Appendix 1

An agent chooses between consumption and savings as a decision variable by comparing maximized two-period utilities under different decision variables. V^s denotes a maximized utility by choosing optimal *savings* and V^c denotes a maximized utility by choosing optimal *consumption.*

$$V^s = \max_z E^s{}_y \left(y - z - \frac{(y-z)^2}{2} + \beta \left(z - \frac{z^2}{2} \right) \right)$$

$z_s{}^* = \frac{E(w)+\beta-1}{1+\beta}$ is the value of savings that maximizes utility.

$$V^c = \max_z E^c{}_y \left(z - \frac{z^2}{2} + \beta \left(y - z - \frac{(y-z)^2}{2} \right) \right)$$

$z_c{}^* = \frac{\beta E(w) - \beta+1}{1+\beta}$ is the value of consumption that maximizes utility.

Firstly, I plug the optimal consumption choice $z_c{}^*$ in V^c and simplify the expression:

$$E\left(\left(\frac{1-\beta+(y)\beta}{1+\beta} - \frac{(1-\beta+E(y)\beta)^2}{2(1+\beta)^2} \right) + \beta \left(y - \frac{1-\beta+E(y)\beta}{1+\beta} \right) - \frac{\left(y - \frac{1-\beta+E(y)\beta}{1+\beta} \right)^2}{2} \right) =$$

$$\frac{(1-\beta)(1-\beta+\beta E(y))}{1+\beta} + \beta \left(E(y) - \frac{E(y^2)}{2} \right) - \frac{\left(1-\beta+\beta E(y)\right)^2}{2(1+\beta)^2} + \frac{\beta E(y)(1-\beta+\beta E(y))}{1+\beta} - \frac{\beta\left(1-\beta+\beta E(y)\right)^2}{2(1+\beta)^2} =$$

$$\frac{\left(1-\beta+\beta E(y)\right)^2}{1+\beta} - \frac{\left(1-\beta+\beta E(y)\right)^2}{2(1+\beta)} + \beta \left(E(y) - \frac{E(y^2)}{2} \right) = \beta \left(E(y) - \frac{E(y^2)}{2} \right) + \frac{\left(1-\beta+\beta E(y)\right)^2}{2(1+\beta)}.$$

Secondly, I do the same procedure for $z_s{}^*$ and V^s:

$$E\left(y - z - \frac{(y-z)^2}{2} + \beta \left(z - \frac{z^2}{2} \right) \right) = E(y) - \frac{E(y)+\beta-1}{1+\beta} - \frac{\beta E(y^2)}{2} + \frac{E(y)(E(y)+\beta-1)}{1+\beta} - \frac{(E(y)+\beta-1)^2}{2(1+\beta)^2} +$$

$$\frac{\beta(E(y)+\beta-1)}{1+\beta} - \frac{\beta(E(y)+\beta-1)}{2(1+\beta)^2} = E(y) - \frac{E(y^2)}{2} + \frac{(E(y)+\beta-1)^2}{2(1+\beta)}.$$

Then, I compare values V^c and V^s. Savings as a decision variable are better if:

$$E(y) - \frac{E(y^2)}{2} + \frac{(E(y)+\beta-1)^2}{2(1+\beta)} > \beta \left[E(y) - \frac{E(y^2)}{2} \right] + \frac{\left(1-\beta+\beta E(y)\right)^2}{2(1+\beta)}.$$

$(\beta - 1)\left(E(y) - \frac{E(y^2)}{2} \right) < \frac{(E(y)+\beta-1-1+\beta-\beta E(y))(1+\beta)E(y)}{2(1+\beta)}$ and $\beta < 1$.

$E(y^2) < E(y)^2$ – never holds if y is a random variable. Therefore, in this problem, an agent will always choose consumption as a decision variable.

Appendix 2

In the quadratic-gaussian case and the absence of information costs, an agent maximizes the following expression:

$$\max_{p(s|y)} \int_s \int_y p(s|y)g(y)(-(y-s)^2 - \beta s^2)ds\, dy$$

An agent receives the signal x, which contains noise: $x = y + \epsilon$. After receiving the signal, optimal savings choice is $s = \frac{E[y|x]}{1+\beta}$. I plug the optimal choice in the optimization problem:

$$\int_x \int_y p(x|y)g(y)\left(-\left(y - \frac{E[y|x]}{1+\beta}\right)^2 - \beta\left(\frac{E(y|x)}{1+\beta}\right)^2\right) dx\, dy = \int_x p(x) \int_y p(y|x) \left(-y^2 + \frac{2yE(y|x) - E(y|x)^2}{1+\beta}\right) dy\, dx = \int_x p(x)\left(-E(y^2|x) + \frac{E(y|x)^2}{1+\beta}\right)dx = \int_x p(x)\left(-E(y^2|x) + E(y|x)^2 - E(y|x)^2 + \frac{E(y|x)^2}{1+\beta}\right)dx = \int_x p(x)\left(-\sigma^2{}_{y|x} - \frac{\beta E(y|x)^2}{1+\beta}\right) dx = -\sigma^2{}_{y|x} - \frac{\beta\left(\mu_y^2 + \sigma_y^2 - \sigma_{y|x}^2\right)}{1+\beta}.$$

For the first equality, I use the fact that $p(x|y)g(y) = p(x)p(y|x)$. For the last equality, I use the expression for the conditional mean and variance under Gaussian distribution: $E(y|x) = \mu_y + \rho\frac{\sigma_y}{\sigma_x}(x - \mu_x)$ and $\sigma_{y|x}^2 = \sigma_y^2(1-\rho^2)$.

I square the conditional mean $E(y|x)^2 = \mu_y^2 + 2\mu_y\rho\frac{\sigma_y}{\sigma_x}(x-\mu_x) + \rho^2\frac{\sigma_y^2}{\sigma_x^2}(x-\mu_x)^2$ and plug it in the last but one equality: $\int_x p(x)E(y|x)^2 dx = \mu_y^2 + \rho^2\frac{\sigma_y^2}{\sigma_x^2}\sigma_x^2 = \mu_y^2 + \rho^2\sigma_y^2 = \mu_y^2 + \sigma_y^2 - \sigma_{y|x}^2$.