Contracts and Markets: Risk Sharing with Hidden Types

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Abstract

I study two-way effects between financial markets and contractual agreements, such as compensation packages within a firm, or mortgages and loans. I construct a model with many Units, in each of which one of the contracting individuals, the Agent, has private information, while the uninformed individual, the Principal, has the opportunity to trade with the Principals in other Units. I give general conditions under which financial markets induce a transfer of risk from Agents to Principals. These conditions can be reduced to a limited degree of correlation among Units' returns. I show, under the same conditions, that markets induce a transfer of welfare from the best Agents to Principals. Conversely, the information asymmetry within Units leads to excessive aggregate risk in the asset market. However, this problem vanishes in a large economy.

1 Introduction

Risk sharing obtains in different ways. For example, by trading assets on financial markets, or by contractual arrangements among individuals. I study the interaction between these institutions.

These interactions are relevant in a modern economy. Consider how labor compensation is arguably the most relevant expense for a corporation. In the US, for example, more than 60% of the payment to factors group in the 2009 GDP in US was to labor.[1] Since the stock of a company is a claim to its profits, the firm's decisions on workers' compensation affect the returns of its stock. In the aggregate this affects financial markets. On the other hand, diversification opportunities offered by markets influence the design of compensation packages.

Another interesting case of the interaction of these two modes of risk sharing is the securitization of individual contracts, like insurance policies and mortgage loans. In the past decades, these contracts have been increasingly often pooled together and sold as financial assets. The recent financial crisis taught us that this evolution of financial markets can and will affect non-market risk sharing agreements such as mortgages,[2] and that these changes can at times feedback into financial markets with spectacular consequences.

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[1] Economic Report of the President, 2010

[2] There is a vast body of empirical evidence (see for example Agarwal, Chang and Yavas, 2010) that the spread of Mortgage Backed Securities, changed many practices in the underlying sector.

In both cases, it is important for the firm, the insurer or the loan issuer, to assess certain qualities of the counterparty, in order to maximize their profits. The way to screen individuals is to offer them different contracts to choose from. This paper shows how, under some general conditions, financial markets change the tradeoffs involved in designing contracts. This results in risk and being shifted from all non-trading individuals to their trading counterparts, and, at the same time. Welfare is also redistributed, from the non trading individuals of higher qualty, to trading individuals.

To study the interactions between contracting and financial markets, I construct a model with private information in which all individuals are paired into Units generating returns, which are shared by means of a contract. In each Unit, one of the contracting individuals, the Agent, has private information, while the uninformed individual, the Principal, has the opportunity to trade with those in the other Units. In the context of firms, Principals and Agents would be Investors and Employees. In the context of securitization, Agents are individuals taking out a loan or an insurance contract, Principals are the financial institutions securitizing those contracts. This generates two-way effects from markets to Units, and vice versa.

I use this model to answer two questions.

1. How does the existence of asset markets affect risk sharing agreements within individual Units?
2. How does asymmetric information in these agreements affect asset markets?

First, since financial markets help diversify risk, one expects that it would make Principals act as if they were less risk averse. This changes the terms of risk sharing and make contracts less risky for Agents, the "Insurance Effect". Several examples show that this intuitive property need not hold (even in the simple case of symmetric information, in which Agents' types are common knowledge). I give sufficient conditions which rule out these unexpected effects of markets. Roughly speaking, these conditions boil down to markets providing enough diversification opportunities. This is always the case if there is a large number of Units with a limited level of correlation across them. These are fairly general assumptions, which correspond to features of real financial markets.

Second, Principals solve the screening problem they face inside Units by designing contracts offering different levels of risk, to different types of Agents. The need to screen distorts risk sharing within Units. This translates to an inefficiently high level of risk held by Principals, which they consequently trade on asset markets. Excess market risk is hence a byproduct of asymmetric information in agency relationships within Units.

Conclusions can also be drawn for welfare. A large market, offering enough diversification possibilities, reduces informational inefficiencies within Units. However, and perhaps more interestingly, some of the individuals who do not access markets will bear a cost. *All* Agents will see their utility pushed to the reservation level, even if they enjoyed some information rent in absence of markets, or in presence of a less developed market. In particular, those who will lose will be the "better" Agents. Going back to the applications discussed earlier, this last result means that introducing financial markets decreases the reward to a good worker, or to a safe loan applicant.

While markets naturally increase the welfare of Principals', by allowing them to adjust their portfolios, they also improve their ability to extract welfare from Agents.

While these are new results, the study of financial markets as means to share risk in relation to other types of risk sharing, and their effects on each other has been touched upon by the economic and financial literature in different ways.

I do not study the effects of asymmetric information *in* markets, but rather the effects that asymmetric information resolved elsewhere has *on* markets. This marks the first difference from the General Equilibrium works on insurance markets, starting from the seminal paper of

Rothschild and Stiglitz. Another important difference is that, in those papers, the fact that some individuals are risk neutral and act as firms is usually an assumption (An exception is Dubey and Geneakoplos, 2002), where individuals endogenously form pools to share risk. In this paper there are many risk averse investors, who access financial markets to trade away part of the risk they are exposed to. Traditionally, the assumption of risk neutrality of a principal is motivated by the existence of diversification opportunities. The present work also enquires when the usual motivation, the opportunity to trade risks on a financial market, really provides a justification for the risk neutrality assumption and its implications.

A strand of the finance literature looks at asset pricing in the presence of delegated portfolio management (for a survey, see Stracca, 2003, for a more recent example, see Ou Yang, 2005). These studies look at the effect on prices and returns of the classical informational asymmetry problems. There are studies on moral hazard and hidden type problems, studied in a CAPM or APT setting, in which a representative principal delegates his investing decisions to an agent. In this literature inefficiencies take the form of deviations from the non-delegated case equilibrium which take the form of changes in asset prices and optimal portfolio composition. Besides the different object of interest, the perspective in these works is in a sense opposite of the one taken here. There we have informed parties trading, whereas in the present work it is the uninformed parties who access markets.

This paper fits best in spirit with other works making less general assumptions to reach more specific conclusions. Legros and Newman (1996) analyze the internal organization of firms, in relation to the distribution of wealth in the economy and the imperfections of credit markets. Legros and Newman (2008) use markets to show how shocks to individual firms can cause restructuring in a sector. Similarly, in this paper the internal agreements in a firm impact those in other firms, but the channel is here that of financial markets. Gibbons, Holden, and Powell (2010) take firms formation as exogenous, as in the present work, but they focus on the interaction of market and non-market modes of information acquisition rather than risk sharing, as a determinant of firms' internal structure. Legros and Newman (2007) use endogenous matching as market mechanism to analyze efficiency within a pair and within an economy and apply their results to risk sharing problems.

Finally, the works by Magill and Quinzii (2005) and Parlour and Walden (2009), bear similarities to this one. They also take firm formation as an exogenous process, abstracting from labor market considerations, and they allow for contracts inside firms and financial markets across firms. However they use their model to study economies with hidden action. Magill and Quinzii study how available securities affect economic incentives, whereas Parlour and Walden consider effort in the moral hazard problem a human capital investment, and use their model to derive testable implications (for example on size effect) on the cross of section returns. I address the problem of moral hazard in a separate paper (Maretto, 2010), in which I focus on economic performance and volatility.

2 The Model

To capture the feedback between contracting and trading on financial markets, I construct a model where risk sharing obtains both through one-to-one contracting and through the exchange of assets on a competitive market.

2.1 Primitives

There are $2N$ individuals with identical expected utility preferences $U(X) = E[u(X)]$, with quadratic utility function $u(x) = x - \frac{b}{2}x^2$, where b is small enough that utility is increasing.

N individuals are Principals, and N are Agents. Principals are all identical. An Agent can be of type $t \in \{L, H\}$, which is his private information and influences the Agent's performance. Types are drawn from a commonly known distribution $\mathbf{F}$ on $\{L, H\}^N$.

There are N Units, each unit n is formed by a Principal P^n and an Agent A^n, whose type is $t(n)$. Unit n generates random binary returns $X(t(n))$, whose distribution is common knowledge and depends on the Agent's type. Any pattern of correlation between Units' is admissible.

Principals are entitled to the returns generated by the unit. Agents obtain a reservation utility of $\overline{u}$ if they do not participate in production, while Principals obtain zero.

2.2 Contracting

Within each Unit, sharing of surplus takes place through contracts. A contract $C^n = (\alpha^n, \beta^n)$ consists of a cash transfer α^n and a fraction of output β^n. I will call $\mathbf{C}$ a vector of N contracts $\{C^n\}_{n=1}^N$.

A contract leaves Principal P^n holding the random variable $\alpha^n + \beta^n X(t(n))$, and Agent A^n holding $-\alpha^n + (1 - \beta^n) X(t(n)))$.

Each Principal P_n offers to agent A_n a menu of contracts $M^n = [C_H^n, C_L^n] = [(\alpha_H^n, \beta_H^n), (\alpha_L^n, \beta_L^n)]$. A_n picks his preferred contract, or decides to not partecipate and obtain his reservation utility $\overline{u}$. Without loss of generality, we can restrict attention to Incentive Compatible menus, such that Agents of type H would pick C_t

$$E[u(-\alpha_t + (1 - \beta_t) X_t)] \geq E[u(-\alpha_s + (1 - \beta_s) X_t)], \forall s, t \in [L, H]$$

Let $\mathbf{M} = M^1, ...M^n$ be the menus of all Units. Let $c^n(M^n)$ be the contract C^n chosen by Agent A^n out of menu M^n. Denoting with $\mathbf{c} = [c_1(\cdot) \ldots c_N(\cdot)]$ the vector of Agents' choices, and with with , we have that $\mathbf{c}(\mathbf{M})$ is the vector of contracts chosen by agents facing menus $\mathbf{M}$.

2.3 Market

Principals can access a market for financial assets. Principals are endowed with their claims to profits $\alpha_{t(n)}^n + \beta_{t(n)}^n X(t(n))$. These claims are the assets available for trade at price q^n, together with a riskless asset available in zero net supply at price q^0. A portfolio of financial assets is denoted by $\boldsymbol{\theta} = (\theta_0|\theta_1, ..., \theta_N)$. θ_0 is the position an investor holds in the riskless asset. $\theta_1, ..., \theta_N$ are the holdings of securities of each of the N Units. The solution concept used here is the classic Arrow-Debreu competitive equilibrium. All Principals choose a porfolio $\boldsymbol{\theta}^n$ they can afford to maximize their expected utility, taking prices as given. Equilibrium prices will be such that all markets clear.

2.4 Timeline

The model plays out as follows

- **Time 0** Nature randomly draws the types of each Agent $\mathbf{t} = t(1) ...t(N)$ from distribution $\mathbf{F}$.
- **Time 1** Each Principal makes a take-it-or-leave-it offer to Agent he is matched with in the form of a menu of Incentive Compatible Contracts.
- **Time 2** Each Agent A^n chooses a contract from the menu he is offered, or his resservation utility.

- **Time 3** Principals trade on the asset market.

Finally, uncertainty is realized and contracts pay off.

When Principals design contracts at time 1, they are also designing their endowment for the market at time 3. Conversely, the asset market equilibrium is determined by risk sharing within units. These effects create the feedback between markets and contracts.

3 Equilibrium

Given the sequential nature of the model, it is useful to describe payoffs (and then the equilibrium concept) working backwards from the final stage.

Asset Market

The Principal's utility from a portfolio $\boldsymbol{\theta}$, a type realization $\mathbf{t}$, and contracts $\mathbf{C}$, is given by

$$U^3_{P^n}\left(\theta, \mathbf{C}, \mathbf{t}\right) = E\left[u\left(\theta_0 + \Sigma_{m=1}^N \theta_m \left[\alpha^m + \beta^m X\left(t\left(m\right)\right)\right]\right)\right]$$

Principals maximize their expected utility from their portfolio given prices, and markets clear. Since securities payoffs are determined by contracts $\mathbf{C}$ and by agents' skills $\mathbf{t}$, the equilibrium portfolios θ_* and prices $q_* = q^1_*, \ldots q^n_*$ will be a function $\left(\boldsymbol{\theta}_*, q_*\right)\left(\mathbf{C}, \mathbf{t}\right)$.

Contracting, Agents' turn

Agent A^n pick a contract out of the menu they are offered, maximizing their utility.

$$U^2_{A^n}\left(C^n, t\left(n\right)\right) = E\left[u\left(-\alpha^n + \left(1 - \beta^n\right) X\left(t\left(n\right)\right)\right)\right]$$

Contracting, Principals' turn

At time 1 the *expected* utility of a Principal holding portfolio θ, when menus are $\mathbf{M}$, Agents choose contracts $\mathbf{c}\left(\mathbf{M}\right)$ is

$$U^1_{P^n}\left(\mathbf{M}, \mathbf{c}\left(\cdot\right), \theta\right) = E_{\mathbf{t}}\left[U^3_{P^n}\left(\boldsymbol{\theta}, \mathbf{c}\left(\mathbf{M}\right), \mathbf{t}\right)\right]$$

Each principal offers a menu, without knowing the Agents' types. They can correctly foresee the strategies of each agent, and the outcome of asset markets, for any possible menu. In other words, they can forecast the equilibrium path for all possible offered menus $\mathbf{M}$ and realizations of types $\mathbf{t}$. Principals at this stage are playing a game against each other. Their mixed strategies are lotteries on menus.[3] For Principal P^n, $\tilde{M}^n \in \Delta\left(\mathcal{M}^n\right)$.

Based on this timeline we can write the utility in the first stage in this form:

$$V^n\left(\tilde{M}^n | \tilde{\mathbf{M}}^{-n}\right) = E_{\tilde{\mathbf{M}}}\left[U^1_{P^n}\left(\tilde{M}^n | \tilde{\mathbf{M}}^{-n}, \mathbf{c}\left(\tilde{M}^n | \tilde{\mathbf{M}}^{-n}, \tilde{\mathbf{t}}\right), \theta\left(\mathbf{c}\left(\tilde{M}^n | \tilde{\mathbf{M}}^{-n}, \tilde{\mathbf{t}}\right)\right)\right)\right] \quad (1)$$

[3] Allowing for mixed strategy is necessary, as the space of Incentive Compatible Menus is not convex. However, in the less general economy discussed in Section 4, there is a unique equilibrium in pure strategies. The proof is available upon request.

3.1 Definition

An Equilibrium consists of

- Portfolios $\boldsymbol{\theta}_*^n(\mathbf{C},\mathbf{t})$ for each Principal P_n and prices $\mathbf{q}_*(\mathbf{C},\mathbf{t})\in\mathbb{R}^{N+1}$ such that $[\theta_*,q_*](\mathbf{C},\mathbf{t})$ is an Arrow-Debreu Equilibrium for the symmetric information asset market taking place after contracting. Each principal is endowed with one unit of her asset so that the endowment of principal n is $w^n=[0,0,...,1,...,0,0]$ with 1 being in the nth position, and its value q_*^n.

$$\begin{aligned}
&\boldsymbol{\theta}_*^n(\mathbf{C},\mathbf{t})\in arg\max_{\boldsymbol{\theta}\in\mathbb{R}_+^{N+1}} U_{P^n}^3(\boldsymbol{\theta},\mathbf{C},\mathbf{t})\\
&s.t.\\
&\mathbf{q}_*(\mathbf{C},\mathbf{t})\cdot\boldsymbol{\theta}(\mathbf{C},\mathbf{t})\leq q_*^n(\mathbf{C},\mathbf{t})\\
&\sum_{n\in N}\boldsymbol{\theta}_{*0}^n=0\\
&\sum_{n\in N}\boldsymbol{\theta}_{*m}^n=1,\forall m=1,...,N
\end{aligned}$$

- For each agent A^n a strategy $c_*^n(M,t)$ such that

$$c_*^n(M_*^n,t(n))\in arg\max_{C\in\mathcal{C}^n} U_{A^n}^2(C,t(n))$$

- For each principal P^n, a lottery $\tilde{M}_*^n$ of menus such that

$$\text{supp}\left[\tilde{M}_*^n\right]\subseteq arg\max_{M^n\in\mathcal{M}} V^n(M^n|\mathbf{M}_*^{-n})$$

3.2 Existence

Theorem 1. *There is an equilibrium.*

The proof, in the appendix, follows the traditional pattern of using the maximum theorem to guarantee that individuals' best responses are well behaved enough to apply a fixed point theorem. There are two tricky steps in the process of applying the maximum theorem. For Principals' payoffs to be continuous in their own strategies (menus), it has then to be the case that (i) The contracts chosen by Agents change continuously with the menus offered and (ii) The Equilibrium correspondence of the asset market stage is a continuous function of the prevailing contracts. The first is achieved by noting that the economy has the same equilibria if we restrict Principals to offering Incentive Compatible Menus. The second point is trickier, as the Walrasian Equilibrium correspondence is in general neither continuous, nor single-valued.

However, the assumptions on preferences and the existence of a riskless asset assure that, for a given distribution of returns, the asset market equilibrium exhibits the properties of a Capital Asset Pricing Model (CAPM) equilibrium, which are

- **Portfolio Separation.** Every individual in the economy holds a fraction of the same portfolio of risky assets, the "Market Portfolio". In this case, all traders have the same preferences, and will hence be holding the *same* fraction of the Market Portfolio. Differences in initial wealth are accounted for by a short or long position in the riskless asset.

- **"Beta" Pricing.** The price of a financial asset is a linear function of its expected payoff and the sum of its variance plus its correlation with the market portfolio.

Because of these two properties, all final holdings are uniquely determined as a continuous function of endowments. As endowments are determined by contracts, and final holdings determine utilities, the maximum theorem can be applied.

4 Markets and Contracts with Diversifiable Risk

For the remainder of the paper, I will require type H to be "better" than type L. The Single Crossing Property provides a natural way of ranking individuals.

Definition 1. *An economy satisfies Single Crossing Property if and only if*

$$\frac{\frac{\partial E[u(-\alpha+(1-\beta)X_H)]}{\partial(1-\beta)}}{\frac{\partial E[u(-\alpha+(1-\beta)X_H)]}{\partial(-\alpha)}} > \frac{\frac{\partial E[u(-\alpha+(1-\beta)X_L)]}{\partial(1-\beta)}}{\frac{\partial E[u(-\alpha+(1-\beta)X_L)]}{\partial(-\alpha)}}, \forall\alpha,\beta$$

Assumption 1. *Single Crossing Property is satisfied.*

To substantiate the claim that SCP amounts to H being better than L, the following fact (proven in the appendix) shows that a few reasonable notions of "better" are encompassed by SCP.

Fact 1. *With quadratic utility the following cases imply SCP:*

1. *Both types generate the same mean returns but H does so with lower variance.* $\sigma_H^2 < \sigma_L^2$
2. *Both types generate the same variance of returns, but H provides higher mean returns.* $\mu_H > \mu_L$
3. *Both types can generate the same two outcomes, but H has a higher probability of the best outcome.*

4.1 Insurance

Having estabilished existence of equilibrium in the model, we can now turn to the relationship between financial markets and contracting. If Principals can diversify some of their risk through the asset markets, they will be in a position to take on more risk at the contracting stage. In this model, this amounts to Principals retaining a larger share of the Unit's return β. Conversely, Agents end up with less risk, when markets are present, even though they have no direct access to diversification by means of trading.

To guarantee enough diversification, for now I restrict attention to economies, in which Units' returns are not correlated.

Assumption 2. *Units' returns are stochastically independent.*
$\forall m, n \leq N$ $X(t(n))$ *and* $X(t(m))$ *are independently distributed.*

This implies that for a large number of Units risk can be entirely diversified away.

Let $M^n(N)$ be the equilibrium menu in unit n, in an economy with N Units. $M(1)$ is hence the optimal menu of contracts for an economy with a single Unit (this is also the optimal menu in any economy when Principals do not trade).

$$\mathbf{M}(N) = [M^n(N)]_{n=1}^N = [C_H^n(N) \;\; C_L^n(N)]_{n=1}^N = [(\alpha_H^n(N), \beta_H^n(N)) \;\; (\alpha_L^n(N), \beta_L^n(N))]_{n=1}^N$$

Lemma 1. *Under Assumptions 2, and 1,*

- $\beta_H^n(N) < 1$, $\beta_L^n(N) \leq 1$
- $\lim_{N\to\infty} \beta_t^n(N) = 1, \forall n, \forall t$
- $\exists \overline{N} s.t. \beta_L^n(N) = 1, \forall N \geq \overline{N}$

Lemma 1 establishes the convergence of any economy to full insurance for all types. As a consequence we can conclude that adding enough Units, and hence securities, creates diversification opportunities for Principals to "insure" Agents.

Theorem 2. *Under Assumptions 2 and 1, increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \overline{N} : \beta_t^n\left(N'\right) \geq \beta_t^n(N), \forall n, \forall t \in \{L, H\}, \forall N' \geq \overline{N}$$

This seemingly natural "Insurance Effect" is not a foregone conclusion. The previous theorem shows that Agents are always "insured" by markets in economies with a large number of units, with independent returns. In the appendix I extend this result in various directions and in the next discussion discuss the effects of dropping the assumption of independence, whose role is making sure that markets provide *enough* diversification opportunities to make Principals almost risk neutral in a large but finite economy. So that the complicated expression found in Equation 1 gets closer and closer to the expected value of the Menu offered by that particular Principal.

$$\lim_{N\to\infty} V^n\left(M^n|\tilde{\mathbf{M}}^{-n}\right) \to E_t\left[M^n\right]$$

Assumption 1 makes sure that the problem of a Principal is well behaved,[4] and that convergence of the utility function leads to convergence of the solution.

4.2 Diversification and Welfare

The previous result highlights how markets change equilibrium contracts. This change has implications for the efficiency of Units' and for welfare distribution.

The natural efficiency benchmark is to be found among economies where Agents' types are common knowledge. Let $\hat{M}(N)$ be the first best menu, arising as an equilibrium of such economies.[5]

$$\hat{M}(N) = \left[\hat{C}_H(N), \hat{C}_L(N)\right] = \left[\left(\hat{\alpha}_H(N), \hat{\beta}_H(N)\right), \left(\hat{\alpha}_L(N), \hat{\beta}_L(N)\right)\right]$$

The conclusions of Theorem 2 apply also to this symmetric information case.

Lemma 2. *Under Assumption 1*

- $\hat{\beta}_t^n(N) < 1$
- $\lim_{N\to\infty} \hat{\beta}_t^n(N) = 1, \forall n, \forall t$

Theorem 3. *Increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \overline{N} : \hat{\beta}_t^n\left(N'\right) \geq \hat{\beta}_t^n(N), \forall n, \forall t \in \{L, H\}, \forall N' \geq \overline{N}$$

[4] The proof uses techniques, similar to those of in Maskin and Riley (1984)

[5] Existence of an equilibrium in the symmetric information economy is guaranteed by identical arguments as those necessary to prove Theorem 1.

Both the first and second best economy converge to an identical equilibrium, in which both types are offered the same full insurance contract $(\overline{\alpha}, 1)$ with $\overline{\alpha}$ such that $u(-\overline{\alpha}) = \overline{u}$. However, there are some important differences.

Single Crossing Property impliesthat type H values a marginal increase of his fraction of the Unit $(1-\beta)$, more than type L does

Agents' obtain the same utility only for full insurance contracts, consisting only of a cash transfer $(\beta = 1)$. For any given contract with $\beta < 1$, H obtains more utility. C_L guarantees L a utility of $\overline{u}$. The positive difference $\Delta u = E[u(-\alpha_L + (1-\beta_L) X_H)] - \overline{u}$ is the "information rent" of type H. C_H has to guarantee type H at least $\overline{u} + \Delta u$, otherwise it will not be chosen over type C_L. Naturally this contract leaves the principal worse off than the first best contract $\hat{C}_H$, which pushed H to his reservation utility.

To avoid giving up too much utility when the Agent is of type H, the Principal distorts C_L to make it less palatable for type H, while yielding the same utility to type L. Distortion takes the form of overinsuring type L, by giving him more cash and less of the Unit's return.

However, distortion also reduces Principal's utility, when the Agent is of type L. The optimal contract strikes the right balance between two extremes. A contract accepted by both types, with minimum distortion for Agent L and maximum rent for Agent H, and a menu in which L is fully insured, so that H can be pushed to his reservation utility. We will see below that markets make these two extremes closer, allowing a more favorable tradeoff for the Principal.

The discussion above suggests that equilibrium contracts are in general different from the first best ones, and less efficient.

Proposition 1. *Asymmetric information Equilibria are weakly Pareto Dominated*

The proof amounts to solving a modified first best problem in which type H is guaranteed the same utility he obtains in an identical economy with asymmetric information $E[u(-\alpha_H + (1-\beta_H) X_H)] = \overline{u} + \Delta u$. The resulting contracts $\hat{\hat{M}} = \left(\hat{\hat{C}}_H, \hat{\hat{C}}_L\right)$ (weakly) Pareto dominate the equilibrium contracts.

The loss of efficiency is due to the distortion in the allocation of type L, who is insured more than he would be at the optimum.

Conversely Principals' take on more risk than they would without incentive compatibility considerations. Because this risk is then poured onto the market, we can conclude that asymmetric information within units generates excess aggregate risk on the security market.

Proposition 2. *Aggregate Risk on markets is higher than in the symmetric information Pareto Optimal economy.*

$$\sum_{1=1}^{N} \left(\beta_{t(n)}^{n}\right)^2 \sigma_{t(n)}^2 \geq \sum_{1=1}^{N} \left(\hat{\hat{\beta}}_{t(n)}^{n}\right)^2 \sigma_{t(n)}^2, \forall \mathbf{t} \in [L, H]^T$$

Having established the existence and nature of inefficiency, we can now assess the effects of the size of markets on this problem. Lemma 1 and 4 show that the information structure change the dynamics of convergence for the contract of type L. β_L reaches 1 (full insurance) at a finite number of units $\overline{N}$, whereas in the first best economy none of the two types is ever fully insured.

Using the distortion with respect to the Pareto dominating contracts $\Delta_t(N) = \beta_t(N) - \hat{\hat{\beta}}_t(N)$ as a measure of inefficiency,[6] the following result shows that financial markets have a beneficial effect on contracting within Units.

[6]This result would hold true if we choose to measure inefficiency with $\beta_t(N) - \hat{\beta}_t(N)$.

Proposition 3. *Increasing the number of units in an economy, eventually makes contracts more efficient.*

$$\forall N, \exists \overline{N} : \forall N' \geq \overline{N}, \Delta_t \left(N'\right) < \Delta_t \left(N\right)$$

This result relies on the convergence of all contracts to full insurance for agents of type H. and the monotonicity condition $\beta_H < \beta_L$ implied by Single Crossing Property. These together force the convergence of equilibrium contracts to the Pareto Optimal contracts $\hat{\hat{C}}$.

Diversification opportunities make Principals' behave as if they were less risk averse. As a result they can better exploit Agents' own risk aversion to screen them. It is hence possible for a Principal to achieve a more efficient outcome, by offering a menu closer to $\hat{\hat{M}}$. This efficiency need not coincide with a Pareto Improvement. In fact, while Principals are naturally made better off by the extra opportunities provided by markets, and Agents of type L see no change in their utility level ($\overline{u}$), Agents' of type H can be made worse off.

We have seen that H Agents enjoy a non negative rent Δu. The following result shows that this rent *is* zero for a large enough market.

Proposition 4. *With large enough markets, no agent enjoys an information rent.*

$$\exists \overline{N} : \forall N \geq \overline{N}, U\left(C_t\left(N\right), t\right) = \overline{u}, \forall t$$

To optimally screen types, a Principal faces a trade off between the loss of utility due to the distortion of the contract of type L, and the loss of utility in favor of type H (the information rent). In a large market where also first best contracts get approximately close to $(\overline{\alpha}, 1)$, the cost of distorting is very small, and the Principal will optimally offer a full insurance contract to type L ($\beta_L = 1$). This contract leaves both types at their reservation utility, so that the Incentive Compatibility constraint of type H is binding exactly where his Individual Rationality constraint binds, and no rents are possible in equilibrium.

The result is significant because someone is made worse off by financial markets, but also that it is the "best" among those who do not access markets (Agents), who pay the price of the benefits reaped by those who trade (Principals).

5 Markets and Contracts with Undiversifiable Risk

One of the key motives behind financial markets is the pooling and diversification of risk. Naturally this is a reason for trading only if risks are not perfectly correlated. It should not come as a surprise that a certain degree of independence and a certain number of Units is needed for Principals to be able to diversify away enough risk to act as if they were less risk averse. What is less expected is that enough positive correlation can generate the "opposite" effect to that of Theorem 1, as the following example shows.

Considering an economy with two Units, and the following parameters.

$\left(\mu_H, \sigma_H^2\right)$	$\left(\mu_L, \sigma_L^2\right)$	r	$\overline{u}$	ρ
$(3, 0.1)$	$(3, 1)$	0.1	0.3	0.9

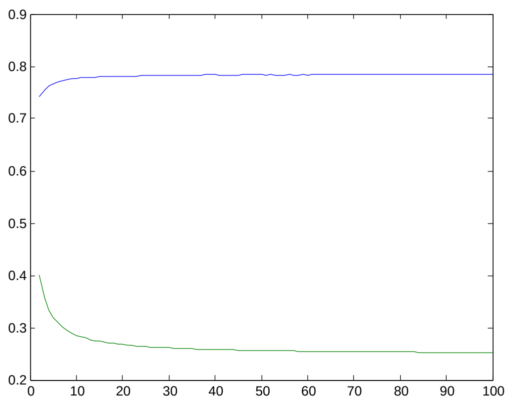
The equilibrium contracts without markets will be

$$-\alpha_H + (1 - \beta_H) X = -1.4068 + 0.5711X$$
$$-\alpha_L + (1 - \beta_L) X = -1.3866 + 0.5693X$$

Those with markets will be

$$-\alpha_H^M + \left(1 - \beta_H^M\right) X = -1.4451 + \mathit{0.5838}X$$
$$-\alpha_L^M + \left(1 - \beta_L^M\right) X = -1.1569 + 0.4913X$$

This is not a violation of Theorem 1, which is a limiting statement. So one would hope that adding enough units is enough to restore the expected insurance effect, by bringing about more diversification. However, it need not be the case. The graph shows in blue $1 - \beta_L$, and in red $1 - \beta_H$ as functions of N. The sensitivity to returns and hence the variance of the contract of Agent H actually increases with the size of markets.



Why does increasing markets make the variance of contract H higher? The existence of non diversifiable risk, modeled as positive correlation ρ between Units returns, creates a strong incentive to issue securities which get as close as possible to hedging returns when the economy does bad.

To get an intuition, it is useful to think in terms of underlying states. Units's returns are binary $X_H \in \{\underline{x}_H, \overline{x}_H\}$ and $X_L \in \{\underline{x}_L, \overline{x}_L\}$, with $\mu_H = \mu_L$ and $\sigma_H^2 < \sigma_L^2$, so that $\underline{x}_L < \underline{x}_H < \overline{x}_H < \overline{x}_L$. Suppose ρ is equal to one, so that a space $S = \{\underline{s}, \overline{s}\}$ with 2 states is sufficient to represent the economy.

$$X_t\left(\underline{s}\right) = \underline{x}_t$$
$$X_t\left(\overline{s}\right) = \overline{x}_t$$

Whenever an L unit returns $\underline{x}_L$ all other L units will do the same, and H units will return $\underline{x}_H$.

In absence of markets both Principals and Agents have the same risk attitude and the optimal contracts will strike some balance of risk sharing. Now suppose trading is possible. When Principals designs C_H she knows, that when she is matched with H she will then trade with some L and some H units, and end up holding a portfolio of these. She also knows, as an H unit, she has an advantage in providing returns in state $\underline{s}$, since $\underline{x}_L < \underline{x}_H$. Units H can hedge returns in $\underline{s}$, The markets provides them with the incentives to do so, by making returns in that state particularly valuable. On the other side, the cost she bears by giving less returns to Agent in $\underline{s}$, in the form of a higher $1-\beta_H$ and a lower $-\alpha_H$ is not changed by markets. The optimal contract will reflect this and put more risk in the hands of type H Agents.

The extent of this effect hence depends on the difference between $\underline{x}_L < \underline{x}_H$ and of course on ρ. We can however conclude that for small enough levels of correlation the insurance effect still holds. Under the assumption of independence (Theorem 1) we saw how many Principals acted as if they were risk neutral as N got large. In general this need not be true, because correlation across Units generates undiversifiable risk. What is always true is that in a large economy, Principals' behave as if they were maximizing the price of their Unit q.

$$\lim_{N\to\infty} V^n\left(M^n|\tilde{\mathbf{M}}^{-n}\right) \to E_{\mathbf{t}}\left[q^n\right]$$

If returns are independent, it is the case that the price of an asset coincides with its expected value. The next result puts a bound on correlation to control the difference $|E_{\mathbf{t}}\left[q^n\right] - E_t\left[M^N\right]|$ and reach the same conclusion as Theorem 1.

Proposition 5. *Suppose Assumption 1 holds, then there is a $\overline{\rho}$, such that, if $\rho_{st}^{mn} < \overline{\rho}$ for all m, n, s, t, increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \overline{N} : \beta_t^n\left(N'\right) \geq \beta_t^n\left(N\right), \forall n, \forall t \in \{L, H\}, \forall N' \geq \overline{N}$$

Furthermore, $\exists \overline{N} s.t. \beta_L^n\left(N\right) = 1, \forall N \geq \overline{N}$

All results from Section 4.2 apply. In particular, the last point implies that Proposition 9 carries through as well.

6 Conclusions

This paper tackles the question of how financial markets interacts with other forms of risk sharing. In particular risk sharing problems with asymmetric information problems which are dealt with outside the markets. To do so I integrate a model of principal-agent interaction with hidden type with asset markets. A Unit is formed by a Principal and an Agent. Each pair produces random returns, whose distribution is known only to the agent at the contracting stage. What marks the difference from the standard contracting model is that Principals have access to an asset market on which they trade their shares of returns, and a riskless asset.

I present a general framework and define a notion of equilibrium, for which I prove existence.Under standard assumptions of contract theory, I study the interactions of financial markets on contracts. The existence of markets, can have an *Insurance Effect*, inducing less risky compensation for agents. However I show how this seemingly natural effect need not obtain, and how high level of undiversifiable risk, might induce the opposite effect for some types of Agents.

For contracts to exhibit less variance, two ingredients must be present in the economy. A large number of traders, and little systemic risk. As soon as one of these assumptions is dropped, counterexamples can be constructed.

In the case of asymmetric information, one must also add a generalization of the familiar condition of Single Crossing Property to the case of many Units.

Equilibria are inefficient, and it will be the case that contracting inside Units induces excessive aggregate risk in markets. The size of inefficiency is small when markets are large. However, Agents might bear a cost with the introduction of markets. Large markets will of course increase the utility of Principals, who access them, but will push all Agents, including those who enjoyed an information rent when Units exist in isolation or when markets are small. Introducing markets in some economies, does not result in a Pareto Improvement, and it is the "better" Agents who will be worse off.

Two potential extensions seem particularly relevant. One is the study of the effects of non diversifiable risk besides the example presented. A more challenging one is studying the interactions between labor market and financial markets in a similar framework. The negative welfare effects in this paper indicate that this is a direction worth pursuing.

7 Appendix

7.1 The General Model

The model of Section 2 can be extended to a larger type space $\mathcal{T}$ can be of arbitrary finite size T, rather than two, and to the case of heterogenous Principals.

To accomodate for the added generality, we need some extra notation. As Units are now different, I will denote the returns and contracts of Unit n when Agent is of type t by X_t^n and $C_t^n = (\alpha_t^n, \beta_t^n)$.

Requiring the same ranking of types in all Units would be both unnecessary and unrealistic, so I adapt Single Crossing Property to get around this undesirable restriction.

Let $\tau(n)(t) \in \{1, .., T\}$ be the position occupied by type t in a permutation of $\mathcal{T}$, $\tau(n)$.

Definition 2. *An economy satisfies Single Crossing Property in Every Unit ($SCP2$) if and only if there is a function τ, as defined above such that*

$$\begin{aligned}
&\forall t : \tau(n)(t) \leq T - 1, \forall n, \\
&\frac{\frac{\partial E\left[u\left(-\alpha+(1-\beta)X^n_{\tau(n)(t)}\right)\right]}{\partial(1-\beta)}}{\frac{\partial E\left[u\left(-\alpha+(1-\beta)X^n_{\tau(n)(t)}\right)\right]}{\partial(-\alpha)}} > \frac{\frac{\partial E\left[u\left(-\alpha+(1-\beta)X^n_{\tau(n)(t)+1}\right)\right]}{\partial(1-\beta)}}{\frac{\partial E\left[u\left(-\alpha+(1-\beta)X^n_{\tau(n)(t)+1}\right)\right]}{\partial(-\alpha)}}, \\
&\forall \alpha, \beta
\end{aligned}$$

Assumption 3. *Single Crossing Property in Every Unit ($SCP2$) is satisfied.*

Finally, we can turn to the distribution of returns. Since Units' returns are binary, a single coefficient $\rho_{s,t}^{m,n} \in [-1, 1]$ describes all covariation between Units m and n with agents of type $t(m) = s$ and $t(n) = t$. Covariance is

$$Cov\left(X_s^m, X_t^n\right) = \rho_{s,t}^{m,n} \sigma_s^m \sigma_t^n$$

7.1.1 Some implications of Single Crossing Property

Fact 2. *In the case of quadratic utility, SCP amounts to*

$$\mu_t - \frac{b\sigma_t^2 (1-\beta)}{1 - b(-\alpha) - b\mu_t (1-\beta)} > \mu_{t+1} - \frac{b\sigma_{t+1}^2 (1-\beta)}{1 - b(-\alpha) - b\mu_{t+1} (1-\beta)}$$

Proof. The claim follows by differentiating the quadratic utility functions of Agents, and taking the ratios. $\square$

The following two implications of SCP will be useful in the proof Theorem 2

Fact 3. $\forall t, (\alpha, \beta), U(-\alpha, 1-\beta|t) > U(-\alpha, 1-\beta|t+1)$

Fact 4. $\forall t < s, \forall (\alpha, \beta), (\alpha', \beta') : \beta \leq \beta'$

$$U(-\alpha, (1-\beta)|\, t) - U\left(-\alpha', \left(1-\beta'\right)\middle|\, t\right) > U(-\alpha, (1-\beta)|\, s) - U\left(-\alpha', \left(1-\beta'\right)\middle|\, s\right)$$

Fact 5. *SCP implies that $\mu_t \geq \mu_{t+1}$*

Proof. The claim follows immediately by setting $\beta = 1$ in the expression of SCP for quadratic utility . □

This is the proof of Fact 1

Proof.

1. $\mu_t = \mu_{t+1}, \sigma_t^2 < \sigma_{t+1}^2$. As all identical parts cancel from the condition above, we are left with $-\sigma_t^2 > -\sigma_{t+1}^2$. Multiplying both sides by -1 concludes the proof of 1.
2. $\mu_t > \mu_{t+1}, \sigma_t^2 = \sigma_{t+1}^2$.
   U_2, the derivative with respect to $(1-\beta)$ is given by
   $\mu + b\alpha\mu - b(1-\beta)\mu^2 - b(1-\beta)\sigma^2$
   we want to show that $U_2(\cdot|t) > U_2(\cdot|t+1)$, that is
   $\mu_t + b\alpha\mu_t - b(1-\beta)\mu_t^2 - b(1-\beta)\sigma^2 > \mu_{t+1} + b\alpha\mu_{t+1} - b(1-\beta)\mu_{t+1}^2 - b(1-\beta)\sigma^2$
   $\mu_t(1 + b\alpha - b(1-\beta)\mu_t) > \mu_{t+1}(1 + b\alpha - b(1-\beta)\mu_{t+1})$
   We have left to show that $U_1(\cdot|t+1) > U_1(\cdot|t)$. To see this note how U_1 amounts to $1 + b\alpha - b(1-\beta)\mu$.
   Inspection shows that if $\mu_t > \mu_{t+1}$, we have that $U_1(\cdot|t+1) > U_1(\cdot|t)$, which concludes the proof of 2.
3. Without loss of generality suppose that the outcome can be either 0 or 1, and let p_t be the probability of success of an agent of type t with, and consider $p_t > p_{t+1}$. The returns from employing agent t , will have mean p_t and variance $p_t(1-p_t)$. Again, let's start by proving that $U_2(\cdot|t) > U_2(\cdot|t+1)$. That is
   $p_t + b\alpha p_t - b(1-\beta)p_t^2 - b(1-\beta)p_t(1-p_t) > p_{t+1} + b\alpha p_{t+1} - b(1-\beta)p_{t+1}^2 - b(1-\beta)p_{t+1}(1-p_{t+1})$
   which readily simplifies to
   $p_t + b\alpha p_t - b(1-\beta)p_t > p_{t+1} + b\alpha p_{t+1} - b(1-\beta)p_{t+1}$
   and finally $(p_t - p_{t+1})(1 + b\alpha - b(1-\beta)) > 0$.
   This has to be true, as $p_t > p_{t+1}$ by assumption and the second term in brackets cannot be negative, otherwise individual utility $x - \frac{b}{2}x^2$ would be decreasing for $x = 1$, which violates the assumption of monotonicity of preferences.
   The proof that $U_1(\cdot|t) < U_1(\cdot|t+1)$ is identical to the one of case 2, since $\mu = p$ and $p_t > p_{t+1}$.

□

7.2 Mean and Variance

As previously noted, the assumptions on preferences and availability of a riskless asset, pay off in terms of tractability. First let's note that quadratic expected utility preferences on random varaiables can be equivalently represented as preferences over mean and variance of random variables.

$$\begin{aligned} E[u(X)] &= \\ E[X] - \frac{b}{2}E\left[X^2\right] &= \\ E[X] - \frac{b}{2}E[X]^2 - \frac{b}{2}Var[X] \end{aligned}$$

I will call the Mean-Variance representation $F\left(\mu_X, \sigma_X^2\right)$, where the first argument is the mean of X and the second argument its variance.

Linear contracts also allow a handy representation in terms of Mean and Variance.

$$\mu_{\alpha+\beta_X} = \alpha + \beta\mu_X$$
$$\sigma^2_{\alpha+\beta_X} = \beta^2\sigma_X^2$$

7.3 CAPM

To study the problem, we need to incorporate the outcome of markets in the Principals' objective functions. The final holdings are in equilibrium can be expressed analytically, because our assumptions imply the existence of a CAPM equilibrium in the asset market.[7]

Every individual will hold the same risky portfolio, an equal fraction $\frac{1}{N}$ of the aggregate endowment, and will spend the rest on the riskless asset (or short it if their remaining endowment is negative). With this in mind the mean and variance of the portfolio held by the agent is readily computed as a function of contracts. For a general principal n we have that

- The holding of riskless asset is $q^n - \frac{1}{N}\sum_{m=1}^N q^m$
- The mean of the risky portfolio is $\frac{1}{N}\sum_{m=1}^N \left(\alpha^m + \beta^m\mu^m\right)$
- The variance of the risky portfolio is $\frac{1}{N^2}\sum_{m=1}^N \left(\beta_m^2\sigma^{m2} + \sum_{k\neq m}\rho^{mk}\beta^m\beta^k\sigma^m\sigma^k\right)$

Since $q^n = \alpha^n + \beta^n\mu^n - \frac{b}{N}\beta^{n2}\sigma^{n2}\sum_{m\neq n}\rho^{mn}\beta^m\beta^n\sigma^m\sigma^n$, we have that the mean of P^n's holdings simplifies to

$$\alpha^n + \beta^n\mu^n - \frac{b}{N}\left(\beta^{n2}\sigma^{n2} + \sum_{m\neq n}\rho^{mn}\beta^n\beta^m\sigma^n\sigma^m\right) + \frac{b}{N^2}\sum_{m=1}^N\left(\beta^{m2}\sigma^{m2} + \sum_{k\neq m}\rho^{mk}\beta^m\beta^k\sigma^m\sigma^k\right)$$

and the variance is of course the variance of the risky part $\frac{1}{N^2}\sum_{m=1}^N \left(\beta_m^2\sigma^{m2} + \sum_{k\neq m}\rho^{mk}\beta^m\beta^k\sigma^m\sigma^k\right)$

7.4 The Objective Function

If $V\left(\alpha^n, \beta^n\right) = F\left(\alpha^n + \beta^n\mu^n, \beta^{n2}\sigma^{n2}\right)$ is the utility a Principal obtains from contract α, β when no markets are available, markets will change this into

$$\begin{aligned}
&V_M\left(\alpha^n, \beta^n\right) = \\
&F^M\left(\alpha^n + \beta^n\mu^n, \beta^{n2}\sigma^{n2}\right) = \\
&F\left(\alpha^n + \beta^n\mu^n - \frac{b}{N}\left(\beta^{n2}\sigma^{n2} + \sum_{m\neq n}\rho^{nm}\beta^n\beta^m\sigma^n\sigma^m\right) + \frac{b}{N^2}\sum_{m\in N}\left(\beta^{m2}\sigma^{m2} + \sum_{k\neq m}\rho^{mk}\beta^m\beta^k\sigma^m\sigma^k\right)\right. \\
&\left., \frac{1}{N^2}\sum_{m\in N}\left(\beta^{m2}\sigma^{m2} + \sum_{k\neq m}\rho^{mk}\beta^m\beta^k\sigma^m\sigma^k\right)\right)
\end{aligned}$$

It is useful for the following proof to explicitly write the partial derivatives of V_M with respect

[7] See the proof of Theorem 3

$$\frac{\partial V_M}{\partial \alpha^n}\left(\alpha^n,\beta^n\right)=F_\mu\left(\cdot,\cdot\right)$$

$$\frac{\partial V_M}{\partial \beta^n}\left(\alpha^n,\beta^n\right)=F_{\mu^n}\left(\cdot,\cdot\right)\left[\mu^n-\frac{b}{N}\left(2\sigma^{n2}\beta^n+\sum_{m\neq n}\rho^{nm}\sigma^n\sigma^m\beta^m\right)+\frac{b}{N^2}\left(2\sigma^{n2}\beta^n+2\sum_{m\neq n}\rho^{nm}\sigma^n\sigma^m\beta^m\right)\right]$$
$$+F_{\sigma^2}\left(\cdot,\cdot\right)\left[\frac{1}{N^2}\left(2\sigma^{n2}\beta^n+2\sum_{m\neq n}\rho^{nm}\sigma^n\sigma^m\beta^m\right)\right]$$

When principals are allowed to trade their claims, they will act at the first stage, as if their utility functions were

$$\begin{aligned}
E_{\mathbf{t}}V_M\left(\alpha_t^n\beta_t^n\right)&=\\
E_{\mathbf{t}}F^M\left(\alpha_t^n+\beta_t^n\mu_t^n,\beta^{n2}\sigma^{n2}\right)&=\\
E_{\mathbf{t}}F\left(\alpha_t^n+\beta_t^n\mu_t^n-\frac{b}{N}\left(\beta_t^{n2}\sigma_t^{n2}+\sum_{m\neq n}\rho^{nm}_{tt(m)}\beta_t^n\beta^m_{t(m)}\sigma_t^n\sigma^m_{(m)}\right)\right.&\\
+\frac{b}{N^2}\sum_{m\in N}\left(\beta^{m2}_{t(m)}\sigma^{m2}_{t(m)}+\sum_{k\neq m}\rho^{mk}_{t(k)t(m)}\beta^m_{t(m)}\beta^k_{t(k)}\sigma^m_{t(m)}\sigma^k_{t(k)}\right)&\\
\left.,\frac{1}{N^2}\sum_{m\in N}\left(\beta^{m2}_{t(m)}\sigma^{m2}_{t(m)}+\sum_{k\neq m}\rho^{mk}_{t(m)t(k)}\beta^m_{t(m)}\beta^k_{t(k)}\sigma^m_{t(m)}\sigma^k_{t(k)}\right)\right)&
\end{aligned}$$

7.5 Existence in the General Model

Like in the two-type symmetric economy, monotonicity of preferences is enough for existence of an equilibrium. However, allowing for an arbitrary type distribution, heterogenous Units' returns, and correlation between Units, does not allow to rule out mixed equilibria.

Theorem 4. *There is an equilibrium.*

Proof. I am going to use a well known fixed point result by Glicksberg (1952) to show that there is an equilibrium in the first stage of the game, given that the asset market develops as predicted by the CAPM model.

I need to show that

1. The strategy space of each Principal $\Delta(\mathcal{M}^n)$ is a convex, compact subset of a locally convex Hausdorff space.
2. The best response correspondence of all principals is upper hemi-continuous, convex valued, and nonempty.

For the first part note that the space of Incentive Compatible menus $\mathcal{M}^n$ is a subset of a Euclidean space. It is closed because it is defined by a finite number of weak inequalities, and it is bounded because it is included in the larger set of feasible contracts, which is also bounded. Hence it is compact.

The space of lotteries (identified with Borel probability measures) over these Menus is of course convex. It is also compact with respect to the weak* topology. This space of probabilities is a subset of the space of continuous functions $\mathcal{C}(\mathcal{M}^n)$, which is locally convex (and Hausdorff) with respect to the weak* topology.[8]

For the second part, convexity of the best response correspondence follows from preferences on random variables being represented by expected utility. I will use Berge's Maximum theorem to show that it is non empty, compact-valued and upper hemi-continous.

To apply the maximum theorem to individuals' best response, it has to be that constraints vary continuously with other principals' strategies, and that the payoff function is continuous in one's own actions.

First note how the constraints correspondence is constant with respect to other principals strategies, and is therefore continuous. Also note how the constraints correspondence maps to the space of Borel probability measures on menus, which is a Hausdorff space as noted above.

We also need to make sure that the payoff function of a principal is continuous in menus. To do this we need to show that

1. Payoffs at the market stage are a continuous function of the contracts chosen by agents.
2. The contracts chosen by agents are a continuous function of the menus offered.

Claim 1 By Lemma 4, if the preferences are monotonic for (μ, Ω), they are going to be monotonic for the asset markets resulting from all possible contracts C. Under the present assumptions a CAPM equilibrium exists once contracts are chosen.[9] Because in equilibrium the price of a security can be expressed as $q^n = \alpha^n + \beta^n \mu^n - \frac{b}{N}\left(\beta^{n2}\sigma^{n2} + \Sigma_{m \neq n}\rho^{mn}\beta^m\beta^n\sigma^m\sigma^n\right)$ The indirect utility from a contract profile in the CAPM function is continuous in contracts.

Claim 2 Without loss of generality, we can restrict attention to Incentive Compatible menus, from the set $\mathcal{M}^n_{IC}$. If a principal makes a small change to the menu he offers while remaining in this set, every type of agent t , will still find it optimal to pick the contract intended for him, C_t. Hence any small change, will correspond to a small change in the contract picked by each type of agent.

We can conclude that the indirect utility for a principal facing type t is a continuous function of the menus offered.

Taking expectation with respect to $\mathbf{F}$ over these indirect utilities yields a continuous functional on the domain of lotteries on IC menus $\Delta(\mathcal{M}^n_{IC})$.

By the maximum theorem the best response correspondence of each player is now UHC and compact valued, which implies that the game best response is as well.

By Glicksberg's theorem there is a fixed point, which is an equilibrium by construction. □

7.6 Insurance and Welfare

The results of Section 4 extend to this setting provided the economy satisfies $SCP2$ and the level of correlation is limited.

Lemma 3. *Under Assumption 3, there is a $\overline{\rho}$, such that, if $\rho < \overline{\rho}$*

- $\beta^n_1(N) < 1$, $\beta^n_t(N) \leq 1, \forall t \in \{2, ..., T\}$
- $\lim_{N\to\infty} \beta^n_t(N) = 1, \forall n, \forall t$
- $\exists \overline{N} s.t.\ \ \beta^n_t(N) = 1, \forall t \in \{2, ..., T\}, \forall N \geq \overline{N}$

[8]For a treatment of these and other results on weak topologies, and also to see the theorems of Berge and Glicksberg, see Aliprantis, Border (2005)

[9]In the literature briefly reviewed by Nielsen (1990), one can find many sufficient conditions for the existence of a CAPM equilibrium, most of them deal with the possibility of satiation of preferences. Things are particularly simple when returns are bounded (which includes this model): monotonicity and local non satiation are guaranteed by a low enough risk aversion.

Proof.

$$\max_{(\alpha_t,\beta_t)_{t=1}^T} E_{\mathbf{t}} V^M(\alpha_t, \beta_t|t)$$

$$s.t.\ \ U(-\alpha_t, 1-\beta_t|t) \geq \overline{u},\ \ \forall t \in \{1, ...T\} \tag{IR t}$$

$$U(-\alpha_t, 1-\beta_t|t) \geq U(-\alpha_{t'}, 1-\beta_{t'}|t),\ \ \forall t, t' \in \{1, ..., T\} \tag{IC t t'}$$

SCP and IC constraints imply by usual arguments that yield that

$$(1-\beta_s^*) \geq (1-\beta_t^*), \forall s < t \in \{1, ...T\} \tag{2}$$

Note how this implies that $\beta_1 \leq \beta_t, \forall t$

Now we have to solve for the contract of type 1. To do this I show that the IC_{12} constraint will always be binding, and that this is enough to attain the desired result.

The first thing to do is to reduce the set of relevant constraints.

Fact 3 implies that

$$U(-\alpha_t, 1-\beta_t|t) \geq U(-\alpha_t, 1-\beta_t|T)$$

This together with IR holding for type T and IC holding for type t with respect to the contracts of type T, implies that IR holds for type t.

In other words, if

$$\begin{aligned} U(-\alpha_T, 1-\beta_T|T) &\geq \overline{u} \\ U(-\alpha_t, 1-\beta_t|t) &\geq U(-\alpha_T, 1-\beta_T|t) \end{aligned}$$

we will also have that

$$U(-\alpha_t, 1-\beta_t|t) \geq \overline{u}$$

We can hence solve the problem without worrying about any of the IR constraints except that of type T .

We can also infer that $IC_{t-1,t}$ will be binding at an optimum for any t. Suppose that it were not binding,

$$U(-\alpha_{t-1}, 1-\beta_{t-1}|t-1) > U(-\alpha_t, 1-\beta_t|t-1)$$

Since we are at an optimum it has to be that the IC constraints are satisfied

$$U(-\alpha_t, 1-\beta_t|t) \geq U(-\alpha_k, 1-\beta_k|t), \forall k$$

By Fact 4 it has to be that

$$U(-\alpha_t, 1-\beta_t|s) > U(-\alpha_k, 1-\beta_k|s), \forall k \geq t, \forall s < t$$

Consider an alternative incentive scheme $\{\alpha_t', \beta_t'\}_{t=1}^T$, which gives a smaller fixed payment less transfer $-\alpha_s' < \alpha_s$ to all types s lower t. Because their IC constraints for contracts C_k, with $k > t$ ($IC_{s,k}$) are not binding, we are increasing the maximand while remaining in the admissible set of contracts, which contradicts the original scheme $\{\alpha_t, \beta_t\}_{t=1}^T$ being an optimum.

This and SCP imply that constraints $IC_{t,t-1}$ will not be binding.

It also implies that no other IC constraint will bind at the optimum. Fact 3, $IC_{t-1,t}$, and $IC_{t,t+1}$ imply that $IC_{t-1,t+1}$ is satisfied with a strict inequality.

This means that the only relevant constraints for determining the optimal β_1 are in the form

$$U(-\alpha_1, 1-\beta_1|1) = U(-\alpha_2, 1-\beta_2|1)$$

I now have to solve a simpler problem

$$\begin{aligned}
&\max_{(\alpha_t,\beta_t)_{t=1}^T} E_{\mathbf{t}} V^M(\alpha_t, \beta_t|t) \\
&\quad s.t.\ U(-\alpha_T, 1-\beta_T|t) = \overline{u} && \text{(IR T)} \\
&\qquad\quad U(-\alpha_t, 1-\beta_t|t) = U(-\alpha_{t+1}, 1-\beta_{t+1}|t), \quad \forall t \in \{1, ..., T-1\} && \text{(IC t, t+1)}
\end{aligned}$$

The only first order conditions involving α_1 and β_1 are given by

$$\mathbf{F}(\mathbf{t}|t(n)=1) V_\alpha - \lambda_1 U_{-\alpha} = 0$$
$$\mathbf{F}(\mathbf{t}|t(n)=1) V_\beta - \lambda_1 U_{(1-\beta)} = 0$$

Dividing both sides of each equation by $\mathbf{F}(\mathbf{t}|t(n)=1)$, and solving for $\frac{\lambda_1}{\mathbf{F}(\mathbf{t}|t(n)=1)}$ (where λ_1 is the Lagrange Multiplier associated with IC_{12} yields

$$\frac{V_\alpha}{U_{-\alpha}} = \frac{V_\beta}{U_{(1-\beta)}}$$

$$\frac{E_{\mathbf{t}|t(n)=1}\left\{F_\mu^M\left(\alpha_1 + \beta_1\mu_1, \beta_1^2\sigma_1^2\right)\right\}}{F_\mu\left(-\alpha_1 + (1-\beta_1)\mu_1, (1-\beta_1)^2\sigma_1^2\right)} =$$
$$\frac{E_{\mathbf{t}|t(n)=1}\left\{(\mu_1 - bR(N)) F_\mu^M\left(\alpha_1 + \beta_1\mu_1, \beta_1^2\sigma_1^2\right) + bS(N) F_{\sigma^2}^M\right\}}{\mu_1 F_\mu\left(-\alpha_1 + (1-\beta_1)\mu_1, (1-\beta_1)^2\sigma_1^2\right) - b(1-\beta_1)\sigma_1^2}$$

where

$$R(N) = \frac{1}{N}\left(2\sigma^{n2}\beta^n + \sum_{m\neq n}\rho^{nm}\sigma^n\sigma^m\beta^m\right) - \frac{1}{N^2}\left(2\sigma^{n2}\beta^n + 2\sum_{m\neq n}\rho^{nm}\sigma^n\sigma^m\beta^m\right)$$
$$S(N) = \frac{1}{N^2}\left(2\sigma^{n2}\beta^n + 2\sum_{m\neq n}\rho^{nm}\sigma^n\sigma^m\beta^m\right)$$

Solving above for β_1 we have

$$\beta_1(N) = \Big[2\sigma_1^2 F_{\sigma^2} - F_\mu\left(-\alpha_1 + (1-\beta_1)\mu_1, (1-\beta_1)^2\sigma_1^2\right) F_{\sigma^2}^M bR(N) +$$
$$F_\mu\left(-\alpha_1 + (1-\beta_1)\mu_1, (1-\beta_1)^2\sigma_1^2\right) F_\mu^M\left(\alpha_1 + \beta_1\mu_1, \beta_1^2\sigma_1^2\right) S(N)\Big] /$$
$$2\sigma_1^2 F_{\sigma^2} F_\mu^M\left(\alpha_1 + \beta_1\mu_1, \beta_1^2\sigma_1^2\right)$$

As N gets large, this converges to

$$1 - \lim_{N\to\infty} R(N) F_\mu\left(-\alpha_1 + (1-\beta_1)\mu_1, (1-\beta_1)^2\sigma_1^2\right)$$

Letting $\rho_1 \equiv \max_{m,n,s,t,}\rho_{st}^{mn}$, and observing that F_μ is always strictly smaller than one, we can solve the following inequality to find a bound for ρ_1.

$$\beta_1(M) < 1 - \rho_1\sigma_1 \lim \sum_{m\neq 1} \frac{\sigma_m}{N}$$

Which is satisfied whenever $\rho_1 < \frac{1-\beta_1(N)}{\sigma_1 \lim \sum_{m\neq 1} \frac{\sigma_m}{N}} = \overline{\rho}_1$. Whenever all correlations are bounded above, we have the desired result for type 1.

To conclude that *every* contract is less risky, I need to rule out the case in which some β_t^M goes from 1 to some number in $\left(\beta_1^M, 1\right)$, and then converges to 1, without hitting 1 in a finite time).

Consider the simplified optimization problem with markets described above. Consider the contract of some type t greater than 1. Let λ_t be the Lagrange multiplier of the downward Incentive Compatibility constraint of type t, $IC_{t,t+1}$. That is that type t prefers the contract designed for him over that designed for type $t+1$. Let λ_{t-1} be the Lagrange multiplier associated with the downward Incentive Compatibility constraint of type $t-1$, $IC_{t-1,t}$. The first order conditions determining the contract of type t .

$$E_{\mathbf{t}|t(n)=t}\left\{F(t) F_\mu^M\left(\alpha_t + \beta_t\mu_t, \beta_t^2\sigma_t^2\right)\right\} +$$
$$-\lambda_t F_\mu\left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right) +$$
$$+\lambda_{t-1} F_\mu\left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2\sigma_{t-1}^2\right) = 0$$
$$E_{\mathbf{t}|t(n)=t}\left\{F(t)\left[(\mu_t - R(N)) F_\mu^M\left(\alpha_t + \beta_t\mu_t, \beta_t^2\sigma_t^2\right) + S(N) F_{\sigma^2}^M\right]\right\}$$
$$-\lambda_t\left[\mu_t F_\mu\left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right) - b(1-\beta_t)\sigma_t^2\right]$$
$$+\lambda_{t-1}\left[\mu_{t-1} F_\mu\left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2\sigma_{t-1}^2\right) - b(1-\beta_t)\sigma_{t-1}^2\right] = 0$$

Solving for λ_t we obtain

$$E_{\mathbf{t}|t(n)=t}\left\{F(t) F_\mu^M\left(\alpha_t + \beta_t\mu_t, \beta_t^2\sigma_t^2\right)\right\}$$
$$+\lambda_{t-1} F_\mu\left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2\sigma_{t-1}^2\right) /$$
$$F_\mu\left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right) =$$
$$E_{\mathbf{t}|t(n)=t}\left\{F(t)\left[(\mu_t - bR(N)) F_\mu^M\left(\alpha_t + \beta_t\mu_t, \beta_t^2\sigma_t^2\right) + S(N) F_{\sigma^2}^M\right]\right\}$$
$$+\lambda_{t-1}\left[\mu_{t-1} F_\mu\left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2\sigma_{t-1}^2\right) - b(1-\beta_t)\sigma_{t-1}^2\right] /$$
$$\mu_t F_\mu\left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right) - b(1-\beta_t)\sigma_t^2$$

Solving for β_t we obtain

$$\beta_t^* = 1 + \left[\lambda_{t-1} F_\mu \left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2 \sigma_t^2\right) F_\mu \left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2 \sigma_{t-1}^2\right)(\mu_{t-1} - \mu_t) - F_\mu \left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2 \sigma_t^2\right) E_{\mathbf{t}|t(n)=t} \left\{F(t)\, bR(N)\, F_\mu^M M \left(\alpha_t + \beta_t \mu_t, \beta_t^2 \sigma_t^2\right)\mu_t + F(t)\, S(N)\, F_{\sigma^2}\right\}\right] / (B)$$

where

$$B \equiv b\left[E_{\mathbf{t}|t(n)=t}\left\{F(t)\, F_\mu^M \left(\alpha_t + \beta_t\mu_t, \beta_t^2\sigma_t^2\right)\sigma_t^2\right\} + \lambda_{t-1}\left(F_\mu\left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2\sigma_{t-1}^2\right)\sigma_t^2 - F_\mu\left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right)\sigma_{t-1}^2\right)\right] \geq 0$$

By Fact 5 The second addend in the numerator is positive. Since $F_{\sigma^2}^M$ is negative, the second addend in the denominator is also positive, butit will go to zero in a large enough market, and β_t^* will converge to

$$\begin{aligned} 1 + \Big[\lambda_{t-1} F_\mu \left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right) F_\mu\left(-\alpha_t + (1-\beta_t)\mu_{t-1}, (1-\beta_t)^2\sigma_{t-1}^2\right)(\mu_{t-1} - \mu_t) \\ -E_{\mathbf{t}|t(n)=t}\left\{F(t)\,\mu_t R(N)\, F_\mu\left(-\alpha_t + (1-\beta_t)\mu_t, (1-\beta_t)^2\sigma_t^2\right) F_\mu^M\left(\alpha_t + \beta_t\mu_t, \beta_t^2\sigma_t^2\right)\right\}\Big] / B \end{aligned} \tag{3}$$

By putting a bound on correlation ρ_{st}^{nm} we can arbitrarily bound $\lim R(N)$, to the point in which this solution is greater than 1, which cannot be. It has to be that $\beta_t^*(N) = 1$. Since t was arbitrarily chosen between 1 and T, there is a $\overline{N}$ such that $\beta_t^*(N), \forall N \geq \overline{N}$ and types $t > 1$ will receive a contract with zero variance. Let this bound be called $\overline{\rho}_t$.

By letting $\overline{\rho} = \min_{\{1,\ldots T\}} \overline{\rho}_t$, we found a bound for correlation for which all conclusions hold. This concludes the proof.

□

The other results follow from the construction of the optimal contracts in the previous proof.

The first best equilibrium menus in the more general economy are

$$\hat{M}^n(N) = \left[\hat{C}_t^n(N)\right]_{t=1}^T = \left[\left(\hat{\alpha}_t(N), \hat{\beta}_t(N)\right)\right]_{t=1}^T$$

Lemma 4. *Under Assumption 3, there is a $\overline{\rho}$, such that, if $\rho_{st}^{mn} < \overline{\rho}$ for all m, n, s, t*

- $\hat{\beta}_t^n(N) < 1$
- $\lim_{N\to\infty} \hat{\beta}_t^n(N) = 1, \forall n, \forall t$

Theorem 5. *Increasing the number of Units in an economy, eventually induces Principals to retain more risk at the contracting stage.*

$$\forall N, \exists \overline{N} : \hat{\beta}_t^n(N') \geq \hat{\beta}_t^n(N), \forall n, \forall t \in \{L, H\}, \forall N' \geq \overline{N}$$

Proposition 6. *Asymmetric information Equilibria are weakly Pareto Dominated*

Proof. While the contract of type 1 is efficient by construction, the contracts of type $t > 1$ are all inefficient and can be improved upon. In fact every agent can be left at the same utility u_t while the principal utility can be improved upon by issuing a first contract solving separated problems

$$\max_{(\alpha_t,\beta_t)} U^M(\alpha_t,\beta_t|t)$$
$$s.t.\ \ U(-\alpha_t, 1-\beta_t|t) \geq u_t$$

The resulting Contracts $\left\{\hat{\hat{M}}^n\right\}_{n=1}^N = \left\{\left[\hat{\hat{C}}_t^n\right]_{t=1}^T\right\}_{n=1}^N$ are Pareto Efficient by construction, and give at least the same utility to Principals, and exactly the same to Agents. □

Proposition 7. *Aggregate Risk on markets is higher than under the Pareto Optimal contracts described in the proof of Proposition 6.*

$$\sum_{1=1}^N \left(\beta_{t(n)}^n\right)^2 \sigma_{t(n)}^2 \geq \sum_{1=1}^N \left(\hat{\hat{\beta}}_{t(n)}^n\right)^2 \sigma_{t(n)}^2, \forall \mathbf{t} \in [L,H]^T$$

Proof. The contract (and hence the security) of a Unit n employing its "best" possible agent $(t : \tau(n)(1) = 1)$ is optimal, so the claim is trivially true. For types $t : t(n) = 2, ...T$ the claim follows from Equation 3. □

Proposition 8. *Increasing the number of Units in an economy, eventually makes contracts more efficient. Letting $\Delta_t^n(N) = |\beta_t^n(N) - \hat{\hat{\beta}}_t^n(N)|$.*

$$\forall n, \forall N, \exists \overline{N} : \forall N' \geq \overline{N}, \Delta_t^n\left(N'\right) < \Delta_t^n(N)$$

Proof. The proof leverages on the fact that both first and second best contracts are arbitrarily close to 1 for large enough markets. The claim is trivially satisfied for $\beta_1^n(N)$ (the second best solution, with markets), since it is a Pareto Efficient Contract by construction. For $\beta_t^n(N), t > 1$, note how $\beta_t^n(N)$ and $\hat{\hat{\beta}}_t(N)$ both go to one as N gets large, there must be a $\overline{N}$ such that $\Delta_t^n(N) < \Delta_t^n(1)$ for all $N > \overline{N}$. □

Proposition 9. *With large enough markets, no agent enjoys an information rent.*

$$\exists \overline{N} : \forall N \geq \overline{N}, U(C_t(N), t) = \overline{u}, \forall t$$

Proof. For a given economy consider an N' which makes $\beta_t^n(N) = 1, \forall t > 1$, in every Unit, as per Equation 3. The contracts for these types t, will be identical for all types $t > 1$, in the form $C_t^n = (\alpha^u, 1)$ where α^u is such that $-\alpha^u - \frac{b}{2}(-\alpha^u)^2 = \overline{u}$. This contract leaves all types, including 1 at their participation constraint. This implies immediately that all types $2, ..., T$ will be at their reservation utility. Furthermore the optimal contract for type 1 now will also yield him only his reservation utility, because that's the most he can obtain by mimicking another type. □

7.7 Counterexamples

These two examples, combined with the correlation example in Section 5 show that the assumptions (independence of returns and ordering of types), and conclusions of Theorem 1 are "tight".

7.7.1 A Type Space violating SCP

Consider the following set of parameters

(μ_H, σ_H^2)	(μ_L, σ_L^2)	b	$\overline{u}$
$(2.1, 1)$	$(2, 0.1)$	0.1	0.3

$$-\alpha_H + (1 - \beta_H) X = -0.9301 + 0.6163X$$
$$-\alpha_L + (1 - \beta_L) X = 0.3122$$

Those with markets will be

$$-\alpha_H^M + \left(1 - \beta_H^M\right) X = 0.3083 + 0.0018X$$
$$-\alpha_L^M + \left(1 - \beta_L^M\right) X = -0.9301 + \mathit{0.4913}X$$

We can see that the contract of type L becomes riskier as markets are introduced. This is because in this example SCP fails, so that there is not a "better" type. The "better" type depends on which part of the contract space we are looking at. The diversification possibilities available to the Principal, change the portion relevant in equilibrium.

7.7.2 Small Market

The insurance results are formulated as convergence results rather than Comparative Statics result. The following example shows that a stronger comparaatives statics would not be true. Adding a Unit does not necessarily induce insurance for Agents.

Consider the set of parameters.

(μ_H, σ_H^2)	(μ_L, σ_L^2)	r	$\overline{u}$
$(3, 0.3)$	$(2, 1)$	.01	.5

The equilibrium contracts without markets will be

$$-\alpha_H + (1 - \beta_H) X = -0.2483 + 0.2499X$$
$$-\alpha_L + (1 - \beta_L) X = -0.5368 + 0.5197X$$

Those with markets will be

$$-\alpha_H^M + \left(1 - \beta_H^M\right) X = -0.7713 + \mathit{0.4243}X$$
$$-\alpha_L^M + \left(1 - \beta_L^M\right) X = -0.3529 + 0.4275X$$

In this economy, the contract of type H is now riskier, when markets are present.

To have an intuition for what is going on, compare the parameters with those from the previous example and note that now

- *Risk Aversion is very low.* Markets do not change much the risk taking attitude, as Principals are close to being risk neutral.

- *The distributions of returns are very different.* Entering the market amounts to selling half of a Unit to buy a fraction of the other, plus/minus a transfer of riskless asset. This introduce a big change in the risk held by a principal.

In this example, the Principal owning the safer Unit ends up holding more risk than her agent does, when the possibility of trading is introduced. She is of course compensated with a transfer of riskless asset, but at the contracting stage she acts as if her risk attitude has increased

Consider what happens to the contracts when markets are present. If we consider a replica economy with 10 Units, we will have that

$$
\begin{aligned}
-\alpha_H^M + \left(1 - \beta_H^M\right) X &= 0.0279 + 0.1578X \\
-\alpha_L^M + \left(1 - \beta_L^M\right) X &= 0.1835 + 0.1589X
\end{aligned}
$$

Theorem 1 starts biting and both contracts are again less risky than they would have been without markets. This is because the large number of Units makes diversification and the ensuing change in risk aversion enough to countervail the effect of selling part of the less risky endowment.

References

[1] Sumit Agarwal, Chang Yan, and Yavas, Abdullah, "Adverse Selection in Mortgage Securization", Paolo Baffi Centre Research Paper No. 2009-67, 2010.

[2] Aliprantis, Charalambos D., Kim C. Border, "Infinite Dimensional Analysis: A Hitchhiker's Guide", Springer, 2006.

[3] Bolton, Patrick, Mathias Dewatripont, "Contract Theory", MIT press, 2005.

[4] Dekel, Eddie, "An axiomatic characterization of preferences under uncertainty: Weakening the independence axiom", *Journal of Economic Theory*, 40, 1986, 304-318

[5] Ellickson, Bryan, Birgit Grodal, Suzanne Scotchmer and William R. Zame, "The Organization of Production, Consumption and Learning", Papers in Honor of Birgit Grodal, New York: Springer (forthcoming).

[6] Ellickson, Bryan, Birgit Grodal, Suzanne Scotchmer and William R. Zame, "Clubs and the Market: Large Finite Economies", *Journal of Economic Theory*, 101, 2001, 40-77.

[7] Ellickson, Bryan, Birgit Grodal, Suzanne Scotchmer and William R. Zame, "Clubs and the Market", *Econometrica*, 67, 1999, 1185-1217.

[8] Gibson, Robert S., Richard Holden and Michael Powell, "Organization and Information: Firms' Governance Choices in Rational Expectations Equilibrium", working paper, 2010.

[9] Helpman, Elhanan, Jean-Jacques Laffont "On moral hazard in general equilibrium theory", *Journal of Economic Theory*, 10, 1975, 8-23.

[10] Jensen, Michael C., William H. Meckling, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure", *Journal of Financial Economics*, 3, 1976, 305-360.

[11] Legros, Patrick, Andrew F. Newman, "Wealth effects, distribution, and the theory of organization", *Journal of Economic Theory*, 70, 1996, 312-341.

[12] Legros, Patrick, Andrew F. Newman, "Monotone matching in perfect and imperfect worlds", *Review of Economic Studies*, 69, 2002, 925-942.

[13] Legros, Patrick, Andrew F. Newman, “Beauty is a beast, frog is a prince: assortative matching with nontransferabilities”, *Econometrica*, 75, 2007, 1073-1102.

[14] Maccheroni, Fabio, Massimo Marinacci, Aldo Rustichini, ”Ambiguity Aversion, Robustness, and the Variational Representation of Preferences”,*Econometrica*, 74, 2006,1447-1498.

[15] Machina, Mark, “Dynamic Consistency and Non-Expected Utility Models of Choice under Uncertainty”, *Journal of Economic Literature*, XXVIII, 1989, 1622-1668

[16] Magill, Michael, Martine Quinzii, ”Theory of incomplete markets. Vol. 1”, MIT Press, 2002

[17] Magill, Michael, Martine Quinzii, “An equilibrium model of managerial compensation”, working paper, 2005.

[18] Maretto, Guido “Financial Markets, Contracts, and Moral Hazard”, working paper, 2010.

[19] Maskin, Eric, John Riley “Monopoly with incomplete information”, *Rand Journal of Economics*, Vol. 15, No. 2, 1984, 171-196.

[20] Nielsen, Lars T., ”Existence of Equilibrium in CAPM”, *Journal of Economic Theory*, 52, 1990, 223-231 .

[21] Ou-Yang, Hui, “An Equilibrium Model of Asset Pricing and Moral Hazard”, *Review of Financial Studies*, 18, 2005, 1254-1302.

[22] Parlour, Christine A., Johan Walden, “Capital, Contracts and the Cross Section of Stock Returns”, Working Paper 2008.

[23] Prescott, Edward C. and Robert M. Townsend *Econometrica*, 52, 1984, 21-45

[24] Rahman, David, “Competition with Local Mechanisms: Allocating Residual Claims and Control Rights”, working paper, 2005.

[25] Rahman, David, “Contractual Pricing with Incentive Constraints”, working paper, 2005.

[26] Stracca Livio, “Delegated Portfolio Management: A Survey of the Theoretical Literature”, European Central Bank Working Paper Series, 520, 2005.

[27] Stiglitz, Joseph E., “Monopoly, Non-Linear Pricing and Imperfect Information: The Insurance Market”, *Review of Economic Studies*, 44, 1977, 407-430.

[28] Zame, William R., “Incentives, Contracts and Markets: A General Equilibrium Theory of Firms”, UCLA Economics Working Papers, 843, 2005.