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Inattention, Hand-to-mouth Behavior, and Poverty Trap

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Prague, Czech Republic

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June, 2021

Declaration

1. I hereby declare that I have compiled this thesis using the listed literature and resources only.
2. I hereby declare that my thesis has not been used to gain any other academic title.
3. I fully agree to my work being used for study and scientific purposes.

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Abstract

In this thesis, I study the hand-to-mouth behavior of the poor that originates from inattentive savings behavior and leads to a poverty trap. I assume that in the rational inattention model people acquire costly information about stochastic permanent income and choose a savings plan. I hypothesize that in this context high information costs and low income variance may encourage poor agents to choose a rigid savings plan below an optimal level, persistently undersave, and get stuck in a poverty trap. I construct and solve the two-period rational inattention model of savings decisions. The results show that a poor agent chooses a rigid savings plan below an optimal level when information costs are high and income variance is low. I discuss avenues for future research that are based on solving an infinite-horizon model with rationally inattentive agents, provide the conditions under which a poverty trap occurs, and test the model on consumption-savings data.

Keywords: information costs, hand-to-mouth behavior, poverty trap, savings plan

Abstrakt

Ve své práci zkoumám chování chudých, kteří těsně vyjdou se svými příjmy a jejichž chování pramení z nezodpovědné tvorby úspor a vede k pasti chudoby. Předpokládám, že v modelu s racionální nepozorností lidé získávají informace o náhodném permanentním příjmu a volí svůj plán spoření. Předkládám hypotézu, že v tomto kontextu vysoké náklady na získání informace a nízký rozptyl příjmů mohou motivovat chudé aktéry k volbě rigidního spořicího plánu pod optimální úrovní, persistentně méně spořit a uvíznout v pasti chudoby. Vytvářím a řeším model racionální nepozornosti o dvou periodách s volbou úspor. Výsledky ukazují, že chudí aktéři volí rigidní plány úspor pod optimální úrovní, pokud jsou náklady na získání informací vysoké a rozptyl příjmů je nízký. Diskutuji směry možného budoucího výzkumu, které mohou zahrnovat řešení modelu s nekonečně mnoha periodami, nalezení podmínek pro vznik pasti chudoby a testování modelu na datech o spotřebě a úsporách.

Klíčová slova: náklady na informace, žít z ruky do úst, past chudoby, plán úspor

Project of Master Thesis

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Theme: Inattention, Hand-to-mouth Behavior and Poverty Trap

Research question and motivation

Following the extensive discussion about undersaving by the poor in (Karlan, 2014), I focus on poor people who save non-optimally and therefore face financial precarity and the risk of falling into a poverty trap. According to a US Federal Reserve survey, four in ten American adults have \$400 or less to spend on an emergency if necessary. The share of people who live in persistent poverty is around 10% in the US and even higher in developing countries. Why, then, do poor people fail to accumulate enough savings to insure against income shocks and escape from a poverty trap?

Contribution

This research contributes to the broad literature on hand-to-mouth behavior and the risks of a poverty trap by providing a novel perspective on the trap resulting from inattentive savings behavior. This research may also contribute to the literature on the effects of financial literacy and the career opportunities.

Methodology

In order to explain persistent undersaving, I propose to use the theory of rational inattention, pioneered by Sims (2003). I assume that an agent allocates consumption between periods based on her stochastic permanent income, which is the present value of future income flows, and can learn about that income by acquiring costly information. I assume that poor agents have low variance of permanent income and high costs of acquiring information about their permanent income. I hypothesize that low income variance and high information costs induce a rigid savings plan below an optimal level and may lead to a poverty trap. I construct and solve the two-period rational inattention model of savings choice.

Outline

1. Introduction and Literature Review
2. Theoretical Model
3. Avenues for Future Research
4. Summary
5. References / Bibliography

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Introduction

Imagine the lifestyle of a poor agent who earns enough to cover her basic needs. She probably spends the remainder of her income on entertainment and fails to accumulate enough savings to insure against income volatility, while experiencing severe stress due to financial precarity. Translating an imaginary story into a real one, (Leana, 2019) conducted a study of the savings behavior of over 1,000 short-haul truck drivers. The results showed that two-thirds of employees did not accumulate enough savings to cover 60 days of expenses in case of injury or illness while earning up to \$60,000 per year. This extreme case illustrates the behavior of agents who earn enough to accumulate savings but consume excessively, undersave and face financial precarity. This type of behavior is called *hand-to-mouth* and is widespread. The share of hand-to-mouth consumers who keep no liquid or illiquid assets is around 10% in the USA (Kaplan and Violante, 2014) and is even higher in developing countries. According to a Federal Reserve survey¹, four in ten adults could hardly spend just \$400 in case of emergency. This behavior is mostly pronounced among poor people and has the most acute consequences for them. Poor people may fail to accumulate enough savings to insure against income shocks or invest in future consumption, and, thus, get stuck in a poverty trap.

Following the extensive literature on the undersaving of the poor (Karlan et al., 2014), I focus on poor people who save but do it non-optimally and fall into a poverty trap. The share of people who live in persistent poverty is around 10% in the USA². Why, then, do poor people fail to accumulate enough savings to insure against income shocks and escape from a poverty trap? The definition of poor that I use in the thesis is rather broad and may include the lowest quantile of the middle class or blue-collar workers. My research is focused on the poor who have enough income to save at least something, so I disregard extremely poor people.

¹ Report on the Economic Well-Being of U.S. Households in 2019, Featuring Supplemental Data from April 2020; Board of Governors of the Federal Reserve System

² 2020 Current Population Survey Annual Social and Economic Supplement (CPS ASEC)

I argue that poor people may exhibit *hand-to-mouth* behavior, and thus fall into a *poverty trap*. The analysis includes two main concepts: hand-to-mouth behavior and a poverty trap. In the thesis, an agent exhibits *hand-to-mouth* behavior if she consumes almost all of her income and saves too little. I do not distinguish between liquid or illiquid savings. By definition, a poverty trap is a persistent life in poverty. I claim that poor people live in a poverty trap because they fail to accumulate savings required to escape from the trap.

In order to explain persistent undersaving, I use the theory of rational inattention, pioneered by (Sims, 2003). I assume that an agent allocates consumption between periods based on her stochastic permanent income, which is the present value of future income flows, and can learn about that income by acquiring costly information. By undersaving, I mean a lower level of savings than one would have with perfect knowledge of her permanent income (zero information costs). I also assume that heterogeneity in permanent income implies two features. Firstly, a poor agent has a lower variance of permanent income than a rich agent. This assumption can be rationalized by the fact that labor income is less volatile than capital income, and the share of capital income in total income increases as an agent becomes richer. The permanent income of the rich encompasses a high share of capital and business incomes. Moreover, poor people experience a lack of career opportunities for promotion (Weinger, 2000), whereas the unemployment rate in 2019 was the lowest for 50 years in the US. All these facts combined evidence for the lower volatility of permanent income of the poor than the rich. Secondly, a poor agent has higher costs of information about her permanent income than a rich one. This fact is extensively documented in the literature (Leana et al., 2012; Mani et al., 2013; Schilbach et al., 2016) and can be thought of as a cognitive tax implied by stress, physically demanding work, ignorance of market opportunities, or the lack of financial literacy. This means that a poor agent pays higher costs for an additional unit of information about the present value of her future income flows than a rich one. In the thesis, my main interpretation of information costs is from the perspective of financial literacy, but one should think about this concept more broadly.

Under these assumptions, a poor agent may have low incentives to learn about the true value of future income flows. This behavior is partially in line with the results of (Mackowiak and Wiederholt, 2009), who showed that firms pay more attention to shocks with higher variance. Nevertheless, information costs and income variance may non-trivially interact with each other

and influence savings decisions, and thus the total effect is not obvious. I hypothesize that low income variance and high information costs induce hand-to-mouth behavior and may lead to a poverty trap. In this context, hand-to-mouth behavior is a rigid savings plan below an optimal level, meaning that an agent chooses the fixed amount of savings for any possible income level. By the optimal level, I mean a plan that would be chosen if one knew her permanent income perfectly. A rigid savings plan compels an agent to always save as if she has low permanent income and consume positive income shocks. Moreover, such an agent is sensitive to large negative shocks because she has not accumulated enough savings in the past to insure against them. In this case, an agent preserves hand-to-mouth behavior, fails to accumulate enough savings to insure against negative shocks, and gets stuck in a poverty trap.

I split the decision process into two steps. In the first, an agent chooses between consumption or savings as a decision variable. I show that the choice of consumption as a decision variable is always optimal in a two-period model in the absence of constraints. However, when an agent faces a constraint on savings (for example, budget constraint), she might choose savings as a decision variable to satisfy the constraint on savings with certainty. In the second step, an agent allocates her attention and chooses the level of *savings*.

The existence of poor hand-to-mouth consumers is extensively studied (Kaplan and Violante, 2014; Gomes, 2021; Aguiar et al., 2020) and generally accepted to be of the great importance. Poor hand-to-mouth agents influence the dependence between income and consumption inequalities because they have an enormously high marginal propensity to consume (MPC). It is generally known that consumption inequality is lower than income inequality because people smooth consumption over time. High MPC means that consumption dynamics closely follow income dynamics in low income percentiles, and thus poor people fail to smooth consumption and consumption inequality is high. Nevertheless, although the related literature states the existence of poor hand-to-mouth consumers and discusses the macroeconomic consequences of the phenomenon, it remains almost silent about the reasons. I contribute to the literature on hand-to-mouth behavior by providing a micro-founded rationale for the existence of poor hand-to-mouth consumers.

One could split the large body of rather scattered research on a poverty trap into two streams. The first focuses on external constraints that prevent poor agents from accumulating savings, such

as transaction costs (Dupas et al., 2012; Schaner, 2016), regulatory barriers (Jentzsch, 2009), or the lack of career opportunities (Weinger, 2000). The second focuses on constraints that are internal to individuals by assuming a particular behavioral bias. For instance, people may fall into the trap due to a lack of self-control (Bernheim et al., 2015), present-biased preferences (Bouchard St-Amant and Perrault, 2019), aspirations failure (Dalton et al., 2011), or focusing effect (Canidio, 2015). In my research, I combine the two streams by analyzing a poverty trap resulting from self-evident limited mental capacity (internal constraint) and (external) constraint on income shocks. This allows me to provide a general perspective on a poverty trap unifying the set of scattered explanations. The rational inattention model can be integrated into a macro-level framework jointly with other mechanisms studied in the literature on consumption choice (for an example of such integration see Luo [2008]). Further, unlike the behavioral studies on a poverty trap, I assume rational agents with a mental capacity constraint instead of assuming any type of behavioral bias. I consider hand-to-mouth behavior as a rational reaction to the restrictions on income distribution and information processing. As hand-to-mouth behavior leads to a poverty trap, this approach provides straightforward policy implications for mitigating a poverty trap by changing information costs and the distribution of permanent income instead of fixing behavioral biases.

Under the above assumptions, a policy intervention should jointly follow two directions: introducing higher risk into the distribution of permanent income and decreasing information costs. If information costs remain the same, the higher risk may even exacerbate a poverty trap: due to high information costs people may still have a rigid savings plan, but negative income shocks become more probable. By increasing income variance and decreasing information costs, a policymaker would encourage poor agents to pay more attention to their permanent income and allocate savings more optimally. In other words, a policy intervention that provides opportunities for career promotion and helps people to learn which opportunity matches their skills the best may allow people to avoid a poverty trap.

This research analyses hand-to-mouth behavior and a poverty trap originating from inattentive behavior in the spirit of attention trap (Macaulay, 2021) and savings rigidity (Reis, 2006). Macaulay (2021) shows that low wealth implies low attention to savings opportunities because of low returns to investments. Thus, people with low initial wealth will have low wealth forever. I propose a different perspective on undersaving resulting from inattention to income

rather than investment opportunities, which may be more appropriate in the case of a poverty trap. Poor people just accumulate savings for precautionary motives and searching for investment opportunities may be a second-order problem for them (Karlan et al., 2014). Reis (2006) analyzes hand-to-mouth behavior arising from the costs of updating information. He shows that for high enough costs, one will choose a rigid savings plan and consume all the additional income. While the paper provides an important insight into the role of information costs in the formation of hand-to-mouth behavior, it remains silent about any income trap and discusses only a rigid savings plan. Further, the author simplified the information acquisition problem and allows agents to choose only the time of updating savings choice rather than the time-dependent distribution of savings and income. To the best of my knowledge, there are no studies that analyze hand-to-mouth behavior and a poverty trap originating from inattentive savings behavior in the rational inattention framework.

To study hand-to-mouth behavior, I construct a two-period rational inattention model of consumption choice. Firstly, I solve the model in the quadratic-gaussian case and obtain the analytical solution for the allocation of savings. The solution is close to that of the standard one-period problem, but in the two-period model, the conditional variance of income also depends on the discount factor. Secondly, I numerically solve the model with CRRA utility function, non-gaussian distribution of income, and borrowing constraint and obtain that a poor agent chooses a rigid savings plan below an optimal level when information costs are high and income variance is low. Poor agents have a more rigid savings plan than rich ones. Further, I construct an infinite-horizon model with rationally inattentive agents. In future research, I plan to solve the infinite-horizon model and provide conditions under which a poverty trap occurs. Also, I extensively discuss future empirical work that aims to test the theoretical predictions from the model providing evidence in favor of hand-to-mouth behavior resulted from high information costs and low income variance.

This thesis has the following structure. In Section 1, I extensively discuss the existing studies on hand-to-mouth behavior and a poverty trap from their origins. I start with the theory of rational expectations and finish with two papers by (Macaulay, 2021) and Reis (2006), which are the most relevant for my study. In particular, I discuss in detail the theory of rational inattention as it is the main framework for the analysis of hand-to-mouth behavior and a poverty trap. In Section 2,

I present a two-period rational inattention model and numerical solution with comparative statics. In Section 3, I discuss the avenues for future research: possible modifications of the theoretical model and empirical strategy. In this section, I also discuss the possible contribution of the thesis. This section also contains empirical models that serve to test the theoretical results.

1 Literature Review

The following literature review aims to extensively introduce in the analysis of hand-to-mouth behavior and a poverty trap originating from inattentive savings behavior. To do this, I firstly describe the basic framework of consumption choice under rational expectations (RE) by discussing the permanent income hypothesis (Friedman, 1957), the life-cycle hypothesis (Modigliani, 1966) and the allocation of savings under these hypotheses. I then discuss the contradictions of the RE framework and introduce the concept of inattention in the context of consumption choice. I consider inattention in the decision process from two perspectives: behavioral frictions and bounded rationality. The first assumes that agents have some behavioral bias, such as present-biased utility (Bouchard St-Amant and Perrault, 2019) or utility with “focusing effect” (Canidio, 2015), that force them to deviate from the behavior predicted by the RE framework. The second considers agents as still rational but experiencing some internal restrictions, such as information processing constraint in the theory of rational inattention (Sims, 2003). Under these internal constraints, an agent behaves sub-optimally from the point of the RE framework, but optimally from the agent’s perspective. I mostly focus on the theory of rational inattention as I use it in the analysis of hand-to-mouth behavior and a poverty trap, describing the basics of the theory and possible applications. Further, I discuss separately the phenomena of hand-to-mouth behavior and a poverty trap. Hand-to-mouth behavior is usually considered in the macroeconomic perspective and context of economic inequality, while the analysis of a poverty trap is mostly focused on micro-level rationale of the trap. In the last section of the literature review, I examine in detail the two papers by Macaulay (2021) and Reis (2006) that are most related to this thesis. Both of them study the savings behavior of inattentive agents. My thesis essentially differs from these papers in several aspects, but I use the insights from them.

1.1 Rational Expectations and Consumption-Savings Choice

1.1.1 Models of Rational Expectations

The classical theory of consumption choice originates from the permanent income hypothesis (PIH) by Friedman (1957) and the life-cycle hypothesis by Modigliani (1966). Friedman (1957) claimed that income can be decomposed into permanent and transitory components, and an agent adjusts consumption differently to the change in each component. In theory, consumption reacts one-to-one to any change in permanent income, meaning that an agent will entirely consume positive change in permanent income or decrease consumption by the amount of negative change in permanent income. On the other hand, consumption is insensitive to changes in the transitory component. An agent saves any positive change in the transitory component and covers negative changes from accumulated savings. The reason for such behavior is that an agent does not want to change her consumption dramatically, and thus she smoothes it over time. The life-cycle hypothesis (LCH) (Modigliani, 1966) also used the concept of smoothing consumption but in a broader context. Modigliani (1966) argued that an agent wants to keep consumption constant during her life, and thus she borrows during periods of low income and saves during periods of high income. Roughly speaking, an agent borrows in youth and repays the debt in adulthood. Both hypotheses refer to some fundamental income (permanent income, in the case of PIH; and life-cycle income in the case of LCH), which agents use while making the consumption choice. This income (which is unobserved) has a structural relation with current income (which is observed) and can be inferred from the relation.

The main critique of the structural relation was made by Lucas (1976), who argued that a stable dependence between fundamental and current incomes may not exist. The reason is that any changes in the economic environment could also change the way agents infer fundamental income from their current income. In other words, any policy change that influences current income will also unpredictably influence the dependence between fundamental and current incomes. Thus, one cannot estimate the parameters of the dependence between consumption and income as they are not stable and estimations will not be consistent.

Hall (1978) addressed Lucas' critique by formulating a stochastic version of the PIH. He argued that structural dependence between permanent income and current income emanates from

a utility maximization problem of the representative agent as for any policy change an agent always maximizes her utility. Thus, the structural dependence encompasses the reaction of agents to changes in current income. Hall (1978) assumed that the utility has a quadratic form, and an agent faces stochastic income. The main implication from the model is that the consumption function is a martingale, and thus the best prediction of consumption in the next period is current consumption, given all available information in the current period. This implication is one of the testable predictions from the model and has been extensively tested (Deaton, 1987; Blundell, 2008). In addition, the model implies that current consumption reacts to the expected changes in future income rather than current income only. Thus, an agent changes her consumption in the consequence of policy at the moment of announcement rather than implementation of the policy.

The PIH and LCH have been extensively tested on data and only partially confirmed. In the case of the permanent income hypothesis, several studies have documented excess smoothing (Deaton, 1987) and excess sensitivity (Flavin, 1981) puzzles. The first means that an agent consumes only part of her permanent income; the second means that an agent over-reacts to transitory income (consumes greater than a zero fraction of transitory income). Blundell (2008) has estimated the parameters of consumption sensitivity to permanent and transitory incomes in different social groups and found that consumption indeed reacts differently to income changes than the PIH predicts.

The life-cycle hypothesis states that agent's lifetime profile is constant over time. Further studies have shown (Carroll and Summers, 1991; Fernandez-Villaverde and Krueger, 2007; Tung, 2011) that the life-cycle consumption profile is hump-shaped, contradicting the life-cycle hypothesis. For example, Tung (2011) conducted an extensive study on life-cycle consumption profiles in 23 different countries and showed that the profiles can significantly vary in their shapes, but none of the profiles was constant over time.

1.1.2 Accumulation of Savings

The main motive for the accumulation of savings in the PIH and LCH is a *precautionary motive*. An agent is uncertain about her future income and wants to save in order to insure against income shocks. In the absence of financial markets and the uniqueness of a consumption good, the only mechanism of insurance is self-insurance via accumulated savings. If one takes a step from

the basic model of consumption-savings choice and introduces durable goods, labor supply, financial markets and etc., there are multiple opportunities for an agent to ex-ante insure against income shocks.

Durable or luxury goods may serve as an insurance mechanism as an agent can invest in them during the periods of favorable financial conditions and sell them to cover emergency expenses (Browning et al., 2003). Further, an agent may vary her labor supply instead of the consumption of goods in order to accumulate savings (Low, 2005). In general, there are several different insurance mechanisms on the labor market, such as implicit contracts (Azariadis, 1975; Baily, 1977) or shifting an occupational choice towards a less risky option. If one considers a household (or family) as an economic agent, it can insure against future income shocks by investing in the education of children in the household. In general, reallocation of economic activity inside a household may serve as a self-insurance mechanism.

Financial markets are widely used for insurance purposes. Under free access to financial markets, an agent can borrow during income decline and repay the debt during income increase. Further, an agent can invest money in multiple financial assets and accumulate savings. Financial markets allow an agent to flexibly adjust to income volatility. For example, an agent can reallocate her portfolio of assets toward safer assets in a period of high income volatility; Davis and Willen (2000) highlight this mechanism in the pricing model with incomplete markets. If an agent has limited access to financial markets, she faces a liquidity constraint and cannot perfectly smooth consumption between periods. To overcome the problem, an agent may use informal financial institutions such as private lenders (Kotlikoff and Spivak, 1981), and thus agents redistribute income among each other to smooth consumption. One example of the liquidity constraint is a borrowing constraint, which is an explicit restriction on the amount that an agent can borrow. The problem of liquidity constraint is mostly pronounced for the poor, and I extensively discuss this in the section on a poverty trap.

1.2 Inattention in Consumption-Savings Choice

The corner stone of the PIH and LCH hypotheses is the concept of “rationality in expectations”, which means that an agent behaves optimally conditional on her information set. In other words, errors are orthogonal to her information set. In addition, the information set

includes all current shocks to economic variables. The one possible direction of deviation from rationality in expectations is introducing imperfect information. For example, Pischke (1995) assumes that agents lack information about aggregate income and focus mainly on individual income while making economic decisions. The author considers two cases of limited information about aggregate income. In the first case, an agent does not observe an aggregate income shock, so she cannot decompose her income into individual and aggregate components. In other words, an agent does not know whether her dismissal is a result of her low productivity or overall recession in the economy. In the second case, an agent observes aggregate income with a one-period lag only. The two different assumptions lead to a similar implication for the dependence between aggregate consumption and income: a change in aggregate consumption in response to a change in aggregate income is more sluggish than one predicted by Hall's (1978) model of PIH.

One can motivate incomplete information about aggregate income by a lag in the announcements of economic aggregates. For example, GDP data is usually published quarterly, and thus an agent can observe only lagged information. Importantly, this argument is not relevant in the economy with rational expectations, because prices will instantaneously react to any shock and aggregate information, and thus agents can learn about aggregate income by observing prices and their individual income level. On the other hand, one may consider a deviation from rational expectations economy and a fully revealing equilibrium, so prices will serve as only noisy signals for aggregate income.

Pischke (1995) exogenously restricts the information set of an agent by prohibiting her from knowing all aggregate income shocks or current aggregate income shock. Nevertheless, the author supposes that an agent may deliberately restrict her information set because an aggregate income is much less volatile than an individual one, and thus the former is less relevant for the consumption decision. To justify the conjecture, one needs to construct the model of attention allocation.

Even though Pischke (1995) does not use the concepts of limited attention or inattentiveness, the ignorance of some information naturally reveals the limited attention of agents in the economy. Agents in Pischke's economy behave as if they have the intrinsic costs of collecting information and, as the ignorance of aggregate income does not influence individual consumption significantly, an agent decides to acquire information about individual income only. This mechanism hinges on the self-evident idea of the limited attention of individuals. While rationality in expectations implies infinite mental capacity so that an agent processes all available information, it is far from

being realistic and, as we see in the context of the PIH and LCH hypotheses, may lead to conclusion that does not fit into the real data.

The theory that explicitly takes into account the limited ability to process information is the theory of rational inattention, pioneered by Sims (2003). The theory allows an agent to deliberately allocate her limited attention and optimally choose the information that seems the most relevant.

Mackowiak and Wiederholt (2009) use the theory of rational inattention to study the information processing behavior of firms in the presence of firm-specific and aggregate shocks. The paper provides evidence in favor of Pischke's conjecture that an agent processes more information about a more volatile variable. The authors show that in the context of costly information, firms optimally pay more attention to firm-specific shocks, which have higher variance than aggregate shocks.

1.2.1 Bounded Rationality and Rational Inattention

Before describing the theory of rational inattention in the context of consumption choice, I briefly introduce the main idea of the theory. The utility function of a rationally inattentive agent depends on the true state of the world x and action y . The agent does not observe the state x , but can learn about it by choosing a signal structure s . If states were observed, the agent would choose a direct mapping from the state into the action. As the agent does not observe the true state and receives some noisy signals about the state, she chooses a distribution of actions $f(y|x)$. The form of $f(y|x)$ depends on the structure of signals that the agent can choose. In the basic formulation, the decision process is two-step. In the first step, an agent chooses the signal structure (the conditional distribution of *signals* given a state). In the second step, she chooses an action given the received signal. Importantly, as signals are costly, the agent will never choose two different signals that prescribe the same action, and thus the problem can be formulated in terms of actions only. Consequently, the problem shrinks to the one-step decision, in which an agent chooses the conditional distribution of *actions* given a state.

The first paper that introduced the theory of rational inattention was (Sims, 2003). In this framework, an agent chooses the conditional distribution of action Y given the state of the world X in order to minimize a quadratic difference between action and state, facing the set of linear constraints. States of the world are normally distributed. This type of problems is always called a

linear-quadratic Gaussian (LQG) case. The agent wants to choose the same action as the state of the world but cannot observe the latter.

$$(1) \min_q E[(Y - X)^2] = \int (y - x)^2 q(y|x) p(x) dy dx \quad \text{s.t}$$

$$(2) \int q(y|x) dx = 1$$

$$(3) -E \left[E[\log_2(q(Y|X))|X] \right] + E[\log_2(\int q(Y|x)p(x)dx)] \leq C$$

Constraint (2) states that the choice of conditional distribution $q(y|x)$ should be consistent, meaning that for each state of the world x the probability of choosing some action should sum up to one. Condition (3) is an information flow constraint. The agent aims to decrease the level of uncertainty (entropy) in x by choosing some action y . The crucial assumption in the theory of rational inattention is that an agent has a limited mental capacity to process information, and thus the reduction in entropy is restricted by the capacity level C .

The model is formulated using Shannon Entropy (Shannon, 1948), which is defined as follows: $H(X) = -\sum_x P(X)\log(P(X))$. One can interpret Shannon Entropy as the level of uncertainty about the value of random variable X . One can consider a binary case, in which there are only two possible states of the world (random variable X takes only two values: X_1 and X_2 , and thus as the assigned probabilities $P(X)$). The variable X has a maximum entropy when two outcomes are equiprobable because one is the mostly uncertain whether X_1 or X_2 will realize; a minimum entropy takes place when one of the outcomes occurs with certainty. Furthermore, the conditional Shannon Entropy is defined as $H(X|Y) = -\sum_x P(X|Y)\log(P(X|Y))$, determining the level of uncertainty about the value of X after observing Y . The essence of the conditions (1) and (3) is that an agent chooses $q(y|x)$ but she is restricted in her ability to reduce entropy in y . Ideally, the agent would like to choose some action with certainty for each state of the world ($q(y|x)$ equals to one for some value of y and zeros for any other value), but, to do so, she needs to perfectly observe x . The agent cannot perfectly observe x because she has a limited mental capacity.

It is worth noting that if X is normally distributed, the optimal form of the conditional distribution q is also normal, and thus X and Y are jointly normally distributed. In this case, one can obtain a closed-form solution for the shape of normal distribution. As normal distribution is characterized by two parameters (mean value and variance), closed-form solution means the exact formulas for the mean and variance of the conditional distribution q .

In his next paper, Sims (2006) analyzes a simple two-period consumption choice model without an assumption on normality of random variables. In the model, an agent chooses choosing the joint distribution of consumption c and wealth w by maximizing her utility subject to consistency and information flow constraints. Consistently with the model (1) – (3), consumption c serves as an action y and wealth w serves as the true state of the world x . Sims (2006) assumes that an agent has a logarithmic utility function.

$$(4) \max_f \int_{0 < c < w} \log(c(w - c)) f(c, w) dw dc \text{ s.t}$$

$$(5) f(c, w) \geq 0$$

$$(6) \int_{0 < c < w} f(w, c) dc = g(w)$$

$$(7) \int_{0 < c < w} \log(f(w, c)) f(w, c) dw dc - \int_0^\infty (\log(\int_c^\infty f(c, w) dw) \int_c^\infty f(c, w) dw) dc - \int_0^\infty \log(g(w)) g(w) dw \leq K$$

In this model, an agent chooses the joint distribution, while in the previous model an agent chose the conditional distribution. In this framework, one can interchangeably formulate the problem in two ways because the marginal distribution of states is given. One can restore the joint distribution from the conditional and the marginal ones, or the conditional distribution from the joint and the marginal ones using Bayes' Law.

The interpretation of the model is the following. An agent is uncertain about her wealth because she does not know the market value of her property and assets that constitute the wealth. She may receive signals about the level of her wealth. For example, she may hire an expert who will assess the value of the agent's property and assets. As each signal prescribes only one action, one can imagine that the expert also says how much the agent should consume given the wealth. The agent may hire a cheap expert who will provide a rough assessment of the agent's wealth (send imprecise signals) or an expensive expert who will perfectly assess the market value (send precise signals).

The shadow price on information constraint λ (a multiplier on the information flow constraint (7)) varies the level of uncertainty an agent faces. When λ declines, an agent becomes more certain about the true value of wealth w . In the limiting case, when λ is equal to zero, an agent is fully certain about wealth and chooses $c = \frac{w}{2}$. The higher information costs imply the higher excess $E[w|c] - 2c$, which is the difference between expected wealth given consumption and wealth level in a deterministic case. One can reformulate the solution for the consumption

choice and obtain that, with higher information costs people tend to consume less and accumulate more savings in a precautionary motive.

Sims (2003) formulates the model of rational inattention in the linear-quadratic case. Lewis (2008) went beyond this case and studied the decision of agents with constrained mental capacity outside the linear-quadratic Gaussian framework. Overall, Sims (2003, 2006) gives birth to flourishing literature on consumption choice under constrained information processing that develops in both micro- and macro-economic analyses.

For example, Luo (2008) analyses the permanent income hypothesis with rational expectations (RE-PIH) and under the theory of rational inattention (RI-PIH). He proposes an analytical approach to solve the multivariate permanent income model with RI and examines its implications for optimal consumption, saving, and welfare. The author compares the dynamics of aggregate consumption growth in RE-PIH and RI-PIH models and shows that the growth under RI is “excessively smooth” to unanticipated income growth and “excessively sensitive” to anticipated changes in income. The result is consistent with the US data and could potentially explain the excess smoothness puzzle and the excess sensitivity puzzles that cannot be explained by the permanent income hypothesis with rational expectations. One main drawback of the paper is that Luo (2008) assumes the linear quadratic Gaussian (LQG) framework, which is unrealistic but allows the author to obtain an analytical solution for the consumption choice. While the assumption is useful for practical reasons, it may seem overly restrictive.

Tutino (2013) constructs a fully dynamic rational inattention model with non-Gaussian ex-ante uncertainty. In the model, an infinitely living agent chooses consumption allocation and quality and quantity of information about wealth to process. The main contribution of the paper is methodological as it was the first to propose the solution for a fully dynamic rational inattention framework without restrictions on the distribution of shocks. The model predicts that consumption responds to wealth shocks asymmetrically: positive wealth shocks influence consumption with a delay and smaller magnitude than negative wealth shocks. The model also predicts that the persistence and volatility of consumption behavior increase as information-processing constraints become tighter.

Importantly, the models of rational inattention and rational expectations can be tested using survey data on expectations. Coibion and Gorodnichenko (2015) document that full information rational expectations are inconsistent with the empirical evidence. The authors relate ex post mean

forecast errors to the extent in the average forecast and test the null hypothesis that agents are rational in expectations and have full information. The authors use the data on professional forecasters from academics, commercial banks, and non-financial businesses, and provide results that reject the hypothesis of full-information rational expectations. Moreover, the authors claim that the rejection is unlikely to be driven by departures from rationality (such as adaptive expectations) and instead reflects deviations from the assumption of full information. This speaks in favor of the theory of rational inattention as it justifies the systematic deviations from full-information rational expectations observed in the data.

The results of Coibion and Gorodnichenko (2015) predict slow adjustment of information and sluggish reaction to aggregate shocks. Further, the level of sluggishness depends on economic conditions: once the economy is in a recession, expectations are less sluggish. For example, information rigidity was low during the period of Great Inflation in the 1970s and subsequently rose in the period of Great Moderation. This behavior can be rationalized by the theory of rational inattention, which states that an agent pays more attention to the more volatile economic variable. As there is more uncertainty during economic recessions than, on average, over the business cycle, people track changes in economic aggregates with greater attention and adjust their actions faster.

There are multiple studies on consumption choice under RI that are worth mentioning (Mackowiak, 2015; Luo, 2010, 2017; Yin, 2019) but cannot be discussed in detail due to space limits. The field is growing, and I aim to contribute to it by studying the consumption choice of poor people who are uncertain about their permanent income and face information-processing constraints.

Further, there is a strand of literature on rational inattention that focuses specifically on information costs (Kacperczyk et al., 2014; Hebert and Woodford, 2021). For example, Kacperczyk et al. (2014) develop an information-based general equilibrium model that links capital income derived from financial assets to a level of investor sophistication. In the model, the level of sophistication is captured by information costs. The authors showed that income inequality grows with investors' aggregate and relative sophistication in the market. Hebert and Woodford (2021) develop a new cost function called neighborhood-based cost functions. The functions capture the idea that information structures are more costly the greater the extent to which they discriminate between intrinsically similar states of the world (states that share a “neighborhood”).

The authors show that the neighborhood-based cost function makes more accurate predictions about behavior in perceptual experiments than the standard rational inattention cost function.

1.2.2 Behavioral Biases and Inattention

The theory of rational inattention hinges on the assumption that an agent is rational but faces an additional constraint, namely information-processing constraint. The other approach to model the limited attention of people is to consider a deviation from rationality by introducing behavioral biases. This approach can be called the *bounded rationality* approach as it rules out the agent's full rationality. The bounded rationality approach to the analysis of consumption choice differs from the theories of rational expectations and rational inattention, which both assume an agent's rationality. One of the conceptual differences between the bounded rationality approach and the theory of rational inattention is that the latter does not restrict ex ante the *form* of information that the agent processes, while the bounded rationality approach does.

Importantly, behavioral theories do not claim that people are irrational and thus their behavior cannot be modeled. Instead, such theories argue that people are not rational in a particular direction and analyze consumption behavior following this direction.

There are multiple approaches to analyze the consumption choice under behavioral biases. I pick up several of them to sketch the field of study (for an extensive overview see Gabaix [2019]). For example, Bordalo et al (2013) suggest a salience framework in the absence of uncertainty. In the model, an agent chooses a good x_a from a set C , where x_a is the vector of attributes x_{ai} . The utility of consuming good a has the following form: $u^s(a) = \sum_{i=1}^n b_i m_{ai} x_{ai}$ (1), where b_i is a weight of good i in the utility and m_{ai} is a distortion as an increasing function of the *salience* of attribute i for good a with respect to a reference point \bar{x}_i . The authors provide an example with a choice between two bottles of wine in two locations (store and restaurant), and two attributes (price and quality). They show that in the presence of distortions m_{ai} , the choice depends on the choice context, and the price is the more salient attribute in the store, while at the restaurant quality is. Thus, an agent tends to be more attentive to price in a store and choose a cheaper wine. On the other hand, at a restaurant, an agent focuses more on quality and chooses a higher-end wine. In the absence of distortions m_{ai} , a choice would not depend on context and an agent could be called rational.

Another way to model consumption choice under behavioral friction is a “focusing” model proposed by Koszegi and Szeidl (2013). The authors assume that an agent is more attentive to the attributes along which her options vary the most.

Her utility takes the following form: $u^s(a) = \sum_{i=1}^n m_i x_{ai}$, where $m_i = A(\sigma_i)$, $\sigma_i = \max_a(x_{ai}) - \min_a(x_{ai})$, and $A(\sigma_i)$ is an increasing function.

Gabaix (2019) provides a good example with two payment options (a large one-off and a sequence of small payments) to describe how the “focusing” model works. The attributes in this context are an amount of payment in each period. The large one-off payment has a higher variation of attributes than the sequence of small payments, and thus under a certain condition, an agent tends to choose the option of sequent payments even though it is more expensive.

Consumption choice under behavioral frictions is a flourishing field. Almost any study of consumption choice under behavioral friction has its own deviation from rationality. The possible drawback of multiple modeling approaches is that the results may be mutually inconsistent and can hardly be aggregated into one. In this context, it is appropriate to recall Spiegler (2019) who criticizes behavioral theory in general for the atheoretical style. He decomposes the atheoretical style into three directions: anecdotal style, cost-benefit style, and functional-form style. The studies on the consumption choice under behavioral frictions usually use the functional-form style, which means, in Spiegler’s terminology, taking a functional modification of an agent’s problem that explicitly leads to a bias observed in data. The functional-form style of analysis generates a set of scattered models instead of one general model that could possibly encompass multiple behavioral biases. The theory of rational inattention might serve as this general model as it proposes a flexible framework without any *ex ante* restrictions on the process of information acquisition. For example, increasing attention to a more volatile variable is an *assumption* of the “focusing” model and an *implication* of the theory of rational inattention.

To summarize, the field of studies on consumption choice under bounded rationality is broad and encompasses an extensive list of rather scattered behavioral frictions. The theory of rational inattention proposes a flexible framework for studying consumption choice under the constrained process of information acquisition. The relevance of any approach depends on the purpose of study and research question. In this thesis, I use the theory of rational inattention instead of the bounded rationality approach for two reasons. The first is of a philosophical nature. I believe that people are rational and behave optimally in the presence of multiple constraints. The second

is conceptual. As a researcher, I do not want to restrict ex ante the process of information acquisition because permanent income is a complex object that an agent observes, while a researcher does not. In this thesis, I claim that information about permanent income is a crucial object to avoid a poverty trap, and thus any ex ante restrictions on information acquisition about permanent income would be arbitrary or even deteriorative.

1.3 Hand-to-mouth Behavior and Poverty Trap

In this thesis, I study the hand-to-mouth behavior of the poor that originates from inattentive savings behavior and leads to a poverty trap. Therefore, I consider a poverty trap and hand-to-mouth behavior of the poor jointly as different sides of the same phenomenon. By a poor hand-to-mouth consumer, I understand an agent from the low income decile who consumes a large fraction of her additional income and saves too little. By a poverty trap, I literally understand a persistent life in poverty. The logic of the connection between the two concepts is the following. Poor hand-to-mouth consumers, by definition, consume almost all their income and do not save. Thus, they fail to accumulate savings and face financial precarity. Therefore, they underinvest in future consumption, which may increase via investments in education or career promotion. In the absence of accumulated savings and investments in future consumption, agents fail to get out of poverty and get stuck in a poverty trap.

In this thesis, I argue that hand-to-mouth behavior might originate from inattentiveness to permanent income, but this channel is neither unique nor exclusive. The aim of the thesis is to propose a novel perspective on hand-to-mouth behavior and a poverty trap rather than the “correct” explanation of the phenomena.

1.3.1 Hand-to-mouth Behavior

A hand-to-mouth consumer is an agent who consumes almost all of her additional income and does not save. In other words, her marginal propensity to consume out of income is high (almost equal to 1). The behavior of hand-to-mouth consumers and their impact on the economy have been extensively studied in the macroeconomic context (Kaplan and Violante, 2014; Gomes, 2020; Aguiar et al., 2020). One should distinguish between wealthy and poor hand-to-mouth

consumers because they have different reasons for such behavior. The agents of the first type hold illiquid assets and do not hold any liquid assets. Thus, in the presence of an income shock, they will cover it via a consumption change because illiquid assets cannot be easily transmitted into consumption goods. Kaplan and Violante (2014) study the existence of wealthy hand-to-mouth consumers in the context of tax rebate. The authors claim that off-the-shelf consumption theory (the rational expectations life-cycle models with one asset [Carroll, 1992, 1997; Rios-Rull, 1995; Huggett, 1996]) predicts that marginal propensity to consume (MPC) out of transitory income should be negligible in the aggregate. The only agents who exhibit high MPC in these models are those who are constrained. Nevertheless, the share of constrained agents generated in the models is too small to match aggregate MPC in the real-world data. The authors overcome the complication by introducing differentiated wealth: liquid and illiquid. As illiquid assets can be accessed only with transaction costs, wealthy agents who hold a large amount of illiquid assets would prefer to consume a large fraction of transitory income and keep illiquid assets untouchable. Wealthy consumers who hold a large amount of illiquid assets are constrained from the perspective of Kaplan and Violante's (2014) model but would appear unconstrained in the one-asset model since they own substantial net worth. Kaplan and Violante (2014) manage to generate the fraction of hand-to-mouth consumers that matches the real-world data.

In this thesis, I focus on poor hand-to-mouth consumers. In Kaplan and Violante's (2014) terminology, poor hand-to-mouth consumers are those who do not hold either illiquid or liquid assets. Such agents do not have enough savings to insure against income shocks and smooth consumption over time, and thus their consumption just follows the dynamics of income. Kaplan and Violante (2014) claim that poor hand-to-mouth consumers constitute one-third of the total number of hand-to-mouth consumers and around 10% of the population.

The only paper that considers hand-to-mouth behavior resulting from inattentive behavior is that by Reis (2006). He relates this behavior to the high costs of updating savings or consumption decisions. I discuss the paper in detail in the following section.

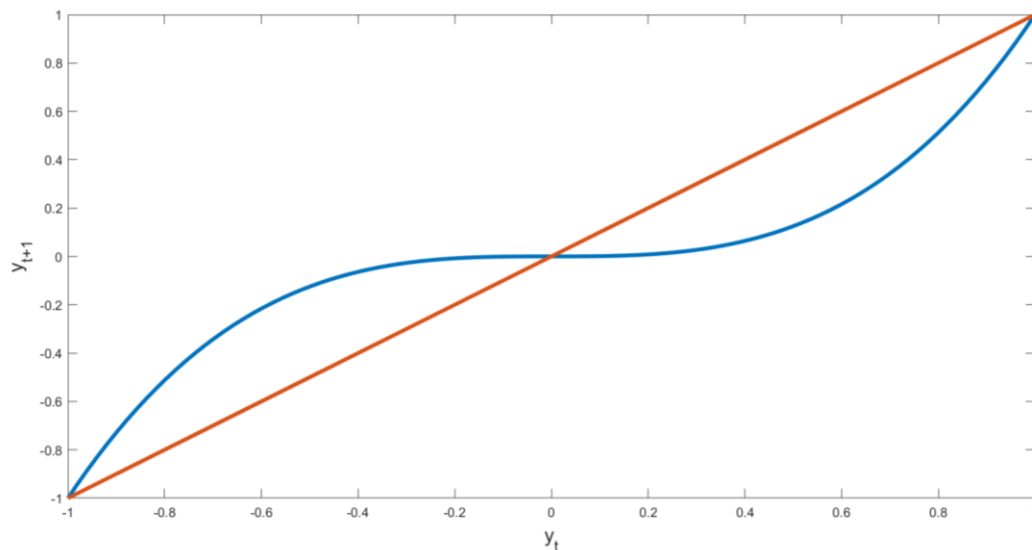
Poor hand-to-mouth consumers fail to accumulate savings and face financial precarity. In the absence of savings, poor agents cannot invest in future consumption (for example, pay for children's education), and thus are stuck in the low income decile. In this way, a poverty trap occurs.

1.3.2 Poverty Trap

The idea of a poverty trap has a long tradition. It emerged in the economic development literature (Rosenstain-Roden, 1943; Nelson, 1956; Banerjii and Newman, 1993) and proceeded with empirical investigations with cross-country data (Graham and Temple, 2006) and micro-studies with observational data (Lybbert et al., 2004; Kraay and McKenzie, 2014). The modern state-of-the-art approach is field experiments that focus on the reasons for a poverty trap (for an extensive overview see Banerjii [2020]).

The essence of a poverty trap is the existence of multiple equilibria or steady states. I depict this phenomenon in Figure 1, where there is a current income y_t on the X-axis and next-period income y_{t+1} on the Y-axis. The blue line represents the production function, which is S-shaped rather than concave. Under the concavity, there will be only one equilibrium. The red line is a 45-degree line. One can see that an agent whose current income is slightly below the zero point will cascade to point -1, while an agent whose current income is slightly above the zero point will increase her income up to point 1. Further, any small deviation from -1 does not change equilibrium as an agent will return to the point in several periods. In essence, this graph reflects the statement that the poor will remain poor forever. There are two stable equilibria in total (the zero-zero point is not stable) with poor and rich agents.

Figure 1: Multiple equilibria – poverty trap



There are two types of constraints that prevent poor people from accumulating savings. The poor may face external constraints, such as transaction costs (Dupas, 2012; Schaner, 2016),

regulatory barriers (Jentzsch, 2009), or the lack of career opportunities (Weinger, 2000). In addition, poor people may be subject to internal constraints, which are typically modeled as a particular behavioral bias that interferes the poor with accumulating savings and leads to a poverty trap. For instance, people may fall into the trap due to a lack of self-control (Bernheim et al., 2015), present-biased preferences (Bouchard St-Amant and Perrault, 2019), aspirations failure (Dalton et al., 2011), or focusing effect (Canidio, 2015). I describe in detail two papers from the list to sketch the modern approach toward the analysis of a poverty trap.

Canidio (2015) aims to explain the blindness of the poor toward investment opportunities: poor people systematically underinvest even though they have profitable investment opportunities available. To do this, he constructs a dynamic consumption-savings model in which the agent's choice is distorted by the "focusing" effect (Koszegi and Szeidl, 2013). The effect was extensively described in the section devoted to behavioral biases and inattention. In the context of Canidio's (2015) paper, poor people might overlook investment opportunities with a high *marginal* return because small initial investments generate a small *total* return. Meanwhile, rich people significantly gain from a high *marginal* return as they can invest much. Canidio (2015) claims that a poverty trap may occur when the *marginal* return on savings is high and disappears when it is low.

Dalton et al. (2011) focus on the psychology of poverty and introduce the concept of aspirations failure. They argue that poor people underestimate the evolution of their aspirations over a lifetime, as a consequence of their effort. As the poor face more downside risk than the rich, they experience a more severe aspiration failure. Significant downside risk affects the effort choice of the poor by lowering their expected benefit of investing. Low effort translates into a high probability of failure and makes the poor more susceptible to an aspiration failure. In this way, poor people get stuck in poverty indefinitely and the trap occurs.

In this thesis, I argue that poor people face a lower variance of their permanent income than their rich counterparts, which leads the poor to pay less attention to changes in permanent income and save only a fixed amount. The presence of aspiration failure might exacerbate the hand-to-mouth behavior because the distribution of permanent income of the poor would have lower variance *and* be skewed to the left. With skewed-to-the-left distribution, poor people would have even fewer incentives to learn about their permanent income and more rigid savings plan. Thus,

even though I do not introduce any behavioral bias into the analysis, an aspiration failure would only strengthen the mechanism behind the occurrence of a poverty trap, which I focus on.

1.4 Inattention Trap and Inattentive Savings

The two main papers I build the thesis on are by Macaulay (2021) and Reis (2006). Both of them use the concept of inattention to study the savings choice. In particular, Macaulay (2021) shows how, in an environment with costly information, persistent inequality may occur. This paper is methodologically close to the thesis as Macaulay (2021) also uses the theory of rational inattention, whereas, in this thesis I focus on the inattentiveness towards permanent income rather than investment opportunities, which enlightens the different aspect of a poverty trap. Reis (2006) does not use the theory of rational inattention but provides great insight into how hand-to-mouth behavior emerges as a consequence of high information costs. The proximity of the thesis to this paper is of a conceptual nature.

Reis (2006) constructs the dynamic model of consumption choice with the costs of acquiring and processing information. The model predicts that the agent will optimally choose to sporadically update information and re-compute the consumption plan, remaining inattentive to shocks between the periods of updating. This result implies that aggregate consumption reacts to shocks with a delay – a fact that is observed in the real-world data and cannot be rationalized by full-information rational expectations models. Thus, Reis (2006) provides a plausible explanation for the excess sensitivity and excess smoothness puzzles. Further, the author differentiates between savings and consumption plans and shows that for high enough information costs an agent chooses savings as a decision variable instead of consumption. In addition, high information costs result in the rare updating of the consumption plan. Therefore, people with information costs above a certain threshold live hand-to-mouth, choosing a rigid savings plan and consuming all the income shocks. It is exactly this mechanism that takes place in my model. Reis (2006) explicitly restricts the process of information acquisition and allows an agent to choose only the timing of updating the choice. It might be the case that the agent's choice is actually not optimal due to the exogenous restriction imposed by the author. To avoid doubts about optimality, in this thesis, I allow an agent to choose the whole distribution of savings, which may evolve over time. In my model, an agent chooses optimally the *form* of information acquisition process. Even though Reis (2006) imposes

the restrictive assumption on the information process, he demonstrates a clear mechanism behind hand-to-mouth behavior resulting from high information costs. I aim to elaborate on the mechanism in this thesis and show that hand-to-mouth behavior may emerge in the model of rational inattention.

Macaulay (2021) shows that a model with *ex ante* identical agents features persistent inequality if information about asset return is costly. The steady-state has a two-agent structure maintained by the complementarity between wealth and attention: a higher wealth level leads to higher attention to asset returns. This complementarity generates an attention trap: rich agents pay more attention to asset prices and thus gain more return on investments than poor agents. In this framework, fiscal expansions are less powerful than in a standard model with heterogeneous discount factors, as the MPC of poor agents is lower in the attention trap model because of the following mechanism. Fiscal expansions incentivize poor households to save more and, consequently, pay more attention to asset returns. Higher attention leads to higher asset returns and incentivizes one to save even more. Therefore, in the attention trap model, poor agents smooth policy shock over a longer time period than a standard model. My thesis is conceptually close to this paper as I also focus on an income trap arising from inattentiveness but bring a different aspect of uncertainty. Instead of asset return, the agent in this thesis is uncertain about her lifetime income level (permanent income) and attempts to acquire costly information about it. Permanent income is a crucial determinant of the consumption-savings decision and may be especially important for the poor. As Karlan (2014) argues, poor people may lack investment opportunities and use only personal mechanisms (savings) to insure against income shocks. Therefore, in the context of a poverty trap, inattention to permanent income might be a more appropriate determinant of hand-to-mouth behavior than inattention to asset returns. In this thesis, the trap emerges because poor people have lower incentives to learn about their *permanent income* as it is less uncertain and costs of learning are higher comparing to their rich counterparts. I aim to build on Macaulay's (2021) paper by developing a mechanism behind a poverty trap resulting from inattention to permanent income.

2 Two-period Rational Inattention Model of Savings Decision

According to the theory of rational inattention, an agent does not observe the true states of the world directly. Instead, she has to learn about them based on costly signals. Compared to standard approaches of modeling the information acquisition process (for an extensive review see Gabaix [2019] and Mackowiak et al. [2020]), the rational inattention approach allows an endogenous choice of information structure. Once an agent has chosen a signal structure, she takes an action. Due to convexity properties of the cost of information, an agent will never choose different signals that prescribe the same action, otherwise she would waste costly information. Thus, the set of signals X is of the same cardinality as the set of savings decisions S : $|X| = |S|$, and the problem can be described in terms of savings decisions only.

In my setup, permanent income y serves as a true state of the world, and thus an agent acquires costly information about her permanent income and chooses savings s . In accordance with the permanent income hypothesis (Friedman, 1957), I consider permanent income y as the present value of future income flows. An agent is uncertain about her permanent income y because she cannot perfectly predict her future, but she may learn about her permanent income and choose savings. In this framework, the savings decision is a joint distribution of savings and permanent income $p(s, y)$, given the marginal distribution of income $g(y)$. I call the conditional distribution of savings given income $p(s|y)$ a savings plan. One can think of the plan as a degree of freedom in the savings disposal. An agent may choose to freely change savings (high variance of conditional distribution $p(s|y)$) or stick to a fixed amount (low variance of conditional distribution $p(s|y)$) for a given income level.

In my model, an agent makes a decision in two steps. In the first step, she chooses between consumption or savings as a decision variable. In doing this, she compares expected lifetime utilities from choosing consumption or savings optimally. In the second step, an agent acquires costly information about her permanent income and optimally sets a decision variable.

2.1 First-step decision

Agent is uncertain about y and investigates whether consumption c or savings s as a decision variable delivers a higher expected utility ($V^c \vee V^s$).

If there are no other constraints on the optimization problem, an agent compares: $V^c = \max_c E [u(c) + \beta u(y - c)]$ and $V^s = \max_s E [u(y - s) + \beta u(s)]$. In the classical problem of allocation between consumption and savings, there is no difference between choosing consumption or savings as a decision variable because an agent is uncertain about her *future* income but knows her *current* income. In my model, an agent is uncertain about her current *permanent* income, and thus she can decide when to bear the "uncertainty losses". If she chooses the amount of consumption today, she will consume the rest tomorrow and she does not know this amount, because $(y - c)$ is uncertain. On the other hand, if she chooses the amount of savings, she will perfectly know the consumption today but will be uncertain about the consumption today. Without any restrictions on the optimization problem and $\beta < 1$, it is always better to postpone "uncertainty losses", and thus an agent will use consumption as a decision variable. I formally prove this statement in Appendix 1 for the case of the quadratic utility function.

Once I add a borrowing constraint, the problem itself and solution become different. Now an agent compares 1) $V^c = \max_c E [u(c) + \beta u(y - c)]$ s. t. $y - c \geq \bar{y}$ and 2) $V^s = \max_s E [u(y - s) + \beta u(s)]$ s. t. $s \geq \bar{y}$. Importantly, the constraint in the first problem ($y - c \geq \bar{y}$) is stochastic, because y is uncertain, while in the second problem the constraint ($s \geq \bar{y}$) is deterministic, because an agent just chooses s .

The solution to the first problem is: $c = c^*$, if $c^* < \min(y) - \bar{y}$ and $c = \min(y) - \bar{y}$, otherwise. An agent cannot violate the borrowing constraint and, thus, she will choose the lowest possible consumption level to surely satisfy the constraint. The solution to the second problem is: $s = s^*$, if $s^* > \bar{y}$ and $s = \bar{y}$, otherwise. One can see that under some conditions on the utility function u and the distribution of income y , $s^* > \bar{y}$ and $c^* < \min(y) - \bar{y}$, and thus an agent will choose savings as a decision variable. Thus, there are agents in the economy who will choose savings as a decision variable in the presence of borrowing constraints.

2.2 Second-step decision

To start with, I construct a two-period model (closely related to that proposed by Sims, [2006]) in which an agent is uncertain about her permanent income in the first period and consumes only savings in the second period. I discuss the extension of the model in the section on avenues for future research. In this model, current income coincides with a permanent one and an agent cannot save more than her current income, but it happens only because of the simplicity of the model.

The optimization problem for an agent who use savings s as a decision variable has the following form:

$$(8) \max_{p_1(s,y)} \int_{y_1} \int_{0 < s_1 < y_1} [u(y_1 - s_1) + \beta u(s_1)] p_1(s, y) dy ds$$

$$(9) p_1(s, y) \in D(s, y)$$

$$(10) \int p_1(s, y) = g_1(y)$$

$$(11) \kappa_1 = \int p_1(s, y) \log \left(\frac{p_1(s, y)}{(\int p_1(\hat{y}, s) d\hat{y}) g_1(y)} \right) ds dy$$

$$(12) g_1(y) \text{ is given,}$$

where $D(s, y) = \{p(s, y): \int p_1(s, y) ds dy = 1, p_1(s, y) \geq 0, \forall (s, y)\}$ and $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$.

An agent maximizes her two-period utility by choosing $p_1(s, y)$ subject to consistency constraints (9)-(10), information processing constraint (11), given the initial marginal distribution of permanent income $g_1(y)$. The constraint (11) defines a decrease in uncertainty about permanent income after choosing a savings plan in terms of reduction in Shannon Entropy (Shannon, 1948). Information costs are captured by a multiplier λ on the constraint (11). The problem can be equivalently formulated in terms of savings plan $p_1(s|y)$, as the marginal distribution of income $g_1(y)$ is given in the first period.

If an agent chooses consumption c as a decision variable, the optimization problem will have a slightly different form:

$$\max_{p_1(c,y)} \int_{y_1} \int_{0 < c_1 < y_1} [u(c_1) + \beta u(y_1 - c_1)] p_1(c, y) dy dc$$

$$(13) p_1(c, y) \in D(c, y)$$

$$(14) \int p_1(c, y) = g_1(y)$$

$$(15) \kappa_1 = \int p_1(c, y) \log \left(\frac{p_1(c, y)}{\left(\int p_1(\hat{y}, c) d\hat{y} \right) g_1(y)} \right) dc dy$$

$$(16) g_1(y) \text{ is given,}$$

where $D(c, y) = \{p(c, y): \int p_1(c, y) dc dy = 1, p_1(c, y) \geq 0, \forall(c, y)\}$.

An agent chooses the joint distribution of consumption and income (equivalent to the choice of the conditional distribution of consumption given an income level) and faces the same four constraints as for savings. In this case, an agent consumes a certain amount today and “postpones” uncertainty as tomorrow she will consume the rest.

2.2.1 Quadratic-Gaussian case

If the agent’s utility function has a quadratic form, income is normally distributed and there is no borrowing constraint, the optimization problem (8) has an analytical solution.

In this case, an agent maximizes the following expression:

$$(17) \max_{p(s|y)} \int_s \int_y p(s|y) g(y) (-y - s)^2 - \beta s^2 ds dy - \lambda I(s; y).$$

By assumption, income y is normally distributed with mean μ_y and variance σ_y^2 : $y \sim N(\mu_y, \sigma_y^2)$.

It is well-known that with quadratic preferences, Gaussian prior uncertainty, and an unbounded action set, Gaussian signals are optimal. As the action set has to be unbounded, I do not impose a borrowing constraint. In the LQG case, information costs have the following formula –

$$\frac{1}{2} \log \left(\frac{\sigma_y^2}{\sigma_{y|x}^2} \right).$$

An agent receives the signal x , which contains noise: $x = y + \epsilon$. After receiving the signal, optimal savings choice is $s = \frac{E[y|x]}{1+\beta}$. In Appendix 2, I show that optimization problem (17)

can be rewritten in terms of the conditional variance of income given the signal $\sigma_{y|x}^2$:

$$(18) L = \max_{\sigma_{y|x}^2} -\sigma_{y|x}^2 - \frac{\beta(\mu_y^2 + \sigma_y^2 - \sigma_{y|x}^2)}{1+\beta} - \frac{\lambda}{2} \log \left(\frac{\sigma_y^2}{\sigma_{y|x}^2} \right)$$

I take the first-order condition:

$$[\sigma_{y|x}^2]: -1 + \frac{\beta}{(1+\beta)} + \frac{\lambda}{2\sigma_{y|x}^2} = 0,$$

and obtain the solution:

$$(19) \sigma_{y|x}^2 = \frac{\lambda(1+\beta)}{2},$$

The solution simplifies to that provided by (Mackowiak et al., 2020) when $\beta = 0$ and the problem has only one period.

Under Bayesian learning and Gaussian signals, the expectation of income after observing the signal is linear in y :

$$(21) E(y|x) = \frac{\sigma_{y|x}^2}{\sigma_y^2} \bar{y} + \left(1 - \frac{\sigma_{y|x}^2}{\sigma_y^2}\right) (y + \epsilon), \text{ and } 1 - \frac{\sigma_{y|x}^2}{\sigma_y^2} \in [0,1].$$

I express the optimal savings choice $s = \frac{E[y|x]}{1+\beta}$ in terms of the basic parameters, income and noise:

$$(22) s = \frac{\lambda}{2\sigma_y^2} \bar{y} + \frac{(y+\epsilon)}{1+\beta} - \frac{\lambda}{2\sigma_y^2} (y + \epsilon).$$

The solution is cumbersome but provides two useful insights. First, when information costs λ increase, an agent puts larger weights on the prior belief about income \bar{y} and smaller weights on the incoming signal, and thus savings become more rigid. Second, when the variance of income distribution σ_y^2 decreases, savings also become more rigid. Therefore, in LQG case, poor agents who have higher information costs and lower income variance will choose a more rigid savings plan than their rich counterparts. In the two-period model, the conditional variance $\sigma_{y|x}^2$ positively³ depends on the discount factor β . However, the allocation of savings itself depends on the discount factor only through the level of savings but not through the precision of signals. I plan to rigorously study the effect of the discount factor on the rigidity of savings plan in future work.

³ $\frac{\partial \sigma_{y|x}^2}{\partial \beta} = \frac{\lambda}{2} > 0$

2.3 Numerical Results

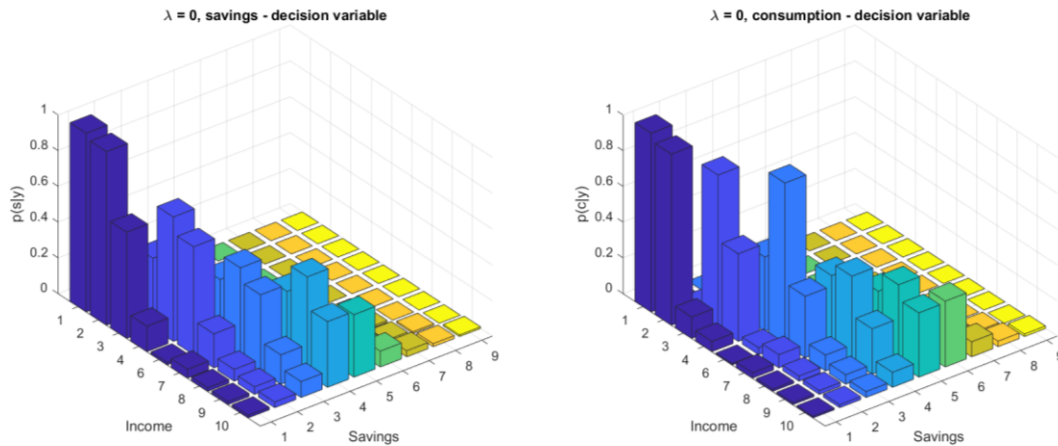
I numerically solve the model (8)–(12) and show that hand-to-mouth behavior might occur due to high information costs and low income variance. I explicitly impose borrowing constraint ($s < y$) and choose such income distribution and the parameter of risk aversion γ that savings as a decision variable provides higher expected utility than consumption.

I calibrate the parameters at the levels $\beta = 0.9$ and $\gamma = 0.5$. I use a discrete grid for possible values of income and savings (the range is 1–10 for income and 1–9 for savings). In future work, I plan to overcome the discrete approximation of the choice set and allow an agent to choose over a continuous grid. However, Matejka (2015) shows that rationally inattentive agents will choose to process a finite number of realizations of signals even though the support of distribution of signals is continuous, and thus the simplification in my solution is less restrictive than it seems.

One should analyze all figures in the following way. The vertical axis depicts the conditional probability of savings given income; two horizontal axes depict income and savings levels. Thus, each bar depicts the probability to choose a particular savings level for a given income level.

The optimization problems are almost identical for consumption and savings as a decision variable (Figure 2). The only matter is the timing of bearing the costs of uncertainty. If an agent chooses the conditional distribution of consumption, she knows with certainty how much she eats today and remains uncertain about consumption tomorrow. On the other hand, savings as a decision variable generates uncertainty about today's consumption. Parameter β drives the dependence between utilities in two periods. If $\beta = 1$, consumption and savings plans are identical and differ otherwise. Figure 1 depicts consumption and savings plans for $\beta = 0.9$. The expected utility under savings as a decision variable is higher than the expected utility under consumption.

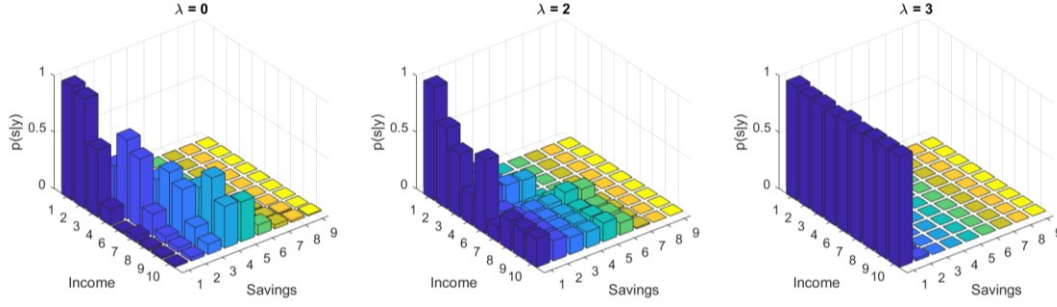
Figure 2 – consumption versus savings as a decision variable



Note: The figure depicts savings plan $p(s|y)$ and consumption plan $p(c|y)$, $\beta = 0.9$.

One can see that with an increase in information costs, savings become more rigid and concentrated around low levels of savings (mostly, around $s = 1$) (Figure 3). Thus, with high information costs, an agent almost always chooses $s = 1$ for any income and allows consumption to adjust. The intuition behind the result is as follows: The agent decides to save a constant amount for any income level because the unit of additional information about the income level is very costly, and thus she cannot distinguish between different income states and chooses only one level of savings. In the limiting case ($\lambda = \infty$), the agent will save the constant amount with probability one. In addition, the agent prefers to undersave than oversave because, otherwise, she may face zero consumption level. Thus, savings are concentrated around the lowest possible value ($s = 1$ in this case).

Figure 3 – comparison between high and low information costs



Note: The figure depicts savings plans $p(s|y)$, which differ with information costs λ . Higher information costs lead to a more rigid savings plan.

In the next step, I compare the average propensity to consume to check whether higher information costs encourage hand-to-mouth behavior. I use the following theoretical formula to calculate average propensity to consume: $APC = \int_w g(w) \int_s p(s|w) \frac{w-s}{w}$. Hand-to-mouth behavior is characterized by a high average propensity to consume and a low propensity to save. The calculations confirm the theoretical reasoning: average propensity to consume grows with information costs: $APC_{\lambda=0} = 0.52$, $APC_{\lambda=2} = 0.56$, $APC_{\lambda=3} = 0.76$. Importantly, the dependence may be non-linear, but it should be positive on average. Overall, a more rigid savings plan is characterized by a higher propensity to consume. This prediction can be tested on the real-world data.

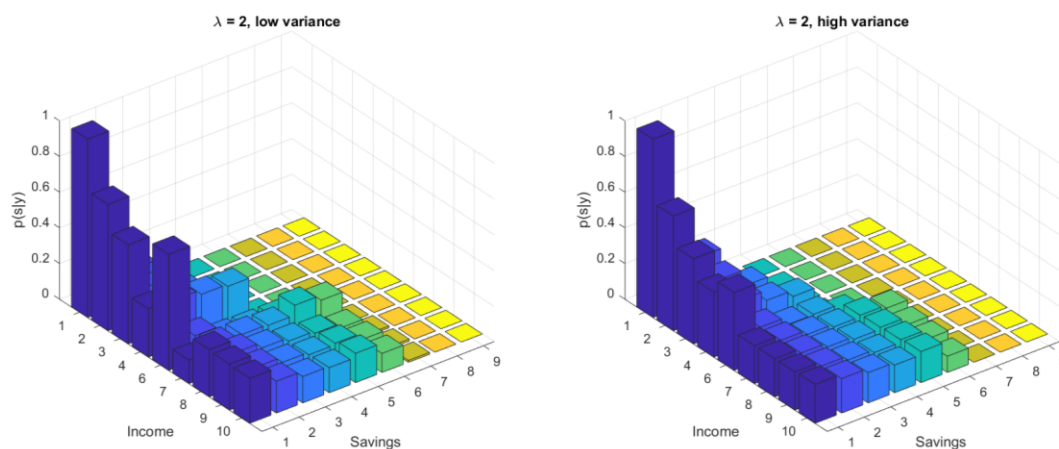
An additional testable prediction from the model is based on the dynamics of unconditional variances. By definition, the unconditional variance of savings is $\sigma^s = \int_y g(y) \int_s p(s|y) (s - E(s|y))^2 ds dy$, and unconditional variance of consumption is $\sigma^c = \int_y g(y) \int_s p(s|y) ((y - s) - E(y - s|y))^2 ds dy$. Unlike conditional variance, a researcher could potentially observe unconditional variance. Numerical results show that the higher information costs imply higher unconditional variance of consumption: $\sigma^c_{\lambda=0} = 3.08$, $\sigma^c_{\lambda=3} = 7.47$, and lower unconditional variance of savings: $\sigma^s_{\lambda=0} = 1.89$, $\sigma^s_{\lambda=3} = 0.02$. In this way when the information costs are high ($\lambda = 3$) the savings plan becomes almost constant.

Importantly, the usage of consumption or savings as a decision variable leads to opposite conclusions about the dependence between a *savings* plan and information costs. If an agent decides on the conditional distribution of consumption, the consumption plan becomes more rigid

with higher information costs and the savings plan becomes more flexible. That is, an agent becomes more uncertain about her income, chooses a fixed level of consumption for any income level, and saves the remainder.

Figure 4 depicts savings plans that differ in the variance of prior distribution $g_1(y)$ only. One can see that with the lower variance of prior distribution, the savings plan becomes more rigid and concentrated around the lowest possible level of savings ($s = 1$). The logic behind the result is that with lower variance an agent has lower incentives to learn about her income because extreme outcomes become less probable. Thus, she almost does not acquire information about income, cannot distinguish between different income states, and chooses a rigid savings plan (fixed level of savings for any income level).

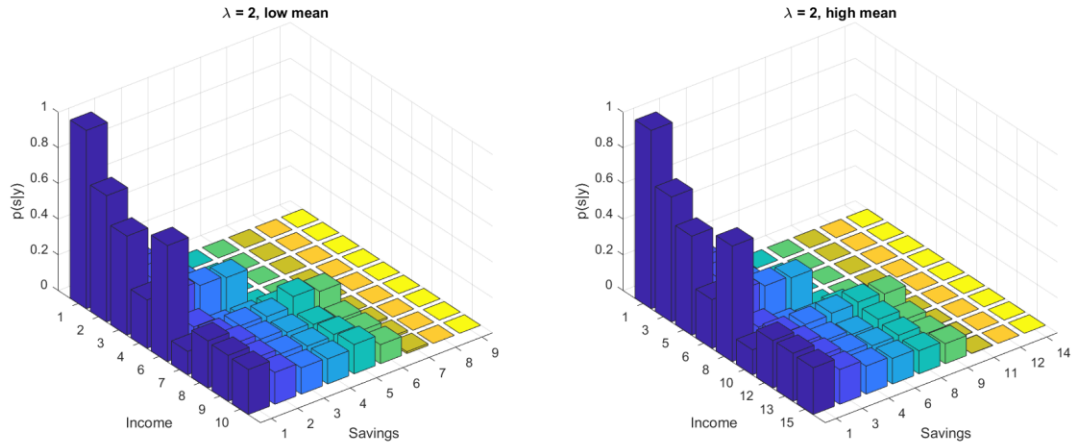
Figure 4 – comparison between high and low variances of income distribution



Note: The figure depicts savings plans $p(s|y)$, which differ with the variance of prior distribution $g_1(y)$. Lower variance leads to a more rigid savings plan.

Figure 5 depicts savings plans that differ in the mean of prior distribution $g_1(y)$ only. One can see that the change in the mean of income distribution does not substantially influence savings rigidity. Therefore, a direct money transfer to a poor agent will not significantly change her savings behavior as she will still have low incentives to learn about her permanent income (still has high information costs and low variance of permanent income). Thus, only a comprehensive policy intervention that influences information costs and income variance will affect the rigidity of savings plan and prevent hand-to-mouth behavior.

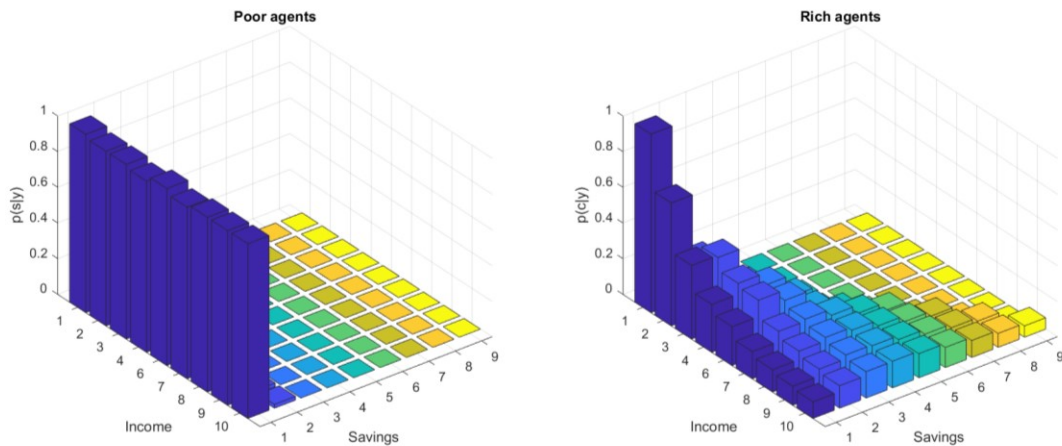
Figure 5 – comparison between high and low means of income distribution



Note: The figure depicts savings plans $p(s|y)$, which differ with the mean of prior distribution $g_1(y)$. The savings plans do not change significantly with changes in means.

By assumption, poor agents experience higher information costs λ and lower variance of prior distribution $g_1(y)$ than their rich counterparts. By definition, poor agents have a lower mean of the income distribution than rich ones. Therefore, I aggregate the three effects depicted in figures 1-3 into one. Figure 6 depicts savings plans of poor and rich agents. One can see that poor people have a more rigid savings plan than rich ones.

Figure 6 – comparison between poor and rich agents



Note: The figure depicts savings plans $p(s|y)$, which differ with mean and variance of prior distribution $g_1(y)$, and information costs λ . Rich agents: high mean and variance, low information costs. Poor agents: low mean and variance, high information costs.

Expectedly, poor agents have a higher average propensity to consume than their rich counterparts: $APC_{poor} = 0.75$ and $APC_{rich} = 0.47$. Exact numbers may vary with parameter values, but the relationship remains the same. The APC of the poor is not exactly unity because they have to eat something in the second period and by assumption, they can only eat their savings. If I add non-zero income in the second period, the APC of the poor may become even greater than unity, meaning that an agent will eat out their future income.

The analysis shows that the poor agents exhibit hand-to-mouth behavior. The poor have high information costs and low income variance, and thus they optimally choose to remain inattentive about their income, save small almost fixed amount for any income level and consume the remainder. Therefore, I claim that the poor undersave and do not accumulate enough savings to escape from a poverty trap.

3 Avenues for Future Research

3.1 Research Prospects in a Theoretical Direction

A poverty trap, by definition, is a persistent life in poverty. Thus, I need a long time horizon to validate the explanation of a poverty trap from the inattention perspective. I propose the following infinite-horizon model to study the emergence of a poverty trap:

$$\max_{p_t(s,y)_{t=0}^{\infty}} E_0[\sum_{t=0}^{\infty} \beta^t \int u(y_t - s_t) p_t(s, y) ds dy | I_0] \text{ s.t.}$$

$$(23) p_t(s, y) \in D(s, y),$$

$$(24) \int p_t(s, y) = g_t(y),$$

$$(25) \kappa_t = \int p_t(s, y) \log \left(\frac{p_t(s, y)}{(\int p_t(\hat{y}, s) d\hat{y}) g_t(y)} \right) ds dy,$$

$$(26) g_{t+1}(y' | s) = \int \tilde{T}_{t+1}(y'; y, s) p_t(y | s) dy,$$

(27) $g_0(y)$ is given,

where $D(s, y) = \{p(s, y): \int p(s, y) ds dy = 1, p(s, y) \geq 0, \forall (s, y)\}$ and $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$.

An agent maximizes her infinite-horizon utility by choosing $p_t(s, y)_{t=0}^{\infty}$ subject to consistency constraints (23)-(24), information processing constraint (25), transition function of beliefs (26), given the initial marginal distribution of permanent income. All the constraints are similar to those from the two-period model except the fourth one. The fourth constraint defines the transition function for the marginal distribution of permanent income. The function $T(\cdot)$ maps current values of consumption and permanent income into the future values of permanent income considering the effect of the income shocks. There is no fourth constraint in the two-period model because there an agent chooses a savings plan in the first period only. The problem can be equivalently formulated in terms of savings plan $p_t(s|y)$, as the marginal distribution of income $g_t(y)$ is given in the period t . I may also add borrowing constraints for the poor, but this will only exacerbate a poverty trap.

I assume that the marginal distribution of income $g_t(y)$ has an income-dependent variance $\sigma_{g_t}(y)$ to match the assumption on different variances of poor and rich agents. The variance is an increasing function of permanent income: $\frac{\partial \sigma_{g_t}(y)}{\partial y} > 0$. I assume that $\lambda_t(y)$ depends on income level to match the assumption on different information costs of poor and rich agents. The costs are a decreasing function of permanent income: $\frac{\partial \lambda_t(y)}{\partial y} > 0$

The model is closely related to that proposed by Tutino (2013) with several important distinctions. First, I use savings as a decision variable instead of consumption. If an agent faces the necessity to save a certain amount (for example, to pay for rent at the end of the month), it is optimal to choose a savings plan rather than a consumption plan, because otherwise, she faces a stochastic constraint on consumption and has to choose a minimum possible consumption level to surely satisfy the constraint. I have formally developed this idea for the two-period model and plan to extend the logic to the infinite-horizon model. Second, I assume that $\lambda_t(y)$ and $\sigma_{g_t}(y)$ depend on income, while Tutino (2013) assumes them to depend on time only. I will also follow the algorithm proposed by Tutino (2013) to solve the model. Since a poverty trap is the feature of equilibrium in the economy, I plan to construct a partial equilibrium model with rationally

inattentive agents who are different in the initial distributions of permanent income. To solve this problem, I plan to follow Boccanfuso and Ferey (2019), who have constructed a partial equilibrium model in the rational inattention framework to study an optimal tax policy.

In my framework, the only motive to save for agents is a precautionary motive. Nevertheless, the model highlights the non-trivial mechanism of insurance as a higher income implies lower information costs. An agent may want to accumulate savings to become richer as, then, she will have lower information costs, know her permanent income better, and accumulate savings more optimally to insure against income shocks.

3.2 Research Prospects in an Empirical Direction

3.2.1 Empirical Model

The general prediction from the model is that the persistent rigid savings plan of poor agents is caused by high information costs and low income variance. If I could test this statement and identify the causal dependence, it would create a space for a range of policy implications. Firstly, I discuss a possible empirical model for the two-period theoretical model that may provide evidence in favor of hand-to-mouth behavior. Secondly, I discuss prospects for testing the infinite-horizon theoretical model.

I plan to test three main predictions from the two-period theoretical model that draw the connection between information costs and hand-to-mouth behavior. First, the expected share of consumption in income positively depends on information costs. Second, unconditional variance of savings $\sigma^s = \int_y g(y) \int_s p(s|y) (s - E(s|y))^2 ds dy$ negatively depends on information costs. Third, unconditional variance of consumption $\sigma^c = \int_y g(y) \int_s p(s|y) ((y - s) - E(y - s|y))^2 ds dy$ positively depends on information costs. The second and third predictions mean that savings become more rigid while consumption becomes more flexible with growing information costs.

In order to test the predictions, I propose the following regressions:

$$(28) c_i = \alpha_0 + \alpha_1 y_i + \alpha_2 \lambda_i + e_i,$$

$$(29) \sigma_i^c = \beta_0^c + \beta_1^c y_i + \beta_2^c \lambda_i + u_i^c,$$

$$(30) \sigma_i^s = \beta_0^s + \beta_1^s y_i + \beta_2^s \lambda_i + u_i^s,$$

where c_i is current consumption of agent i , y_i is disposable income, λ_i is information costs of learning about disposable income, σ_i^c is unconditional variance of current consumption, σ_i^s is unconditional variance of savings.

Positive α_3 would mean that for the given levels of income, current consumption increases with information costs. This suggests a growing average propensity to consume in information costs and provides support for the first theoretical prediction. Negative β_3^s would mean that unconditional variance of savings increases with information costs, and thus savings become more rigid (the second prediction). Positive β_3^c would mean that unconditional variance of current consumption increases with information costs, and thus current consumption becomes more flexible (the third prediction).

To estimate the modes (28)-(30), I assume time-invariant information costs λ_i , which may be an overly restrictive assumption. I plan to relax this assumption in the context of testing the predictions from the infinite-horizon theoretical model.

In order to test the infinite-horizon model, I need to take into account that information costs and income variance vary with income and time. Before describing the data that I plan to use, I suggest imagining an ideal setting that features the patterns of the model. Firstly, I need to observe directly $p_t(s|y)$ for any $\lambda_t(y)$, $\sigma_{g_t}(y)$, and $g_t(y)$, and thus I would imagine observing literally a savings plan. Namely, I observe a commitment plan in which an agent decides on the range of savings for each level of permanent income. Secondly, I need to observe the lifetime flow of income in order to learn about the variance of permanent income $\sigma_{g_t}(y)$ and marginal income distribution $g_t(y)$. Thirdly, I need to observe the level of information costs $\lambda_t(y)$. Agents should be identical in all other dimensions.

3.2.2 Data

The crucial requirement for the real-world data is state-dependent information that contains savings decisions in the different income states (if I rule out the possibility to observe papers with commitment plans). Panel data preserve this feature as I can observe the same individual in

different time periods while her income is changing over time. If I assume that $\sigma_{g_t}(y)$ and $\lambda_t(y)$ change over time slower than the level of permanent income y_t and the panel data are of high frequency, I will obtain the variation in savings choices s_t for different y_t , given $\sigma_{g_t}(y)$ and $\lambda_t(y)$. In order to test causality, I need an exogenous variation in information costs and the distribution of permanent income. Thus, I will look for a policy intervention that generates an exogenous variation in financial training exposure (for example, see Brown et al. [2016]) and career opportunities (for example, see Abubakar and Nura [2020]). If it is also possible to observe the income and savings of individuals before and after the intervention, such programs may generate data that are analogous to the ideal setting. Further, necessary data can be generated in a laboratory experiment (for examples of testing RI theory in lab experiments see Dean [2019]), in which a participant will face income streams of different complexity and decide how much to save for the next period. While such experiments could generate an exogenous variation in information costs, they are costly to conduct and subject to the problem of external validity in the context of the consumption-savings problem.

To be more specific about the data I plan to use, I discuss the estimation procedure of regressions (28)-(30) on the example of RLMS⁴ panel data. The data contain information about consumption and savings levels in different income states, which is crucial for testing the theoretical predictions. The main challenge is to extract information about costs λ_i from the survey. Conceptually, this parameter is responsible for the costs of information about future income. The questions that help to reveal this parameter are “How has the financial situation of the household changed during the last 12 months?” and “Do you think in 12 months you and your family will live better or worse than today?”. These questions contain information about predictions of future income and an error in the predictions. The error is simply the discrepancy between the predicted financial situation and the actual one. I assume that the higher share of errors in the predictions, the more costly is the information about lifetime income (parameter λ_i is higher). The survey also

⁴ The Russia Longitudinal Monitoring Survey (RLMS) is a series of nationally representative surveys designed to monitor the effects of Russian reforms on the health and economic welfare of households and individuals in the Russian Federation. The project has been run jointly by the Carolina Population Center at the University of North Carolina at Chapel Hill, started by Barry M. Popkin and now headed by Klara Peter, and the Demoscope team in Russia, headed by Polina Kozyreva and Mikhail Kosolapov.

contains information about memory skills and the psychological state of individuals that can also serve as a proxy for information costs.

3.3 Contribution

My hypothesis is that hand-to-mouth behavior and a poverty trap occur because high information costs and low income variance lead to low incentives to investigate future income flows, and thus poor people are highly uncertain about their permanent income. Significant uncertainty leads to a rigid savings plan because agents fail to distinguish between different states of permanent income. Further, agents would prefer to choose the plan below an optimal level because they are afraid to oversave in the low state and eat nothing in the next period. A rigid savings plan with a low amount of savings is exactly hand-to-mouth behavior. The hand-to-mouth behavior may lead to a poverty trap because poor agents fail to accumulate enough savings to change the permanent income.

By solving the infinite-horizon model in future, I will derive the conditions on information costs and income variance under which a poverty trap occurs. In addition, I plan to test the model on real-world data in order to check whether the conditions are plausible.

This research primarily contributes to the broad literature on the poverty trap and hand-to-mouth behavior by providing a novel perspective on the phenomena resulting from inattentive savings behavior. The main contribution aims to be of a theoretical nature. To the best of my knowledge, there are no studies that have analyzed hand-to-mouth behavior and a poverty trap originating from inattentive savings behavior in the rational inattention framework. In addition, I examine the conditions under which an agent optimally chooses savings as a decision variable instead of consumption, while researchers typically just assume consumption to be a decision variable.

This research may also contribute to the literature on the effect of financial literacy and career opportunities (Brown et al., 2016; Kaiser and Menkhoff, 2017) on savings accumulation. There is ambiguous evidence on whether an increase in financial literacy causes higher savings, although a high correlation between these two entities is extensively documented (Kaiser and Menkhoff, 2017). In my framework, I show that undersaving is caused by both high information

costs and low income variance. By testing this statement, I may contribute to the discussion on the causal link between financial literacy and undersaving.

Summary

In this thesis, I study the hand-to-mouth behavior of the poor that originates from inattentive savings behavior and leads to a poverty trap. In the first section, I extensively discuss studies related to consumption-saving decisions and, specifically to hand-to-mouth behavior. I explain why classical rational expectation theory may fail to explain the dynamic of consumption over a period and I introduce the concept of inattention in the context of consumption-savings decisions. I compare the theory of rational inattention and bounded rationality models and show that the former is less restrictive than the latter and might be more appropriate for the analysis of hand-to-mouth behavior resulting from inattentiveness. Next, I discuss a range of studies on hand-to-mouth behavior and the poverty trap and extensively elaborate on two studies – Macaulay (2021) and Reis (2006) – that are the most relevant for the thesis. I build on the literature on hand-to-mouth behavior and the poverty trap by providing a novel perspective on the phenomena resulting from inattention to permanent income.

In the second section, I construct the two-period rational inattention model of savings choice. I assume that people acquire costly information about stochastic permanent income and choose a savings plan. Firstly, I solve the model analytically in the quadratic-gaussian case. The solution is close to that from the standard one-period problem, but in the two-period model, savings allocation also depends on the discount factor. Secondly, I numerically solve the model with CRRA utility function, non-gaussian distribution of income, and borrowing constraint. I also assume that poor agents have higher information costs and lower income variance than their rich counterparts. In this framework, a poor agent prefers to choose hand-to-mouth behavior; namely, a persistent rigid savings plan below an optimal level. Such an agent fails to accumulate enough savings to insure against income volatility and escape from a poverty trap. My numerical results show that in a two-period model an agent behaves hand-to-mouth (i.e., chooses a rigid savings plan) when information costs are high and income variance is low. Importantly, I allow agents to choose between savings or consumption as a decision variable in the first step before choosing a

plan. Under savings as a decision variable, higher uncertainty about permanent income leads to a more rigid savings plan, and thus an agent consumes all income shocks and behaves hand-to-mouth. On the other hand, with consumption as a decision variable, an agent chooses a more rigid consumption plan and saves all income shocks. I show that in the presence of the budget constraint, it might be optimal to choose savings as a decision variable instead of consumption. As poor people are exactly those who face a high risk of budget constraint to bind, they will probably choose savings as a decision variable instead of consumption, and then choose a rigid savings plan.

In future work, I plan to construct a partial equilibrium infinite-horizon model with rationally inattentive agents, provide conditions under which a poverty trap occurs, and test the model on consumption-savings data.

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Appendix 1

An agent chooses between consumption and savings as a decision variable by comparing maximized two-period utilities under different decision variables. V^S denotes a maximized utility by choosing optimal *savings* and V^C denotes a maximized utility by choosing optimal *consumption*.

$$V^S = \max_z E^S_y \left(y - z - \frac{(y-z)^2}{2} + \beta \left(z - \frac{z^2}{2} \right) \right)$$

$z_s^* = \frac{E(w)+\beta-1}{1+\beta}$ is the value of savings that maximizes utility.

$$V^C = \max_z E^C_y \left(z - \frac{z^2}{2} + \beta \left(y - z - \frac{(y-z)^2}{2} \right) \right)$$

$z_c^* = \frac{\beta E(w) - \beta + 1}{1 + \beta}$ is the value of consumption that maximizes utility.

Firstly, I plug the optimal consumption choice z_c^* in V^C and simplify the expression:

$$\begin{aligned} E \left(\left(\frac{1-\beta+(y)\beta}{1+\beta} - \frac{(1-\beta+E(y)\beta)^2}{2(1+\beta)^2} \right) + \beta \left(y - \frac{1-\beta+E(y)\beta}{1+\beta} \right) - \frac{\left(y - \frac{1-\beta+E(y)\beta}{1+\beta} \right)^2}{2} \right) = \\ \frac{(1-\beta)(1-\beta+\beta E(y))}{1+\beta} + \beta \left(E(y) - \frac{E(y^2)}{2} \right) - \frac{(1-\beta+\beta E(y))^2}{2(1+\beta)^2} + \frac{\beta E(y)(1-\beta+\beta E(y))}{1+\beta} - \frac{\beta(1-\beta+\beta E(y))^2}{2(1+\beta)^2} = \\ \frac{(1-\beta+\beta E(y))^2}{1+\beta} - \frac{(1-\beta+\beta E(y))^2}{2(1+\beta)} + \beta \left(E(y) - \frac{E(y^2)}{2} \right) = \beta \left(E(y) - \frac{E(y^2)}{2} \right) + \frac{(1-\beta+\beta E(y))^2}{2(1+\beta)}. \end{aligned}$$

Secondly, I do the same procedure for z_s^* and V^S :

$$\begin{aligned} E \left(y - z - \frac{(y-z)^2}{2} + \beta \left(z - \frac{z^2}{2} \right) \right) = E(y) - \frac{E(y)+\beta-1}{1+\beta} - \frac{\beta E(y^2)}{2} + \frac{E(y)(E(y)+\beta-1)}{1+\beta} - \frac{(E(y)+\beta-1)^2}{2(1+\beta)^2} + \\ \frac{\beta(E(y)+\beta-1)}{1+\beta} - \frac{\beta(E(y)+\beta-1)}{2(1+\beta)^2} = E(y) - \frac{E(y^2)}{2} + \frac{(E(y)+\beta-1)^2}{2(1+\beta)}. \end{aligned}$$

Then, I compare values V^C and V^S . Savings as a decision variable are better if:

$$\begin{aligned} E(y) - \frac{E(y^2)}{2} + \frac{(E(y)+\beta-1)^2}{2(1+\beta)} > \beta \left[E(y) - \frac{E(y^2)}{2} \right] + \frac{(1-\beta+\beta E(y))^2}{2(1+\beta)}. \\ (\beta - 1) \left(E(y) - \frac{E(y^2)}{2} \right) < \frac{(E(y)+\beta-1-1+\beta-\beta E(y))(1+\beta)E(y)}{2(1+\beta)} \text{ and } \beta < 1. \end{aligned}$$

$E(y^2) < E(y)^2$ – never holds if y is a random variable. Therefore, in this problem, an agent will always choose consumption as a decision variable.

Appendix 2

In the quadratic-gaussian case and the absence of information costs, an agent maximizes the following expression:

$$\max_{p(s|y)} \int_s \int_y p(s|y)g(y)(-(y-s)^2 - \beta s^2)ds dy$$

An agent receives the signal x , which contains noise: $x = y + \epsilon$. After receiving the signal, optimal savings choice is $s = \frac{E[y|x]}{1+\beta}$. I plug the optimal choice in the optimization problem:

$$\begin{aligned} \int_x \int_y p(x|y)g(y) \left(- \left(y - \frac{E[y|x]}{1+\beta} \right)^2 - \beta \left(\frac{E[y|x]}{1+\beta} \right)^2 \right) dx dy &= \int_x p(x) \int_y p(y|x) \left(-y^2 + \right. \\ &\left. \frac{2yE(y|x) - E(y|x)^2}{1+\beta} \right) dy dx = \int_x p(x) \left(-E(y^2|x) + \frac{E(y|x)^2}{1+\beta} \right) dx = \int_x p(x) \left(-E(y^2|x) + \right. \\ &\left. E(y|x)^2 - E(y|x)^2 + \frac{E(y|x)^2}{1+\beta} \right) dx = \int_x p(x) \left(-\sigma^2_{y|x} - \frac{\beta(E(y|x)^2)}{1+\beta} \right) dx = -\sigma^2_{y|x} - \frac{\beta(\mu_y^2 + \sigma_y^2 - \sigma_{y|x}^2)}{1+\beta}. \end{aligned}$$

For the first equality, I use the fact that $p(x|y)g(y) = p(x)p(y|x)$. For the last equality, I use the expression for the conditional mean and variance under Gaussian distribution: $E(y|x) = \mu_y + \rho \frac{\sigma_y}{\sigma_x}(x - \mu_x)$ and $\sigma^2_{y|x} = \sigma_y^2(1 - \rho^2)$.

I square the conditional mean $E(y|x)^2 = \mu_y^2 + 2\mu_y\rho \frac{\sigma_y}{\sigma_x}(x - \mu_x) + \rho^2 \frac{\sigma_y^2}{\sigma_x^2}(x - \mu_x)^2$ and plug it in the last but one equality: $\int_x p(x)E(y|x)^2 dx = \mu_y^2 + \rho^2 \frac{\sigma_y^2}{\sigma_x^2} \sigma_x^2 = \mu_y^2 + \rho^2 \sigma_y^2 = \mu_y^2 + \sigma_y^2 - \sigma_{y|x}^2$.