theta) = A + \frac{R}{1+\pi} S - S' - \frac{T}{\tilde{P}}. \tag{50}$$

where households' savings are S', assets (=last period savings) are S so that $\frac{R}{1+\pi}S$ is real asset income, $\frac{T}{\tilde{P}}$ are real tax obligations and A is labor income. Solving this equation for real savings S' yields the savings $S'(S, \frac{R}{(1+\pi)}, T/\tilde{P}, ...)$ as a function of assets S, real interest rates and real tax obligations. Imposing the steady state condition $S = S'$ and solving for the fixpoint results in households' steady-state savings schedule

$$S(\frac{R}{(1+\pi)}, T/\tilde{P}, ...). \tag{51}$$

as a function of the steady-state real interest rate $\frac{R}{(1+\pi)}$ real tax payments $T/\tilde{P}$. As in Section 2 the unique steady-state price level P^* equates asset supply and demand while the real interest rate is determined by policy,

$$B/P^* = S(\frac{R}{(1+\pi)}, T/P^*, ...). \tag{52}$$

Again an equivalent representation exists where P^* equates demand and supply in the goods market. Private demand for households with $B/\tilde{P}$ real assets

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) = A + \frac{R}{1+\pi}\frac{B}{\tilde{P}} - \frac{T}{\tilde{P}} - S' \tag{53}$$

Using the government budget constraint $T = G + \frac{R}{(1+\pi)}B - B'$, where B is existing debt and B' is newly issued debt,

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) = A + \frac{B'}{\tilde{P}} - S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...) - \frac{G}{\tilde{P}}, \tag{54}$$

where as above $S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...)$ is savings as a function of assets $B/\tilde{P}$, real interest rates and real tax obligations. Aggregate demand, the sum of private demand D and government consumption $G/\tilde{P}$, equals

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) + \frac{G}{\tilde{P}} = A + \frac{B'}{\tilde{P}} - S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...), \tag{55}$$

which is downward sloping in the price level,

$$\frac{\partial D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) + \frac{G}{\tilde{P}}}{\partial \tilde{P}} < 0. \tag{56}$$

The equilibrium price clears the goods market

$$D(\frac{R}{(1+\pi)}, T/\tilde{P}, ...) + \frac{G}{\tilde{P}} = A, \tag{57}$$

which not surprisingly is equivalent to clearing the asset market,

$$\frac{B}{\tilde{P}} = \frac{B'}{\tilde{P}} = S'(B/\tilde{P}, \frac{R}{(1+\pi)}, T/\tilde{P}, ...) = S. \tag{58}$$

Without a precautionary demand for savings, consumption equals $C = A - G/\tilde{P}$ and aggregate demand is equal to aggregate supply $C + G/\tilde{P} = A$ for all price levels,

$$\frac{\partial D(\tilde{P}) + \frac{G}{\tilde{P}}}{\partial \tilde{P}} = 0. \tag{59}$$

Therefore in this case the price level would not be determined as equating demand and supply but would simply determine the size of the government, $G/\tilde{P}$.

I next characterize the model impulse responses of prices which turn out to be consistent with the responses of steady-state prices to monetary and fiscal policy, technology and liquidity shocks in Figures 7 - 10 in the graphical analysis in Section 2: tightening monetary policy, increases in productivity and a higher demand for liquidity lower prices, and expansionary fiscal policy increases prices. The tractability of my incomplete markets model allows me to go beyond comparing steady states and to characterize the impulse response paths of prices in Section 4.3.

4.3 Impulse Responses

Theorem 1 allows a consideration of the economy's impulse response to monetary and fiscal shocks. While a full analysis will be conducted in a model with sticky prices in Section 5, it is still instructive to compute the impulse responses in the flexible price version of the model.

A theoretical reason why this usually is not done, is that the price level is not determined in both standard sticky and flexible price models when monetary policy is described as setting the nominal interest rate. As this Section has established, this theoretical obstacle is overcome, allowing to conduct such policy experiments.

I consider two policy experiments, a persistent increase in the nominal interest rate and an increase in nominal government spending. I also consider the response of prices to a discount factor and a technology shock. Figures 13, 14 and 15 in Section 5 show not only the impulse responses in the sticky price model but also, as a benchmark, for the flexible price economy. A remarkable feature of these flexible price impulse responses is that prices adjust sluggishly and that monetary policy affects prices without the assumption of prices being sticky.

4.3.1 Monetary Policy

For the monetary policy experiment, I characterize the response of prices to an initial unexpected increase in interest rates $R_0 > R$, which then dies out over time and is back at the steady-state level R from time S onwards, $R_t = R$ for $t \geq S$, that is the interest rate sequence equals

$$R_0 \geq R_1 \geq \ldots \geq R_{S-1} \geq R_S = R, R, \ldots. \tag{60}$$

The resulting price sequence can be precisely characterized:

Proposition 4. *The detrended price sequence in response to a monetary policy as in (60) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \ldots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots, \tag{61}$$

where the inequality between prices at t and $t+1$ is strict whenever it is strict for nominal interest rates, $R_t > R_{t+1}$. For the non-detrended prices

$$\frac{P_{t+1}}{P_t} \geq (1+\gamma) \tag{62}$$

with strict inequality if $R_t > R_{t+1}$.

A more persistent or larger impulse to the nominal interest rate leads to uniformly lower prices.

Proposition 5. *Consider two interest rate sequences*

$$R_0^a \geq R_1^a \geq \ldots \geq R_{S-1}^a \geq R_S^a = R, R, \ldots, \tag{63}$$

$$R_0^b \geq R_1^b \geq \ldots \geq R_{S-1}^b \geq R_S^a = R, R, \ldots, \tag{64}$$

with $R_t^a \geq R_t^b$. Then the prices $\tilde{P}^a$ for policy R^a are uniformly lower than the prices $\tilde{P}^b$ for policy R^b,

$$\tilde{P}_t^a \leq \tilde{P}_t^b. \tag{65}$$

4.3.2 Fiscal Policy

For the fiscal policy experiment, I characterize the response of prices to an initial unexpected increase in nominal government spending for S periods by x percent financed by an increase in taxation, so that

$$\begin{aligned} \hat{G}_t = & \quad (1+x)G_t \quad \text{for } 0 \leq t < S \\ \hat{G}_t = & \quad G_t \quad \text{for } t \geq S. \end{aligned} \tag{66}$$

The response of prices can be precisely characterized:

Proposition 6. *The detrended price sequence in response to a fiscal policy as in (66) is*

$$\tilde{P}_0 \geq \tilde{P}_1 \geq \ldots \geq \tilde{P}_{S-1} \geq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots, \tag{67}$$

and for the non-detrended prices

$$\frac{P_{t+1}}{P_t} \leq (1+\gamma). \tag{68}$$

A more expansive or persistent fiscal policy $G_t^a \geq G_t^b \geq G_t$ leads to stronger price increases

$$\tilde{P}_t^a \geq \tilde{P}_t^b. \tag{69}$$

4.3.3 Discount Factor Shock

I consider a discount factor shock which increases β for S periods:

$$\begin{aligned} \hat{\beta} > &\quad \beta \quad \text{for } 0 \leq t < S \\ \hat{\beta} = &\quad \beta \quad \text{for } t \geq S. \end{aligned} \tag{70}$$

The next proposition shows that prices persistently fall.

Proposition 7. *The detrended price sequence in response to a discount factor shock as in (70) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \ldots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots, \tag{71}$$

A larger discount factor shock, $\hat{\beta}^a > \hat{\beta}^b$ leads to stronger price decreases

$$\tilde{P}_t^a < \tilde{P}_t^b. \tag{72}$$

With one important caveat, discount factor shocks can be neutralized fully by monetary policy through keeping $R\beta$ constant, that is decreasing R by the size of the shock $\hat{\beta}/\beta$. The caveat is that the zero lower bound prevents large cuts in the nominal interest rates in response to large increases in β.

4.3.4 Productivity Shock

Finally I consider a persistent shock to the productivity A:

$$A_0 \geq A_1 \geq \ldots \geq A_{S-1} \geq A_S = A, A, \ldots. \tag{73}$$

The response of prices to this productivity innovation can be characterized precisely as a persistent drop in prices:

Proposition 8. *The detrended price sequence in response to a technology shock as in (73) is*

$$\tilde{P}_0 \leq \tilde{P}_1 \leq \ldots \leq \tilde{P}_{S-1} \leq \tilde{P}_S = P^*, \tilde{P}_{S+1} = P^*, \ldots. \tag{74}$$

4.4 How and when monetary policy controls the inflation rate

A main result of this paper is that fiscal policy is what determines the steady-state inflation rate through controlling nominal spending. If nominal government consumption, nominal taxation and nominal government debt grow at rate γ, then steady-state inflation $\pi = \gamma$. In this case monetary policy has no control over the long-run inflation rate, since fiscal policy does control the nominal anchor.

Yet, monetary policy, as shown above, can still affect the price level. I also show in Section 5 that monetary policy can quite effectively stabilize prices around the steady state. An attempt by fiscal policy to stimulate the economy through generating above steady-state temporary inflation and employment can be neutralized completely by monetary policy. This results in higher government spending, but prices and employment remain at their steady-state values.

In different scenarios monetary policy can affect the long-run inflation rate. One such scenario is where fiscal policy takes the initial level of nominal government debt, B, prices P and monetary policy as given, and fixes the level of real government consumption, g, and the real tax revenue, τ. Some simple algebra can then illustrate how this scenario leads to a different conclusion about long-run inflation.

Fiscal policy has to make two choices. It decides about $s = \tau - g$, the real constant primary surplus, and about the growth rate of nominal government debt, $\frac{B_{+1}-B}{B}$, such that the steady-state flow government constraint holds

$$\frac{Ps}{B} = -\frac{B_{+1} - B}{B} + (R - 1), \tag{75}$$

taking as given P, B and R. Note that in the Fiscal Theory of the Price Level the government effectively picks P through committing to some s and $\frac{B_{+1}-B}{B}$ which satisfy the budget constraint for only one price level P. Generically this makes the price level overdetermined in my incomplete markets model since one variable, the price level, has to satisfy both equations, the government budget constraint and goods market clearing equations, at the same time. Here, in contrast, the budget constraint is satisfied for all P and the price level is uniquely determined as clearing the goods market.

If fiscal policy commits to a real surplus s, then equation (75) implies the debt issuance necessary to balance the budget. In this scenario the growth rate of nominal debt and thus the long-run inflation rate are jointly determined, by fiscal policy through setting s and by monetary policy by setting R:

$$1+\pi = \frac{B_{+1}-B}{B} = (R-1) + \frac{Ps}{B}, \tag{76}$$

where the price level clears the good market.

The reason why now monetary policy and fiscal policy determine the long-run inflation rate jointly is simple. The general principle, that it is fiscal policy which de facto determines steady-state inflation, still holds as it is the evolution of the nominal anchors that matters. However, a government insisting on a certain level of real expenditures and tax revenues has to finance all deficits through issuing debt with a return determined by monetary policy. An increase in the nominal interest rate therefore will lead to an increase in nominal government debt, which with flexible prices materializes immediately in higher inflation rates. In this scenario it is monetary policy and fiscal policy that determine the growth rate of nominal government debt, and therefore the growth rate of G, T and prices P. Future research will show whether and in which episodes monetary or fiscal policy determined the inflation target.

But although monetary policy cannot fully assume control of steady-state inflation, the reasoning illustrates that it still has a large impact on government debt. This channel of monetary policy can be used to make a debt-financed expansionary fiscal policy very expensive for the government. In practice it could be an effective way for monetary policy to prevent inflationary policies.

4.5 Robustness: Adding money to the model

So far I made the standard assumption of a cashless economy. To show that this assumption is inconsequential for the results I now add a motive to hold cash to the model, an extension that does not affect my conclusions. Setting nominal interest rates and fiscal policy is sufficient to determine the price level. Households' nominal money holdings are then endogenously determined to satisfy real money demand. To show this I assume that the transaction services provided by real money balances are represented as an argument of the utility function,

$$u(C_t^h) + \chi(M_t/P_t) + \theta v(c_t^h), \tag{77}$$

where M_t is a family's end of period t_1 money holdings. The family's budget constraint then equals

$$P_t C_t + E_\theta(P_t c_t(\theta)) + b_t^h + M_t = P_t A h_t - T_t + R_t b_{t-1}^h + M_{t-1}, \tag{78}$$

where M_{t-1} is period t initial money holdings carried over from period $t-1$. The government collects seigniorage, which is rebated to households through lower taxation so that its flow budget constraint equals

$$B_{t+1} + M_{t+1} = R_t B_t + M_t + G_t - T_t. \tag{79}$$

The remaining features of the model are unchanged.

The decision problem of a household with initial period bond holdings b_t^h and money holdings M_{t-1} is

$$V_t(b_{t-1}^h, M_{t-1}) = \max_{b_t^h, C_t, c_t(\theta), M_t} \{u(C_t^h) + \chi(M_t/P_t) + E_\theta \theta v(c_t^h) + \beta E_t[V_{t+1}(b_t^h, M_t)]\} \tag{80}$$

subject to the flow budget constraint (78) and the liquidity constraint (23).

Money holdings M_t appear only in the first-order condition of M_t (the small seignorage gains are rebated lump-sum to households),

$$\frac{u'(C_t) - \chi'(M_t/P_t)}{P_t} = \frac{u'(C_{t+1})}{P_{t+1}}, \tag{81}$$

implying that prices P_t and P_{t+1} and consumption C_t and C_{t+1} can be solved independently from M_t. The first-order condition for money is then used to solve for M_t. The only purpose of adding a demand for money to the model is thus to determine the quantity of money that the central bank will need to supply in order to implement its nominal interest rate target.

5 Monetary and Fiscal Policy with Sticky Prices

This section adds two features to the basic model: sticky prices and elastic labor supply. These two features allow me to illustrate the workings of the model and to compute the response of the price level and employment to a tightening of monetary policy and to a fiscal demand stimulus.

5.1 A Sticky Price Model

I now assume that firms are constrained in their price setting to see whether the model can (qualitatively) produce standard impulse responses. I follow Mankiw and Reis (2002) and assume that information is sticky, and I follow the literature in modeling a competitive final good and a monopolistically competitive intermediate sector.

Labor Supply Households provide h_t hours in a perfectly competitive labor market at a fully flexible real wage w_t, which they receive at t_1. The disutility from working h hours is described through GHH preferences:

$$\log\left(C_t^h - \kappa h_t^{1+\phi}\right) + \theta\eta\log\left(c_t^h\right), \tag{82}$$

which implies that an increase in government spending has no negative wealth effect leading to an increase in labor supply. The benefit of this choice of preferences is that an effect of government spending on labor supply is due to demand effects as in the previous sections, and is not confounded with well known wealth effects.

Final Goods

The perfectly competitive, representative, final good producing firm combines a continuum of intermediate goods $Y_t(j)$ indexed by $j \in [0,1]$ using the technology

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{1}{1+\lambda}} dj\right)^{1+\lambda}. \tag{83}$$

Here $\lambda > 0$ and $\frac{1+\lambda}{\lambda}$ represents the elasticity of demand for each intermediate good. The final goods firm takes intermediate good prices $P_t(j)$ of $Y_t(j)$ and output prices P_t of the final output Y_t as given. Profit maximization of intermediate firms implies that the demand for intermediate goods is

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t. \tag{84}$$

The relationship between intermediate goods prices and the price of the final good is

$$P_t = \left(\int_0^1 P_t(j)^{-\frac{1}{\lambda}} dj\right)^{-\lambda}. \tag{85}$$

Intermediate Goods.

Intermediate good j is produced by a monopolist who has access to a linear production technology which uses labor $H(j)$ as the only input:

$$Y_t(j) = AH_t(j). \tag{86}$$

Labor market clearing requires

$$h_t = \int_0^1 H_t(j)dj. \tag{87}$$

Price Setting As in Mankiw and Reis (2002) prices adjust slowly since information disseminates slowly and processing information takes time. In each period, a fraction ω of firms obtains full information, so that their subjective expectation of the contemporaneous values of aggregate variables coincides with the actual values of these variables. The remaining firms base their expectations and decisions on outdated information.

Taking as given nominal wages, final good prices, the demand schedule for intermediate products and technological constraints, firm j chooses its labor inputs $H_t(j)$ and the price $P_t(j)$ to maximize profits

$$\mathbb{E}_t^j\left(\frac{P_{t+s}(j)}{P_{t+s}}Y_{t+s}(j) - \frac{1}{P_{t+s}}W_{t+s}H_{t+s}(j)\right), \tag{88}$$

where the superscript j in the expectation operator $\mathbb{E}_t^j$ indicates that this expectation is formed using firm $j's$ information.

Combining the production technology and the demand schedule implies

$$H_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} \frac{Y_t}{A_t}$$

Thus, the firm maximizes the following objective function with respect to $P_t(j)$:

$$\mathbb{E}_t^j\left(\frac{P_t(j)}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} Y_t - \frac{W_t}{P_t}\left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\lambda}{\lambda}} \frac{Y_t}{A_t}\right), \tag{89}$$

resulting in a price setting rule

$$\mathbb{E}_t^j\left(\frac{P_t(j)}{P_t} = \frac{W_t}{P_t A_t}(1+\lambda)\right). \tag{90}$$

Each firm uses its information set to set the price as a markup $1+\lambda$ over real marginal costs. As information is dispersed across firms and only a fraction ω of firms obtain updated information each period, prices are dispersed as well.

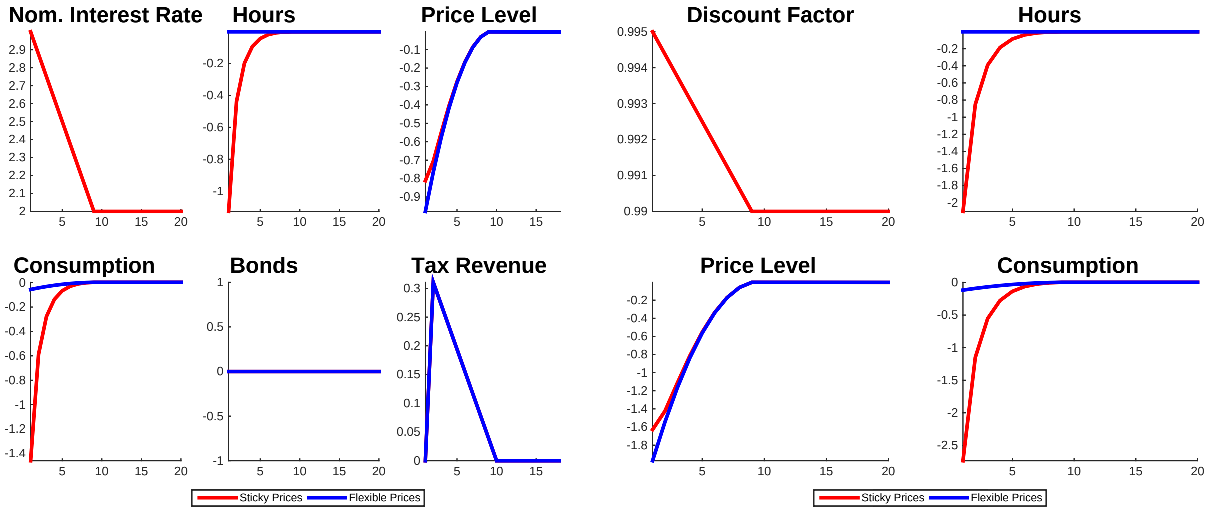
5.2 Numerical Example

Next, I provide some calibrated examples to illustrate the workings of the model, through computing the same impulse responses as characterized analytically in section 4.3. Yet here, prices are sticky and labor supply is elastic. The results should not be considered precise estimates due to the simplifying assumptions of the model, mainly when modeling the consumption and saving decisions.[17]

These numerical illustrations necessitate a choice of parametric forms and parameter values. I choose the following: A model period is a quarter and therefore $\beta = 0.99$. I normalize steady state $A = 1$. The steady-state nominal interest rate is 2% (annualized) and the steady state inflation rate is zero. Following standard choices in the literature I assume $\lambda = 0.2$, $\omega = 0.25$ and $\phi = 2$. I allow households to use a credit line of 10% of their income to pay for emergency consumption, $\bar{b} = 0.1$. The distribution of the preference shocks θ is assumed to be lognormal with parameters $\mu = 0$ and $\sigma = 1$ (the mean and standard deviation of $\log(\theta)$). Kaplan and Violante (2014) document that the median household holds only a small amount of liquid assets, motivating a choice of $B = 0.25$, equal to households' liquid assets to income ratio of 25%. These choices imply that only 1% of households are constrained in emergency expenditures, a fraction much lower than the 17.5% and 35% hand to mouth consumers found in Kaplan and Violante (2014). The model therefore does not overstate the importance of credit constraints. Government expenditures equal $G = 0.2$. Finally I pick the two preference scale parameters $\kappa = 0.833$ and $\eta = 0.079$ to obtain $h = 1$ and $P = 1$ in steady state.

I now use this calibrated model to compute model impulse responses to a monetary policy shock (an increase of R by one (annualized) percentage point), a fiscal policy shock

[17] To obtain analytical tractability I assumed large families with the consequence that the model is not even close to more realistic models of consumption behavior, as for example in Kaplan and Violante (2014). I do so since this assumption has no relevance for the objective of this paper.



(a) Monetary Policy (b) Discount Factor

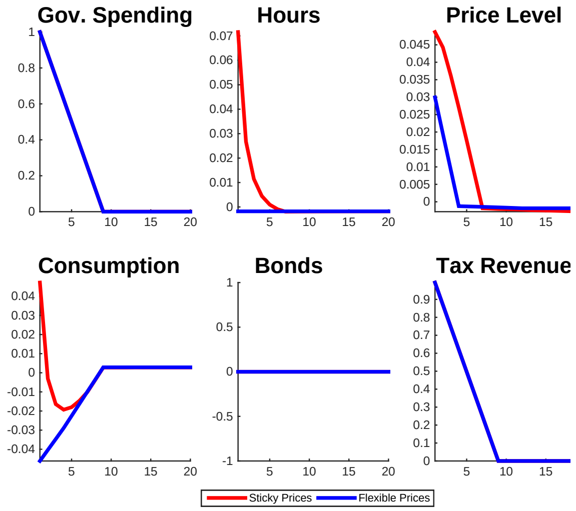
Figure 13: Impulse Responses for a) Monetary Policy and b) Discount Factor of hours, consumption C at t_1 and prices for both the sticky and flexible price economies.

(an increase in nominal government spending G by 1%), a technology shock (an increase of A by 1%) and a discount factor shock (an increase of β from 0.99 to 0.995). All shocks are unexpected. In all experiments except for the fiscal policy one, I assume that 80% of government spending, 0.16, is fixed in real terms to that nominal spending equals $0.16P + 0.04$. Real spending thus equals $0.16 + 0.04/P$, which increases if the price level P falls. To the extent that government spending serves as a nominal anchor, real spending automatically increases in response to price-decreasing shocks. I explore this automatic stabilization feature in more detail in the next section.

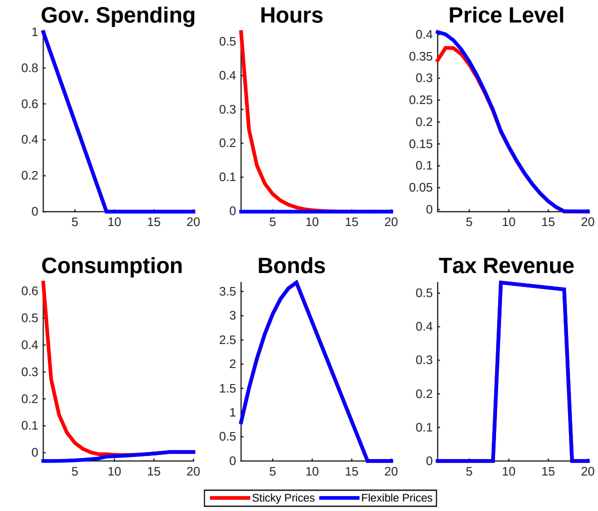
The monetary policy shock has the expected negative effects on prices and employment (panel a) of Figure 13) since higher nominal interest rates increase the incentives to save and thus decrease consumption. Interestingly, the aggregate price level response is very close with flexible and sticky prices, whereas hours do not move if prices are flexible but drop quite a bit if prices are sticky. This is because with sticky prices many firms charge too high a markup and as a result hire too little.

The similar logic applies to the discount factor shock where again prices and hours fall.

The increase in government spending stimulates aggregate demand and therefore leads to an increase in prices, both with flexible and sticky prices. Hours increase only in the latter case, since firms charge too low a markup and as a result hire more to satisfy the additional demand. With flexible prices firms always charge the right markup and as a result employment does not respond. In panel a) of Figure 14 the increase in spending is



(a) Fiscal Policy (Tax financed)

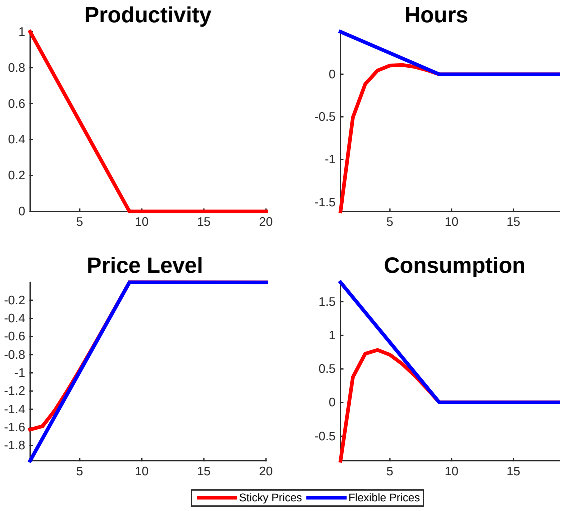


(b) Fiscal Policy (Debt financed)

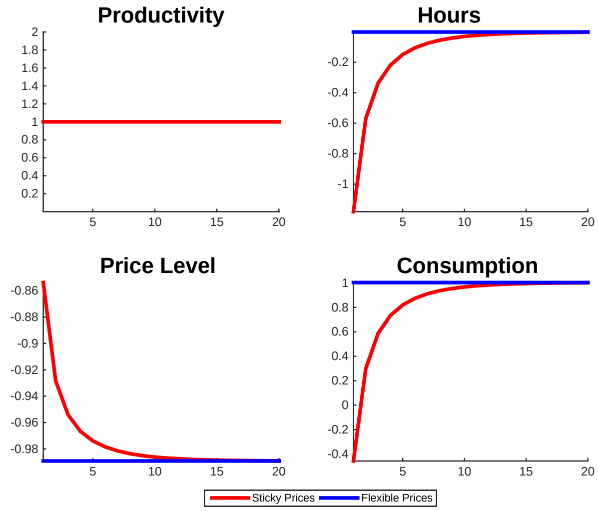
Figure 14: Fiscal Policy: Impulse Responses of hours, consumption C at t_1 and prices for both the sticky and flexible price economies.

financed with an increase in taxes only, which by itself contracts private demand. However, the tax increase lowers private demand by less than the increase in government demand. Total demand still increases, but the positive effects on prices and hours are muted. In panel b) of Figure 14 the increase in spending is financed by issuing government debt which is paid back only later through higher taxes. The initial deficit financing avoids the contractionary demand effect of the tax financing as shown in panel a). As a result the response of hours and prices is, larger, almost by an order of magnitude, with deficit spending than with tax financing.

The technology policy shock has negative effects on prices and employment (panel a) of figure 15) operating through two channels. First, the increase in technology raises aggregate supply and the price level has to fall to stimulate aggregate demand. Second, a non-permanent increase in technology leads to higher savings to smooth consumption between initial periods when technology is high and later periods when the shock is dying out. The savings channel is quite strong such that even though prices are pretty sticky the aggregate price level response when prices are sticky is very close to the flexible prices case. Hours increase instead if prices are flexible, but drop quite a bit if prices are sticky. This is again because with sticky prices many firms charge too high a markup and then hire too little, whereas for firms with flexible prices the increase in productivity leads to more employment. When the technology shock is permanent, the savings channel is eliminated and the impulse responses change accordingly as shown in panel b) of figure 15. In this



(a) Technology



(b) Technology (Permanent)

Figure 15: Technology Shock: Impulse Responses of hours, consumption C at t_1 and prices for both the sticky and flexible price economies.

experiment I make balanced growth assumptions[18] such that with flexible prices the price level drops permanently by one percent and aggregate nominal supply is unchanged. With sticky prices the absence of consumption-smoothing incentives makes prices and hours move less than when the technology shock is temporary. Prices would not adjust at all if fiscal policy keeps the spending output ratio, G/Y, and the debt GDP ratio, B/Y, constant in response to an increase in Y as this equates nominal demand and supply.

5.3 Nominal Fiscal Anchor as Automatic Stabilizer

As explained above, when computing the impulse responses I assumed that 80% of government spending is real and only 20% is nominal, so that changes in the price level move only 20% of real government spending. The larger the fraction of fixed nominal spending is, the larger is the change in real spending as an automatic response to changes in prices. The larger the nominal anchor, the larger then the automatic demand response. I established above that a fiscal policy stimulus increases prices and employment. Not surprisingly, the size of this automatic demand response matters for the reaction of the economy.

I now consider the same discount factor shock as above, and again compute the impulse response of consumption, prices and hours but with one change. Government spending is

[18] The utility function is $\log\left(C_t^h - A_t \kappa h_t^{1+\phi}\right) + \theta\eta \log\left(c_t^h\right)$, credit constraints are $A_t \bar{b}$ and government policy is fully nominal.

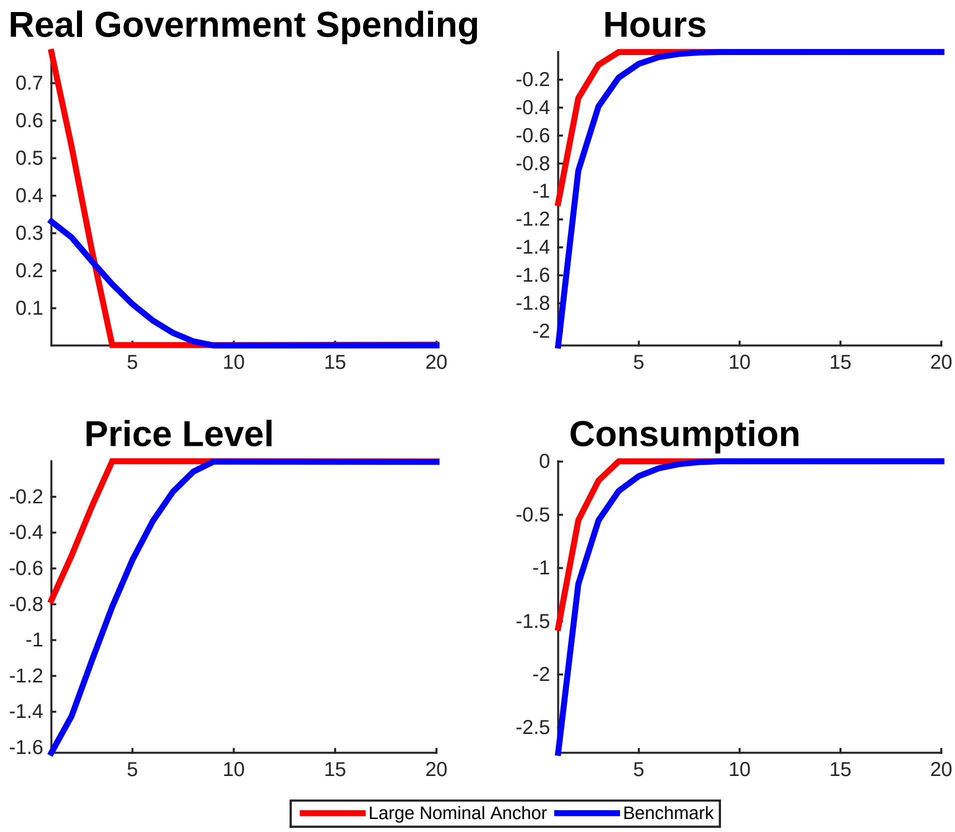


Figure 16: Response to Discount Factor Shock: a) Benchmark (Partial Nominal Anchor) and b) Full Nominal Anchor

now fully nominal, so that real spending as a function of the price level equals G/P. Since the discount factor induces a fall in prices, G/P increases automatically and significantly more than in the benchmark where only 20% of spending is nominal. This additional demand increase partly offsets the discount-factor shock induced drop in demand, and therefore also alleviates the negative consequences on hours worked. The result is shown in Figure 16.

5.4 Zero Lower Bound and Fiscal Policy

Even when government spending is fully nominal and therefore the automatic stabilizer is most effective, increases in the discount factor still lead to a contraction in employment. There are, however, a number of ways to neutralize the effect of a discount factor increase. As long as the zero lower bound is not binding, the solution is particularly simple. An increase in the discount factor from β to $\hat{\beta}_t$ can be neutralized basically through monetary policy alone, by decreasing the nominal interest rate from R to $R_t = R\frac{\beta}{\hat{\beta}_t}$. Fiscal policy uses the (due to the lower R) lower interest rate payments on government debt to lower taxes. This policy is successful in stabilizing employment, consumption and prices at their steady-state values.

If the shock to the discount factor is too large the zero lower bound renders this solution impossible since $R\frac{\beta}{\hat{\beta}_t} < 1$. In this scenario fiscal policy has to step in and increase spending to stabilize employment and prices. Obviously if the goal is employment stabilization, output is stabilized too and therefore private consumption has to fall. This suggests that monetary policy is the preferred policy tool in general as it enables stabilization of all three variables, whereas fiscal policy cannot. Fiscal actions are taken only if the zero lower bound makes monetary policy ineffective.

An increase of $\beta = 0.99$ to 1.01 implies that the zero lower bound is binding. Figure 17 shows the impulse responses when government spending is unchanged and only monetary policy decreases interest rates to zero, as long as the zero lower bound is binding. Since monetary policy is not able to neutralize the increase in β, employment and prices fall. Under these circumstances the goal to stabilize employment and prices requires fiscal policy to become active and increase spending. This policy is successful. Employment and prices remain throughout at their steady-state values. The path of government spending which achieves this is shown in the last panel of Figure 17.

5.5 Monetary and Fiscal Policy: Coordination

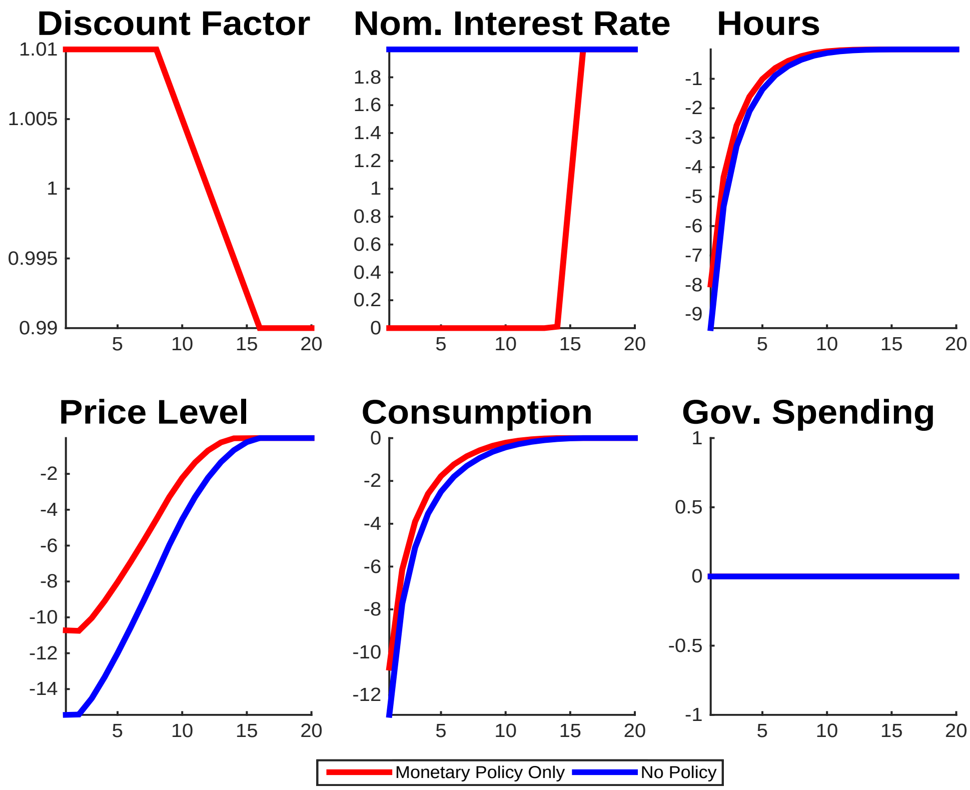
A main result of this paper is that fiscal and monetary policy jointly determine the price level. Understanding how prices move requires a good grasp of the policy coordination by the treasury and the central bank. Since here the price level is determined, policy coordination problems are quite different from those in Sargent and Wallace (1981)'s classic "monetarist arithmetic" or in Leeper (1991)'s active and passive monetary and fiscal policies. In particular the question is not which combination of fiscal and monetary policy leads to local determinacy of the price level and which combination does not. Instead prices are globally unique for any combination of policies.

Policy coordination is therefore very different. Fiscal policy can control the long-run inflation rate through controlling the nominal anchor. I showed in Section 4.4 that monetary policy takes over and sets the steady-state inflation rate when fiscal policy does not exercise its power to control long-run inflation.

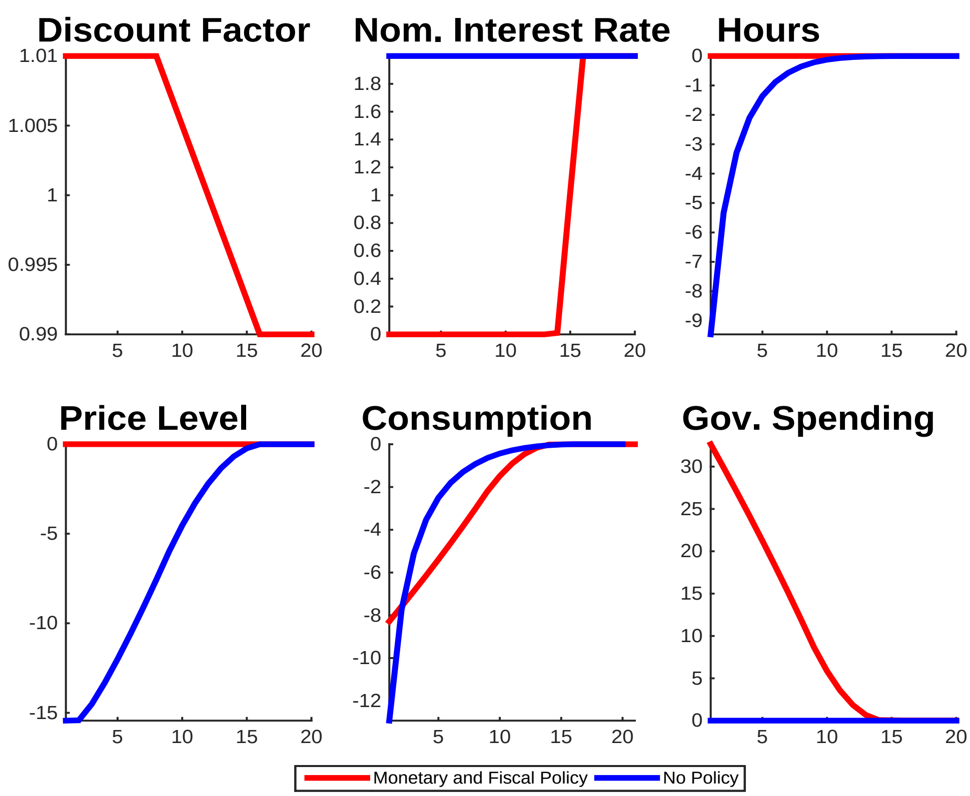
The analysis so far could give the impression that monetary policy is quite powerless and that fiscal policy can always get its way, not only determining fiscal policy but also setting the inflation rate, which in every textbook model is under the control of monetary policy exclusively.

This impression would underestimate the power of a central bank. On the one hand I have shown the effectiveness of monetary policy in stabilizing the economy. I will now illustrate its power by showing that the central bank can undo stimulative policies initiated by the treasury to raise inflation and boost employment. Any such short-run fiscal policy actions not meeting with the central bank's approval, for example because they conflict with the objective of price stability, can be neutralized resulting in no change in prices and employment and only higher taxes or debt. Figure 18 shows the sequence of nominal interest rates, in panel a) for a tax-financing and in panel b) for deficit-financing, which undo the fiscal policy expansion considered in Figure 14. Since a deficit-financed expansionary fiscal policy is more stimulative than a tax-financed one, the offsetting increase in nominal interest rates has to be higher in the former case.

This exercise also reveals another unpleasant consequence of an increase in nominal interest rates for the treasury. In practice this could be the most effective way for the central bank to impose its will on fiscal policy. Increases in nominal interest rates raise the interest payments on government debt, leading to higher debt and eventually higher taxes.

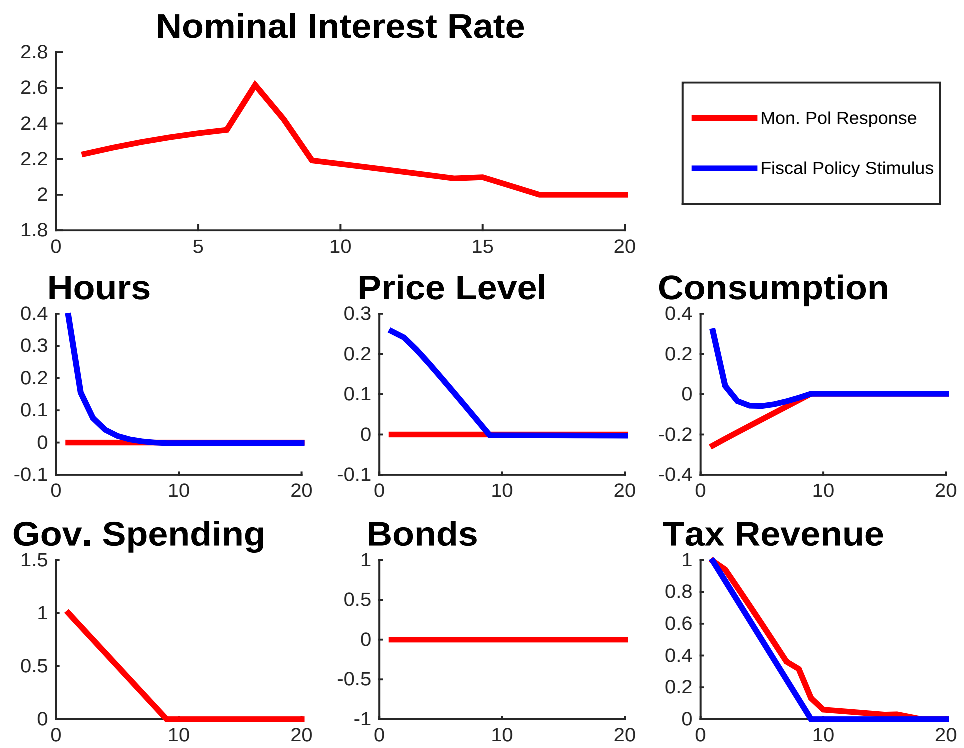


(a) ZLB: Monetary Policy Only



(b) ZLB: Fiscal and Monetary Policy

Figure 17: Zero Lower Bound : a) Monetary Policy Only b) Monetary and Fiscal Policy



(a) Monetary Policy Neutralizing Fiscal Policy

(b) Monetary Policy Neutralizing Fiscal Policy (Debt)

Figure 18: a) Fiscal Policy Stimulus (Tax financed) b) Fiscal Policy Stimulus (Debt financed)

6 Conclusion

This paper has shown that the price level is globally determinate in a model which incorporates the simple empirical finding that a permanent income increase leads to a less than one-for-one increase in consumption, and at the same time to an increase in precautionary savings.

The simplicity of the model keeps it tractable and enables the researcher to better understand the monetary and fiscal transmission mechanism. A key finding is that the price level is determined jointly by monetary and fiscal policy, with long-run inflation determined by the growth rate of nominal government spending. The nominal anchor - nominal government spending - is controlled by fiscal policy, which therefore has the power to set the long-run inflation rate.

Applied to recent attempts by the ECB to increase inflation in the Euro area, the findings in this paper suggest that these efforts are unlikely to be successful. Instead higher inflation would require an expansion of nominal fiscal spending by Euro area member states to stimulate nominal demand, assigning an important role to large countries such as Germany. A fiscal stimulus by a small country would have very little impact on inflation, as it has only a negligible effect on area-wide demand, but would lead to a real appreciation (with likely adverse economic consequences) for this small country.

Applied to the growing concerns that the US or the world economy may be stuck in a liquidity trap with zero nominal and real interest rates for an extended time, the findings in this paper suggest an easy solution. Although the ZLB prevents further cuts of the nominal interest rate, fiscal policy can increase the growth rate of nominal spending and therefore the inflation rate, leading to lower real interest rates, provided that this policy is sufficiently persistent and credible. If instead fiscal policy continues its current austerity plan bringing low inflation rates to around zero, then the real interest rate will hover around zero too, even in the long run.

In a numerical exercise, I show that impulse responses to monetary and fiscal policy shocks, as well as to technology and discount factor shocks, line up with empirical evidence and conventional wisdom. I also establish how government spending serves as an automatic stabilizer, and how monetary and fiscal policy interact, and discuss stabilization policies at the ZLB.

The model used to conduct the quantitative exercises lacks many elements which could be necessary to obtain precise estimates of the policy effects. A full quantitative macroeconomic model would: use the Aiyagari (1994) incomplete market model with capital and elastic labor supply as a starting point; model the consumption behavior better such that the MPC aligns with the data; allow for distortionary taxation such that the cost of an expansionary policy is more realistic; and allow for long-term government debt to discuss quantitative easing. We do so in Hagedorn et al. (2016). Building on the insights in this paper, we use a full Aiyagari (1994) incomplete market model to quantitatively assess the size of the fiscal multiplier in a model with a determinate price level. As explained in the introduction, this approach overcomes Cochrane's criticism of existing approaches to quantify the fiscal multiplier when the zero lower bound is binding. Furthermore, a determinate price level not only allows for policy analyses when the zero lower bound is binding, but more generally for studying policy without using a Taylor rule. In standard complete market models the nominal interest rate has to respond to inflation aggressively enough to guarantee a locally determinate inflation rate. Policy analysis is then restricted to this subset of aggressive monetary policies to avoid indeterminacy. The theory in this paper overcomes these restrictions. Monetary policy can be represented by any arbitrary sequence of nominal interest rates, because for each sequence determinacy of the equilibrium is ensured.

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Appendix

Proof of Proposition 1 For a given level of consumption C at t_1 and a given threshold level $\hat{\theta}$, consumption in period t_2 equals

$$\begin{array}{lll} i) & c(\theta) = (v')^{-1}(\frac{u'(C)}{\theta}) & \text{if } \theta \leq \hat{\theta}, \\ ii) & c(\theta) = B_t/P_t + \overline{b} & \text{if } \theta > \hat{\theta}. \end{array}$$

As a result consumption C at period t_1 solves

$$C = A - \frac{G}{P} - (1 - F(\hat{\theta}))(\frac{B}{P} + \bar{b}) - \int_{\underline{\theta}}^{\hat{\theta}} (v')^{-1}(\frac{u'(C)}{\theta}) dF(\theta), \tag{91}$$

and the threshold level then solves

$$\hat{\theta} = \frac{u'(C)}{v'(\frac{B}{P} + \bar{b})}. \tag{92}$$

This defines $\hat{\theta}$ as a function of P and C. Plugging this expression into the fixpoint equation (91) for C, shows that C is a function of P only and so is $\hat{\theta}$. To see that C is increasing in P, take derivatives of the fixpoint equation

$$C(P) = A - \frac{G}{P} - (1 - F(\hat{\theta}(P)))(\frac{B}{P} + \bar{b}) - \int_{\underline{\theta}}^{\hat{\theta}(P)} (v')^{-1}(\frac{u'(C(P))}{\theta}) dF(\theta). \tag{93}$$

w.r.t P,

$$C'(P) = \frac{G}{P^2} + (1 - F(\hat{\theta}(P)))\frac{B}{P^2} - [\int_{\underline{\theta}}^{\hat{\theta}(P)} (\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))}) dF(\theta)]C'(P) \tag{94}$$

$$+ \hat{\theta}'(P) \underbrace{[(\frac{B}{P} + \bar{b}) f(\hat{\theta}(P)) - (v')^{-1}(\frac{u'(C(P))}{\theta}) f(\hat{\theta}(P))]}_{=0}, \tag{95}$$

so that

$$C'(P) = \frac{\frac{G}{P^2} + (1 - F(\hat{\theta}(P)))\frac{B}{P^2}}{1 + [\int_{\underline{\theta}}^{\hat{\theta}(P)} (\frac{u''(C(P))}{\theta v''((v')^{-1}(\frac{u'(C(P))}{\theta}))}) dF(\theta)]} > 0, \tag{96}$$

since both $u'' < 0$ and $v'' < 0$. Having established that $C(P)$ is increasing in P immediately implies that

$$\hat{\theta}(P) = \frac{u'(C(P))}{v'(\frac{B}{P} + \bar{b})} \tag{97}$$

is decreasing in P since the numerator is decreasing in P and the denominator is increasing in P.

Proof of Proposition 2 Existence and uniqueness of the price level solving the FOC equation (45) implies the existence and uniqueness of all other variables, which can all be expressed as functions of the price level. Rewriting equation (45) yields

$$u'(C(P)) - \int_{\hat{\theta}(P)}^{\infty} \theta v'(B/P + \bar{b}) dF(\theta) = F(\hat{\theta}(P)) \frac{R}{(1+\pi)} \beta u'(C(P)), \tag{98}$$

such that the derivatives of both the LHS and the RHS are negative,

$$\begin{aligned}
\frac{\partial LHS}{\partial P} &= u''(C(P)) \frac{\partial C(P)}{\partial P} + \int_{\hat{\theta}(P)}^{\infty} \theta \frac{B}{P^2} v''(B/P + \bar{b}) dF(\theta) + f(\hat{\theta}(P)) \hat{\theta}(P) v'(B/P + \bar{b}) \frac{\partial \hat{\theta}(P)}{\partial P} \\
\frac{\partial RHS}{\partial P} &= F(\hat{\theta}(P^*)) \frac{R}{1+\pi} \beta u''(C(P)) \frac{\partial C(P)}{\partial P} + f(\hat{\theta}(P)) \frac{R}{1+\pi} \beta u'(C(P^*)) \frac{\partial \hat{\theta}(P)}{\partial P},
\end{aligned}$$

with

$$\frac{\partial LHS}{\partial P} < \frac{\partial RHS}{\partial P} < 0 \tag{99}$$

since $\frac{\partial \hat{\theta}(P)}{\partial P} < 0$, $\frac{\partial C(P)}{\partial P} > 0$, $\hat{\theta}(P) v'(B/P + \bar{b}) = u'(C(P^*)) > \frac{R\beta}{1+\pi} u'(C(P^*))$ and $\frac{R\beta}{1+\pi} F(\hat{\theta}(P^*)) \leq \frac{R\beta}{1+\pi} < 1$. Thus $LHS - RHS$ is a monotonically decreasing function of P and the intermediate value theorem implies that there is at most one P where $LHS(P) = RHS(P)$, that is there is at most one steady state price level.

To establish existence of such a price level, I now show that the LHS is larger than the RHS for small P and the RHS is larger than the LHS for large P, implying a unique P where the LHS is equal to the RHS.

To see this, note that if $P \to 0$, $u'(C(P)) \to \infty$ which since $F(\hat{\theta}(P)) \frac{R}{(1+\pi)} \beta < 1$ and all other terms are bounded implies that $LHS > RHS$ for small P.

If $P \to \infty$, C converges to a number larger than $A - \bar{b}$ and $\hat{\theta}$ converges to a number smaller than $\frac{u'(A-\bar{b})}{v'(\bar{b})}$. Thus using assumption (30) implies

$$LHS \leq u'(A - \bar{b}) - \int_{\frac{u'(A-\bar{b})}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) dF(\theta) < 0 < RHS \tag{100}$$

Proof of Proposition 3

Proposition 2 implies a unique steady state price level such that $\Phi(P) = \frac{R}{1+\gamma} \Gamma(P)$. I now rule out non steady state equilibria, such as cycles etc. To this aim, Step 1 is key as it establishes the monotonicity of Φ and Γ.

Step 1

I first show that both the function

$$\Phi(\tilde{P}_t) = \frac{u'(C(\tilde{P}_t)) - \int_{\hat{\theta}(\tilde{P}_t)}^{\infty} \theta v'(B_t/\tilde{P}_t + \bar{b}) dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \tag{101}$$

is decreasing in $\tilde{P}_t$ if $\Phi(\tilde{P}_t) > 0$ and the function

$$\Gamma(\tilde{P}_{t+1}) = \beta \frac{u'(C(\tilde{P}_{t+1}))}{\tilde{P}_{t+1}} \tag{102}$$

is decreasing in $\tilde{P}_{t+1}$.

This is obvious for Γ with derivative

$$\Gamma'(\tilde{P}) = \beta \frac{u''(C(\tilde{P}))C'(\tilde{P})\tilde{P} - u'(C(\tilde{P}))}{\tilde{P}^2} < 0 \tag{103}$$

since $u', C' > 0$ and $u'' < 0$.

The derivative of Φ equals

$$\begin{aligned}
\Phi'(\tilde{P}) &= \frac{u''(C(\tilde{P}))C'(\tilde{P}) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta \frac{B}{\tilde{P}^2} v''(B/\tilde{P} + \bar{b}) dF(\theta) + \hat{\theta}'(\tilde{P})\hat{\theta}(\tilde{P}) v'(B/\tilde{P} + \bar{b}) f(\hat{\theta}(\tilde{P}))}{\tilde{P} F(\hat{\theta}(\tilde{P}))} \\
&- \frac{[u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta)][F(\hat{\theta}(\tilde{P})) + \tilde{P} f(\hat{\theta}(\tilde{P}))\hat{\theta}'(\tilde{P})]}{(\tilde{P} F(\hat{\theta}(\tilde{P})))^2} \\
&= \frac{u''(C(\tilde{P}))C'(\tilde{P}) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta \frac{B}{\tilde{P}^2} v''(B/\tilde{P} + \bar{b}) dF(\theta)}{\tilde{P} F(\hat{\theta}(\tilde{P}))} \\
&- \frac{u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta)}{\tilde{P}^2 F(\hat{\theta}(\tilde{P}))} \\
&+ \hat{\theta}'(\tilde{P}) \frac{f(\hat{\theta}(\tilde{P}))}{\tilde{P} F(\hat{\theta}(\tilde{P}))^2} \left[\hat{\theta}(\tilde{P}) v'(B/\tilde{P} + \bar{b}) F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta) \right] \\
&< 0,
\end{aligned} \tag{104}$$

since

$$\frac{u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta)}{\tilde{P}^2 F(\hat{\theta}(\tilde{P}))} = \Phi(\tilde{P})/\tilde{P} > 0,$$

$$
\begin{aligned}
& \hat{\theta}(\tilde{P})v'(B/\tilde{P}+\bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P}+\bar{b})dF(\theta) \\
\geq\ & \hat{\theta}(\tilde{P})v'(B/\tilde{P}+\bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + \int_{\hat{\theta}(\tilde{P})}^{\infty} \hat{\theta}(\tilde{P}) v'(B/\tilde{P}+\bar{b})dF(\theta) \\
=\ & \hat{\theta}(\tilde{P})v'(B/\tilde{P}+\bar{b})F(\hat{\theta}(\tilde{P})) - u'(C(\tilde{P})) + (1-F(\hat{\theta}(\tilde{P})))\hat{\theta}(\tilde{P})v'(B/\tilde{P}+\bar{b}) \\
=\ & \hat{\theta}(\tilde{P})v'(B/\tilde{P}+\bar{b}) - u'(C(\tilde{P})) \\
=\ & 0, \qquad (105)
\end{aligned}
$$

and $\hat{\theta}'(\tilde{P}) < 0$.

In steady state $\Phi'(P^*) < \frac{R}{1+\gamma}\Gamma'(P^*)$:

$$
\begin{aligned}
& \Phi'(P^*) - \frac{R}{1+\gamma}\Gamma'(P^*) \\
=\ & \frac{-1}{(P^*)^2 F(\hat{\theta}(P^*))}\Big[u'(C(P^*)) - \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^*+\bar{b})dF(\theta) - \frac{R}{1+\gamma}F(\hat{\theta}(P^*))\beta u'(C(P^*))\Big] \\
+\ & \frac{u''(C(P^*))C'(P^*)}{P^* F(\hat{\theta}(P^*))}[1-\frac{R}{1+\gamma}F(\hat{\theta}(P^*))\beta] \\
+\ & \frac{\int_{\hat{\theta}(P^*)}^{\infty} \theta \frac{B}{(P^*)^2} v''(B/P^*+\bar{b})dF(\theta)}{P^* F(\hat{\theta}(P^*))} \\
+\ & \hat{\theta}'(P^*)\frac{f(\hat{\theta}(P^*))}{F(\hat{\theta}(P^*))}\Big[\hat{\theta}(P^*)v'(B/P^*+\bar{b})F(\hat{\theta}(P^*)) - u'(C(P^*)) + \int_{\hat{\theta}(P^*)}^{\infty} \theta v'(B/P^*+\bar{b})dF(\theta)\Big] \\
<\ & 0,
\end{aligned}
$$

since the first term is zero at the steady state (the term in square brackets is the FOC and thus zero), the second term is negative since $[1-\frac{R}{1+\gamma}F(\hat{\theta}(P^*))\beta] > 0$ and $u'' < 0$, the third term is negative since $v'' < 0$ and the last term was shown above to be negative (equation (105)).

This implies that $\Phi(\tilde{P}) < \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} > P^*$ and $\Phi(\tilde{P}) > \frac{R}{1+\gamma}\Gamma(\tilde{P})$ for $\tilde{P} < P^*$ since there is a unique steady-state price level, $\Phi(\tilde{P}) = \frac{R}{1+\gamma}\Gamma(\tilde{P})$ iff $\tilde{P} = P^*$, as shown in Proposition 2.

Step 2

I now show that if $\tilde{P}_t > P^*$ then the subsequent sequence of prices is monotonically increasing, $\tilde{P}_t < \tilde{P}_{t+1} < \ldots < \tilde{P}_{t+k}, \ldots$ and if $\tilde{P}_t < P^*$ then the subsequent sequence of prices is monotonically decreasing, $\tilde{P}_t > \tilde{P}_{t+1} > \ldots > \tilde{P}_{t+k}, \ldots$. before I finally show in Step 3 that such price sequences do not form an equilibrium.

For every $\tilde{P}_s$ at time s the price at time $s+1$, $\tilde{P}_{s+1}$, is defined as solving

$$\Phi(\tilde{P}_s) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{s+1}) \tag{106}$$

If $\tilde{P}_s > P^*$ then

$$\frac{R}{1+\gamma}\Gamma(\tilde{P}_s) > \Phi(\tilde{P}_s) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{s+1}), \tag{107}$$

which implies that $\tilde{P}_{s+1} > \tilde{P}_s$ since $\Gamma'(\tilde{P}) < 0$.

If $\tilde{P}_s < P^*$ then

$$\frac{R}{1+\gamma}\Gamma(\tilde{P}_s) < \Phi(\tilde{P}_s) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{s+1}), \tag{108}$$

which implies that $\tilde{P}_{s+1} < \tilde{P}_s$ since $\Gamma'(\tilde{P}) < 0$.

Step 3

Step 2 shows that a price $\tilde{P}_t$ different from the steady-state price P^* leads to either an monotonically increasing price sequence (if $\tilde{P}_t > P^*$) or a monotonically decreasing price sequence (if $\tilde{P}_t < P^*$).

The monotonically increasing price sequence is unbounded since otherwise the prices would converge. The limit would be a steady state, contradicting the uniqueness of a steady state established in Proposition 2.

Price levels that are too high are not equilibrium prices however, since for all high enough price levels $\tilde{P}$ it holds that $\hat{\theta}(\tilde{P}) < \frac{u'(A-\bar{b})}{v'(\bar{b})}$ and thus

$$u'(C(\tilde{P})) - \int_{\hat{\theta}(\tilde{P})}^{\infty} \theta v'(B/\tilde{P} + \bar{b}) dF(\theta) \leq u'(A - \bar{b}) - \int_{\frac{u'(A-\bar{b})}{v'(\bar{b})}}^{\infty} \theta v'(\bar{b}) dF(\theta) < 0, \tag{109}$$

implying that such a high $\tilde{P}$ does not form an equilibrium since $\frac{R}{1+\gamma}\Gamma > 0$.

The monotonically decreasing price sequence is not bounded from below by some strictly positive number with a positive consumption level, since otherwise the price sequence would converge to a positive price level. The limit would be a steady state, contradicting the uniqueness of a steady state established in Proposition 2.

Price levels that are too low are not equilibrium prices either, since for a low enough price level $\tilde{P}$ it holds that $G/\tilde{P} > A$, that is real government expenditures exceed real output and consumption is non-positive, implying that such a low $\tilde{P}$ is not an equilibrium either.

Altogether this implies that there is no $\tilde{P}_t \neq P^*$ since an $\tilde{P}_t \neq P^*$ would be consistent only with price expectations which are (eventually) not an equilibrium.

Proof of Theorem 1 From time S on policies are stationary, i.e. for $t \geq S$, $R_t = R$, $G_t = G(1+\gamma)^{t-S}$, $T_t = T(1+\gamma)^{t-S}$ and $B_t = B(1+\gamma)^{t-S}$. The results shown above imply that from time S the economy is characterized by a price level P^* such that prices for $t \geq S$, $P_t = P^*(1+\gamma)^{t-S}$. As before consumption $C_t = C(P^*)$ and $\hat{\theta}_t = \hat{\theta}(P^*)$.

Prices before time S are defined recursively. Given a price P_t^* at time $t \leq S$, the price level at time $t-1$ is

$$P_{t-1}^* = \Phi_{t-1}^{-1}(R_t \Gamma_t(P_t^*)), \tag{110}$$

where

$$\Phi_t(P_t) = \frac{u'(C(P_t, B_t)) - \int_{\hat{\theta}(P_t)}^{\infty} \theta v'(B_t/P_t + \bar{b}) dF(\theta)}{P_t F(\hat{\theta}(P_t))} \tag{111}$$

and the function

$$\Gamma_t(P_{t+1}) = \beta \frac{u'(C(P_{t+1}, B_{t+1})}{P_{t+1}} \tag{112}$$

and Φ is strictly monotone and therefore invertible. Note that these are actual prices P and not detrended prices $\tilde{P}$ as the economy is not in steady state before time S.

This recursive definition yields a price sequence

$$(P_0^*, P_1^*, \ldots, P_t^*, \ldots, P_S = P^*, P_{S+1} = P^*(1+\gamma), P_{S+2} = P^*(1+\gamma)^2, \ldots,) \tag{113}$$

I now show that this the unique equilibrium sequence of prices. Suppose now to the contrary that there is another equilibrium price sequence $\hat{P}$ and let k be the first time when the price levels differ $\hat{P}_k \neq P_k^*$ (and $\hat{P}_m = P_m^*$ for $m < k$). The uniqueness of a steady-state price level implies that $k < S$ since from period S onwards, the equilibrium price equals $P^*(1+\gamma)^{t-S}$).

The price at time k, $\hat{P}_k$, uniquely pins down the full price sequence recursively,

$$\hat{P}_{m+1} = \Gamma_{m+1}^{-1}(\Phi_m(\hat{P}_m)/R_{m+1}), \tag{114}$$

for all $m \geq k$. Since both Γ_t and Φ_t are strictly decreasing functions, the concatenation $\Gamma_{l+1}^{-1}(\Phi_l(P_l^*)/R_{l+1})$ is strictly increasing. This implies that if $\hat{P}_k > P_k^*$, then $\hat{P}_m > P_m^*$ for all $m \geq k$. In particular $\hat{P}_S > P_S^* = P^*$, which is not an equilibrium price.

Similarly, if $\hat{P}_k < P_k^*$, then $\hat{P}_m < P_m^*$ for all $m \geq k$. In particular $\hat{P}_S < P_S^* = P^*$, which is not an equilibrium price either.

Together this implies that $\hat{P}_k \neq P_k^*$ is proven wrong by contradiction, implying that $\hat{P}_t = P_t^*$ for all t is the only equilibrium price sequence.

Proof of Proposition 4 The proof proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}) = \frac{R_S}{1+\gamma}\Gamma(P^*) \geq \frac{R}{1+\gamma}\Gamma(P^*) = \Phi(P^*), \tag{115}$$

which implies that $\tilde{P}_{S-1} \leq P^*$ with strict inequality if $R_S < R$.

Now assume that $\tilde{P}_{t+1} \leq \tilde{P}_{t+2} \leq \ldots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t) = \frac{R_{t+1}}{1+\gamma}\Gamma(\tilde{P}_{t+1}) \geq \frac{R_{t+2}}{1+\gamma}\Gamma(\tilde{P}_{t+1}) \geq \frac{R_{t+2}}{1+\gamma}\Gamma(\tilde{P}_{t+2}) = \Phi(\tilde{P}_{t+1}), \tag{116}$$

which implies that $\tilde{P}_t \leq \tilde{P}_{t+1}$ with strict inequality if $R_{t+1} < R_{t+2}$. The inequality uses that Γ is strictly decreasing and $\tilde{P}_{t+1} \leq \tilde{P}_{t+2}$.

This proves the statement for detrended prices by induction. This immediately implies

$$\frac{P_{t+1}}{P_t} \geq (1+\gamma) \tag{117}$$

for non-detrended prices since the trend is $(1+\gamma)$.

Proof of Proposition 5 The proof proceeds by backwards induction. In period $S-1$, the detrended price levels $\tilde{P}^a_{S-1}$ and $\tilde{P}^b_{S-1}$ solve

$$\Phi(\tilde{P}^a_{S-1}) = \frac{R^a_S}{1+\gamma}\Gamma(P^*) \geq \frac{R^b_S}{1+\gamma}\Gamma(P^*) = \Phi(\tilde{P}^b_{S-1}). \tag{118}$$

which implies that $\tilde{P}^a_{S-1} \leq \tilde{P}^b_{S-1}$ with strict inequality if $R^a_S > R^b_S$ since Φ is decreasing.

Now assume that $\tilde{P}^a_{t+1} \leq \tilde{P}^b_{t+1}$. Then

$$\Phi(\tilde{P}_t^a) = \frac{R_{t+1}^a}{1+\gamma}\Gamma(\tilde{P}_{t+1}^a) \geq \frac{R_{t+1}^b}{1+\gamma}\Gamma(\tilde{P}_{t+1}^a) \geq \frac{R_{t+1}^b}{1+\gamma}\Gamma(\tilde{P}_{t+1}^b) = \Phi(\tilde{P}_t^b), \tag{119}$$

implying that $\tilde{P}_t^a \leq \tilde{P}_t^b$ with strict inequality if $R_{t+1}^a > R_{t+1}^b$.

Proof of Proposition 6

Define

$$\Phi(\tilde{P}_t, G) = \frac{u'(C(\tilde{P}_t, G)) - \int_{\hat{\theta}(\tilde{P}_t,G)}^{\infty} \theta v'(B/\tilde{P}_t + \bar{b})dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \tag{120}$$

$$\Gamma(\tilde{P}_{t+1}, G) = \beta\frac{u'(C(\tilde{P}_{t+1}), G)}{\tilde{P}_{t+1}}, \tag{121}$$

where both functions are strictly decreasing in $\tilde{P}$ and strictly increasing in G.

The proof again proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}, \hat{G}) = \frac{R}{1+\gamma}\Gamma(P^*, G) = \Phi(P^*, G) \leq \Phi(P^*, \hat{G}), \tag{122}$$

which implies that $\tilde{P}_{S-1} \geq P^*$ with strict inequality if $\hat{G} > G$ since Φ is decreasing in $\tilde{P}$.

Now assume that $\tilde{P}_{t+1} \geq \tilde{P}_{t+2} \geq \ldots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t, \hat{G}) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{t+1}, \hat{G}) \leq \frac{R}{1+\gamma}\Gamma(\tilde{P}_{t+2}, \hat{G}) = \Phi(\tilde{P}_{t+1}, \hat{G}), \tag{123}$$

which implies that $\tilde{P}_t \geq \tilde{P}_{t+1}$. The same arguments show that a more expansive fiscal policy leads to bigger price increases.

Proof of Proposition 7 The same arguments as in the proofs of Propositions 4 and 5 apply here.

Proof of Proposition 8 Define

$$\Phi(\tilde{P}_t, A) = \frac{u'(C(\tilde{P}_t, A)) - \int_{\hat{\theta}(\tilde{P}_t,A)}^{\infty} \theta v'(B/\tilde{P}_t + \bar{b})dF(\theta)}{\tilde{P}_t F(\hat{\theta}(\tilde{P}_t))} \tag{124}$$

$$\Gamma(\tilde{P}_{t+1}, A) = \beta\frac{u'(C(\tilde{P}_{t+1}), A)}{\tilde{P}_{t+1}}, \tag{125}$$

where both functions are strictly decreasing in $\tilde{P}$ and A.

The proof again proceeds by backwards induction. In period $S-1$, the detrended price level $\tilde{P}_{S-1}$ solves

$$\Phi(\tilde{P}_{S-1}, \hat{A}) = \frac{R}{1+\gamma}\Gamma(P^*, A) = \Phi(P^*, A) \geq \Phi(P^*, \hat{A}), \tag{126}$$

which implies that $\tilde{P}_{S-1} \leq P^*$ with strict inequality if $\hat{A} > A$ since Φ is strictly decreasing in $\tilde{P}$.

Now assume that $\tilde{P}_{t+1} \leq \tilde{P}_{t+2} \leq \ldots$. The detrended price level $\tilde{P}_t$ solves

$$\Phi(\tilde{P}_t, \hat{A}_t) = \frac{R}{1+\gamma}\Gamma(\tilde{P}_{t+1}, \hat{A}_{t+1}) \geq \frac{R}{1+\gamma}\Gamma(\tilde{P}_{t+2}, \hat{A}_{t+2}) = \Phi(\tilde{P}_{t+1}, \hat{A}_{t+1}) \geq \Phi(\tilde{P}_{t+1}, \hat{A}_t), \tag{127}$$

which implies that $\tilde{P}_t \leq \tilde{P}_{t+1}$.